A structural model for electricity spot and forward for coupled markets

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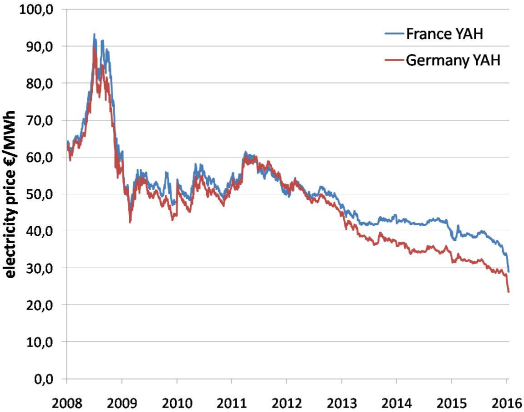
Plan

1. Motivations
   - Context
   - Interconnections
2. An example of existing model for electricity spot/forward for coupled markets
3. Proposed model for electricity spot/forward for coupled markets

Different electricity markets but large similarities

Forward prices movements are close



Source EPEX SPOT

German and French spot price are exactly equal 51% of time in 2014 and 27% in 2015.

Different production fundamentals

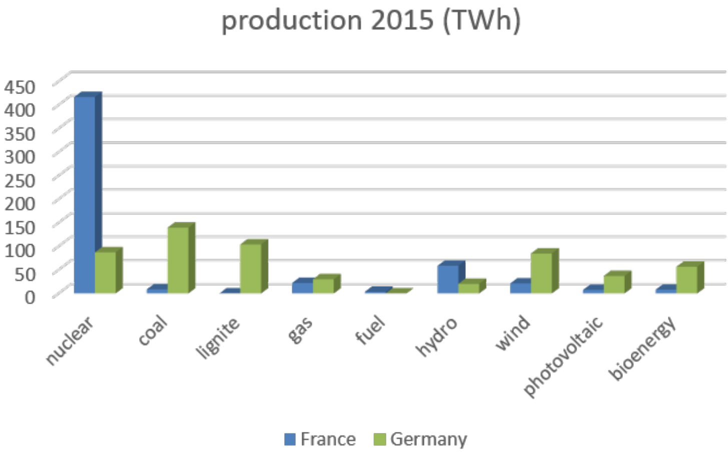
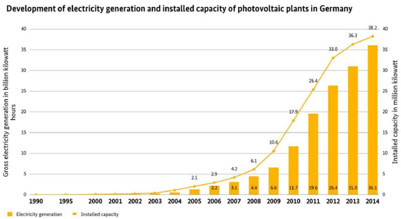


FIGURE – source RTE and Fraunhofer

Strong evolutions of electricity market fundamentals

On example : increase of renewables share in electricity production

- Wind and Solar represent 21,7% of German production in 2015 (production multiplied by 3.4 since 2004)



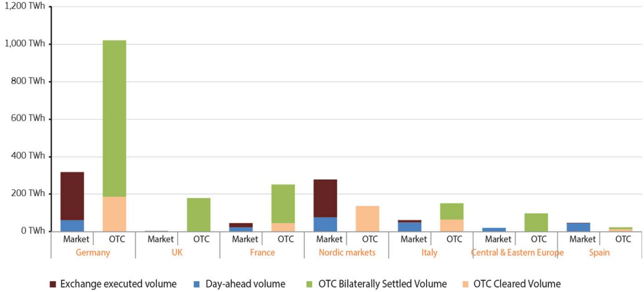
Source : Development of Renewable Energy
Sources in Germany 2014, Federal Ministry for Economic Affairs and Energy

- As a consequence, positive spikes on electricity spot market have decreased in frequency and negative spikes are quite common on spot or intraday markets.

New market conditions can not be modeled using only past price realizations.

Proxy hedging using neighbour countries

German market is the more liquid markets in west Europe

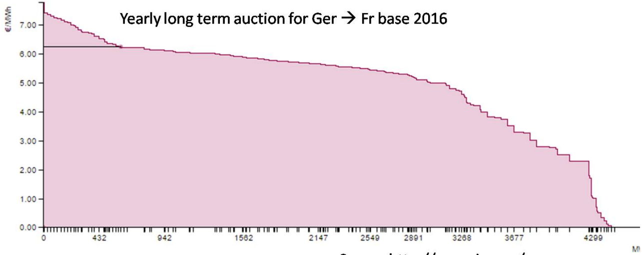


Source https://ec.europa.eu,
Quarterly Report On European Electricity Markets Q3 2015

To use other markets for hedging purpose is attractive !

Interconnection auctions

Explicit auctions for interconnection capacities are organised for year and month delivery : from country A to country B, the payoff of capacity contract : $\sum_{t=0}^{T}\left(S_t^B - S_t^A\right)^+$ (with S for spot price)



Source http://www.jao.eu/

Implicit allocation on day-ahead market with **market coupling** mechanism for most european interconnections

Coupling in Europe for electricity day-ahead market

Europe electricity markets are more and more coupled.

- November 2010 : **coupling of the CWE region** - Central West Europe - Germany/Austria, France, Belgium, The Netherlands, Luxemburg
- February 2014 : **extended coupling to NWE region** - North WE - CWE + Great-Britain, Denmark, Finland, Sweden and Norway.
- February 11st 2014 : every prices of the NWE region, except UK, equal 29.45 euro/MWh
- May 2014 : **Spain and Portugal join** the day-ahead price coupling
- Feb 2015 : **Italy and Slovenia join** the day-ahead price coupling
- 21 May 2015 : flow-based coupling for zone CWE

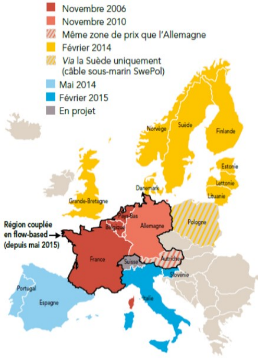
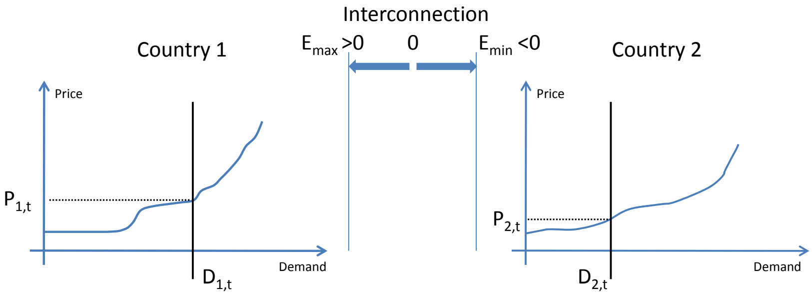


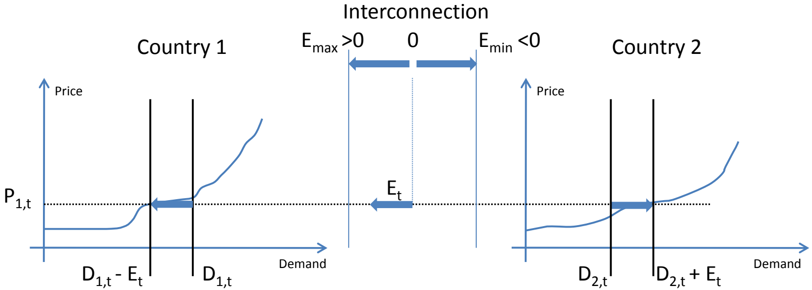
FIGURE – source RTE

How works the coupling ?



$$P_t^1(D_t^1) > P_t^2(D_t^2)$$

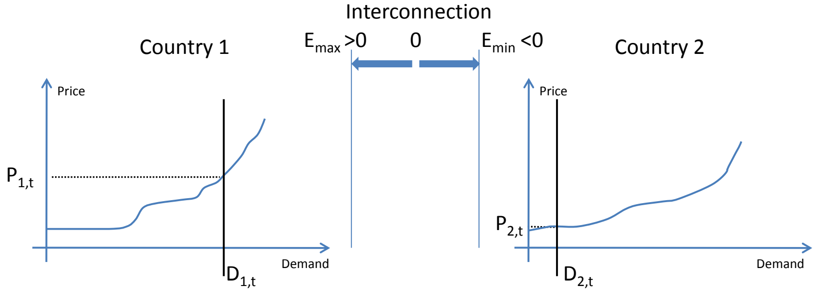
How works the coupling ?



The two markets are coupled

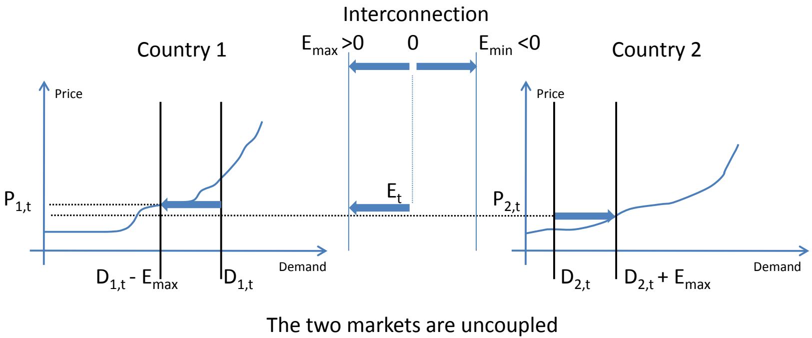
$$P_t^1(D_t^1 - E_t) = P_t^2(D_t^2 + E_t)$$

How works the coupling ?



$$P_t^1(D_t^1) > P_t^2(D_t^2)$$

How works the coupling ?



$$P_t^1(D_t^1 - E_{\max}) > P_t^2(D_t^2 + E_{\max})$$

Objectives summary

For proxy hedging purposes and explicit interconnection auctions, price models for different markets are required with the following characteristics :

1. to represent the strong existing links betwen electricity prices in this different countries
2. to be adapted to changing market conditions
3. to model spot and forward markets at the same time
4. to produce fast simulations

Structural models appear to be a good candidate to fulfill these different objectives (for forward and spot structural model see for example [Carmona et al. (2012)] or [Aïd et al. (2012)]).

[Kiesel (working paper)] and [Füss (working paper)] have proposed model for forward and spot with interconnections but for one technology only.

Plan

1. Motivations
2. An example of existing model for electricity spot/forward for coupled markets
   - Kiesel and Kustermann's approach
3. Proposed model for electricity spot/forward for coupled markets

Model proposed by Kiesel and Kustermann - spot model

For 2 interconnected markets, the respective spot prices are defined by :

$$P_T^1(S_T, D_T^1 - E_T) = S_T e^{a_1 + b_1 (D_T^1 - E_T)} + c \quad (1)$$

$$P_T^2(S_T, D_T^2 + E_T) = S_T e^{a_2 + b_2 (D_T^2 + E_T)} + c \quad (2)$$

with

- the flow at the interconnection $E_T \in [E_{\min}, E_{\max}]$, with $E_{\min} \leq 0$, $E_{\max} \geq 0$, is determined such as to minimise the difference between the two prices
- the log of the commodity S and the demands D^i follow Ornstein Uhlenbech (O.U.) processess,

The following vector is then Gaussian : $\begin{pmatrix} D_T^1 \\ D_T^2 \\ \ln S_T \end{pmatrix}_{|\mathcal{F}_s} \sim \mathcal{N}(\mu, \Sigma)$

Model limit : only one commodity is represented

Model proposed by Kiesel and Kustermann - forward model

In this model, the forward price can be expressed by :

$$F_s^1(T) = \mathbb{E}_s[P_T^1] = \mathbb{E}_s[P_T^1(\mathbb{1}_{\mathrm{A}_1} + \mathbb{1}_{\mathrm{A}_2} + \mathbb{1}_{\mathrm{A}_3})]$$

with

- market 2 exports at maximum of the interconnection to market 1 :
  $\mathrm{A}_1 = \{P_T^1(S_T, D_T^1 - E_{\max}) > P_T^2(S_T, D_T^2 + E_{\max})\}$
- market 1 exports at maximum of the interconnection to market 2 :
  $\mathrm{A}_2 = \{P_T^1(S_T, D_T^1 - E_{\min}) < P_T^2(S_T, D_T^2 + E_{\min})\}$
- the markets are coupled :
  $\mathrm{A}_3 = \mathrm{A}_1^c \cap \mathrm{A}_2^c$

A useful result : truncated Laplace transform

If $X \sim \mathcal{N}(\mu, \Sigma)$ is a Gaussian vector of size n, $\lambda, a \in \mathbb{R}^n$ and $b \in \mathbb{R}$, then

$$\mathbb{E}[e^{\lambda^T X}\mathbb{1}_{a^T X \leq b}] = e^{\frac{\lambda^T \Sigma \lambda}{2} + \lambda^T \mu}\mathbb{P}(a^T \tilde{X} \leq b) \qquad (3)$$

with $\tilde{X} \sim \mathcal{N}(\mu + \Sigma\lambda, \Sigma)$.

Plan

1. Motivations
2. An example of existing model for electricity spot/forward for coupled markets
3. Proposed model for electricity spot/forward for coupled markets
    - Spot Price
    - Forward Price

Model notations and assumptions 1/2

- Two countries are linked by an interconnection whose flow E_t is bounded in $\left[\underline{E}, \overline{E}\right]$, $E > 0$ means country A exports to country B.
- In each country, demand D_t^*, with $* \in \{A, B\}$ is described by an O.U. process with a deterministic component to represent the seasonnality
- Each country has n_*, $* \in \{A, B\}$ possible marginal commodities. The *log* of their marginal prices $S_t^{*,k}$ also follow an O.U. process
- The total number of commodities is $n \leq n_A + n_B$
- The capacity $C_t^{*,k}$, with $k \in [1, n_*]$, of every technology is constant (no failures)
- $\bar{C}_t^* = \sum_{i=1}^{n_*} C_t^{*,i}$ the maximum capacity production in country $*$

We have

$$\begin{pmatrix} \log S_t^A \\ \vdots \\ \log S_t^N \\ D_t^A \\ D_t^B \end{pmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \\ \mu_D^A \\ \mu_D^B \end{bmatrix}, \Sigma \right)$$

Model notations and assumptions 2/2

Let's take this form for the offer curve

$$P^*(\boldsymbol{S}_t, \boldsymbol{C}_t, d)) = \sum_{k=1}^{n_*} f^*(S_t^{*,k}, \bar{C}_t^*, d)\mathbb{1}_{d \in I_t^{*,k} = \left[\sum_{i=0}^{k-1} C_t^{*,i}\ ;\ \sum_{i=0}^{k} C_t^{*,i}\right]} \qquad (4)$$

with

$$f^*(s, c, d) = se^{\alpha^* + \beta^*(c-d)} \qquad (5)$$

and

$\boldsymbol{C}_t^* = \left(C_t^{*,k}\right)_{k=1,\dots,n_*}$ the set of avalaible capacities at time t in the country $*$,

$\boldsymbol{S}_t^* = \left(S_t^{*,k}\right)_{k=1,\dots,n_*}$ the set of production costs at time t in the country $*$,

Conditions to ensure increasing offer curve fuction of demand level : α and β are chosen to be independent of the marginal technology and $\beta < 0$.

Illustration of offer curve

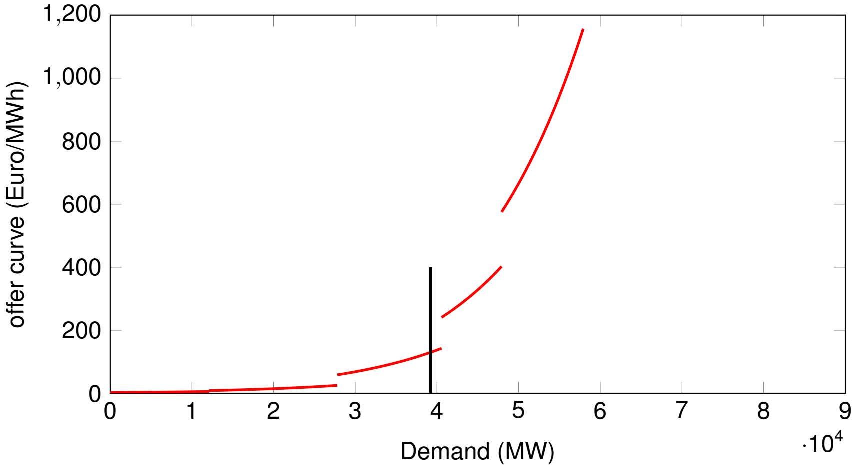


FIGURE – offer curve against demand level

Partition of state space

$E_t \in [\underline{E}, \overline{E}]$ determined such as to minimize the absolute difference between the two spot prices.

As suggested by [Kiesel (working paper)], we consider the following partition :

- $\mathcal{A}_1$ market A exports at maximum of the interconnection to market B : $\mathcal{A}_1 := \{w \in \Omega : P^A(\boldsymbol{S}_t^A, \boldsymbol{C}_t^A, D_t^A + \overline{E}) < P^B(\boldsymbol{S}_t^B, \boldsymbol{C}_t^B, D_t^B - \overline{E})\}$
- $\mathcal{A}_2$ market B exports at maximum of the interconnection to market A : $\mathcal{A}_2 := \{w \in \Omega : P^A(\boldsymbol{S}_t^A, \boldsymbol{C}_t^A, D_t^A + \underline{E}) > P^B(\boldsymbol{S}_t^B, \boldsymbol{C}_t^B, D_t^B - \underline{E})\}$
- $\mathcal{A}_3$the markets are coupled (prices are equal) : $\mathcal{A}_3 := \Omega \backslash (\mathcal{A}_1 \cup \mathcal{A}_2)$

Finally, we use the following partition of the state space :

$$\Omega = \bigcup_{1 \le i \le 3,\ 1 \le k \le, n_A,\ 1 \le l \le n_B} \mathcal{A}_{i,k,l}$$

with $\mathcal{A}_{i,k,l} = \mathcal{A}_i \cap \mathcal{M}_{k,l}$ and $\mathcal{M}_{k,l} := \{\omega \in \Omega :\ D_t^A + E_t \in I_t^{A,k}\ ;\ D_t^B - E_t \in I_t^{B,l}\}$

Spot price

We then define the spot price for each country as

$$\bar{P}^A(\boldsymbol{V}_t) = \sum_{k=0}^{n_A}\sum_{l=0}^{n_B} f^A(S_t^{A,k}, \bar{C}_t^A, D_t^A + \bar{E})\mathbb{1}_{\mathcal{A}_{1,k,l}} + f^A(S_t^{A,k}, \bar{C}_t^A, D_t^A + \underline{E})\mathbb{1}_{\mathcal{A}_{2,k,l}} + f^{A,B}(\boldsymbol{V}_t)\mathbb{1}_{\mathcal{A}_{3,k,l}} \tag{6}$$

$$\bar{P}^B(\boldsymbol{V}_t) = \sum_{k=0}^{n_A}\sum_{l=0}^{n_B} f^B(S_t^{B,l}, \bar{C}_t^B, D_t^B - \bar{E})\mathbb{1}_{\mathcal{A}_{1,k,l}} + f^B(S_t^{B,l}, \bar{C}_t^B, D_t^B - \underline{E})\mathbb{1}_{\mathcal{A}_{2,k,l}} + f^{A,B}(\boldsymbol{V}_t)\mathbb{1}_{\mathcal{A}_{3,k,l}} \tag{7}$$

with $\boldsymbol{V}_t = \left(\boldsymbol{S}_t^A, \boldsymbol{C}_t^A, D_t^A, \boldsymbol{S}_t^B, \boldsymbol{C}_t^B, D_t^B\right)$

Case $\mathbb{1}_{\mathcal{A}_{1,k,l}}$ and $\mathbb{1}_{\mathcal{A}_{2,k,l}}$

These cases are quite trivial :

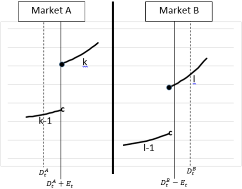
$$\boldsymbol{V}_t \in \mathcal{A}_{1,k,l} \quad \Leftrightarrow \quad \begin{cases} D_t^A + \bar{E} \in I_t^{A,k} \\ D_t^B - \bar{E} \in I_t^{B,l} \\ f^A(S_t^{A,k}, \bar{C}_t^A, D_t^A + \bar{E}) \leq f^B(S_t^{B,k}, \bar{C}_t^B, D_t^B - \bar{E}) \end{cases} \tag{8}$$

and

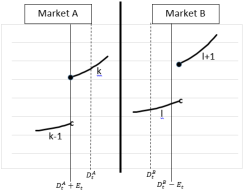
$$\boldsymbol{V}_t \in \mathcal{A}_{2,k,l} \quad \Leftrightarrow \quad \begin{cases} D_t^A + \underline{E} \in I_t^{A,k} \\ D_t^B - \underline{E} \in I_t^{B,l} \\ f^A(S_t^{A,k}, \bar{C}_t^A, D_t^A + \underline{E}) \geq f^B(S_t^{B,k}, \bar{C}_t^B, D_t^B - \underline{E}) \end{cases} \tag{9}$$

Case $\mathbb{1}_{\mathcal{A}_{3,k,l}}$

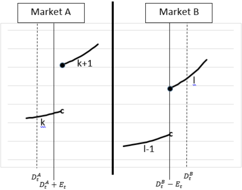
We decompose $\mathcal{A}_{3,k,l}$ into 3 incompatible events $\mathcal{A}^A_{3,k,l}$, $\mathcal{A}^B_{3,k,l}$ and $\mathcal{A}^C_{3,k,l} = \mathcal{A}_{3,k,l} \backslash (\mathcal{A}^A_{3,k,l} \cup \mathcal{A}^B_{3,k,l})$



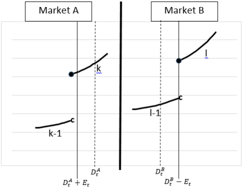
$\mathcal{A}^A_{3,k,l}$ and $E_t > 0$



$\mathcal{A}^A_{3,k,l}$ and $E_t < 0$



$\mathcal{A}^B_{3,k,l}$ and $E_t > 0$



$\mathcal{A}^B_{3,k,l}$ and $E_t < 0$

Fuel switching

This model enables to have fuel switching, i.e. change in the merit order. Let's note $\pi_t = \{\pi_t^1, ..., \pi_t^n\}$, the permutation of the index set $\{1, ..., n\}$ of indices such that $S_t^{\pi_t^1} \leq ... \leq S_t^{\pi_t^n}$ and S^π this event.

We then define the spot price for country A as

$$\bar{P}^A(\boldsymbol{V}_t) = \sum_{\pi \in \Pi} \sum_{k=0}^{n_A} \sum_{l=0}^{n_B} f^A(S_t^{A,k}, \bar{C}_t^A, D_t^A + \bar{E}) \mathbb{1}_{\mathcal{A}_{1,k,l}} \mathbb{1}_{S^\pi} + f^A(S_t^{A,k}, \bar{C}_t^A, D_t^A + \underline{E}) \mathbb{1}_{\mathcal{A}_{2,k,l}} \mathbb{1}_{S^\pi} + f^{A,B}(\boldsymbol{V}_t) \mathbb{1}_{\mathcal{A}_{3,k,l}} \mathbb{1}_{S^\pi} \tag{10}$$

Spot price for country B can be written similarly.

Forward Price

In this model, **forward** and **interconnection prices** can then be calculated using Gaussian cumulative distribution.

Same ideas as [Kiesel (working paper)] (i.e. truncated Laplace transform)

$$
\begin{aligned}
\mathbb{E}_t\left[\bar{P}^A_T(\boldsymbol{V}_T)\right] = \sum_{\pi\in\Pi}\sum_{k=0}^{n_A}\sum_{l=0}^{n_B}\mathbb{E}_t&\left[f^A(S^{A,k}_T,\bar{C}^A_T,D^A_T+\bar{E})\mathbb{1}_{\mathcal{A}_{1,k,l}}\mathbb{1}_{S^\pi}\right]+\\
\mathbb{E}_t&\left[f^A(S^{A,k}_T,\bar{C}^A_T,D^A_T+\underline{E})\mathbb{1}_{\mathcal{A}_{2,k,l}}\mathbb{1}_{S^\pi}\right]+\\
\mathbb{E}_t&\left[f^{A,B}(\boldsymbol{V}_T)\mathbb{1}_{\mathcal{A}_{3,k,l}}\mathbb{1}_{S^\pi}\right]
\end{aligned}
$$

The **interconnection capacity prices** are from country B to A

$$
\begin{aligned}
\sum_{\pi\in\Pi}\sum_{k=0}^{n_A}\sum_{l=0}^{n_B}\mathbb{E}_t&\left[f^A(S^{A,k}_T,\bar{C}^A_t,D^A_T+\bar{E})\mathbb{1}_{\mathcal{A}_{2,k,l}}\mathbb{1}_{S^\pi}\right]-\\
\mathbb{E}_t&\left[f^B(S^{B,k}_T,\bar{C}^A_B,D^A_B-\bar{E})\mathbb{1}_{\mathcal{A}_{2,k,l}}\mathbb{1}_{S^\pi}\right]
\end{aligned}
$$

Illustration of forward prices simulation

Model has been calibrated on French and German 2014 data

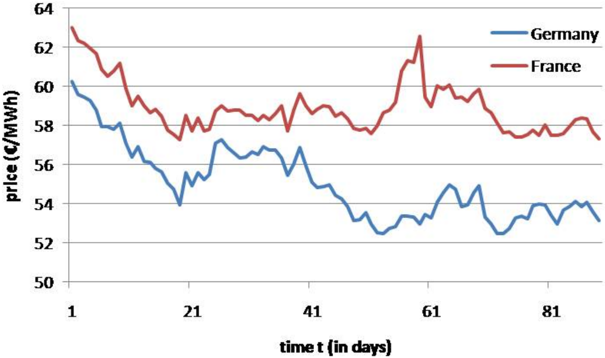


FIGURE – one simulated path 1QAH

Impact of interconnection maximum capacity

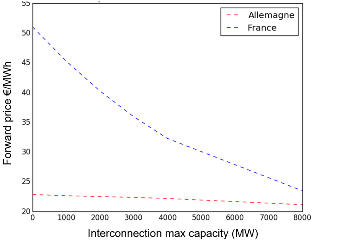


FIGURE – Forward prices for different $\overline{E}$, $T = 100$ days

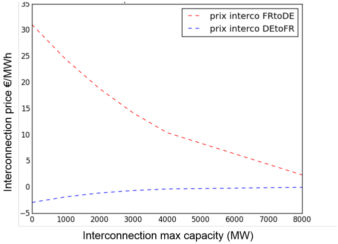


FIGURE – Interconnection prices for different $\overline{E}$, $T = 100$ days

Conclusions and Perspectives

- We proposed a structural model for spot and forward electricity prices taking into account interconnections
- Forward prices are semi-explicit expressions as they only involve Gaussian cumulative distribution
- This model has been calibrated on French/German interconnection
- Next step : improve calibration (more recent data) and use the model for proxy-hedging

References I

R. Aïd, L. Campi and N. Langrené
A structural risk-neutral model for pricing and hedging power derivatives
Mathematical Finance, 2012.

R. Carmona and M. Coulon
A survey of commodity markets and structural models for electricity prices
Financial Engineering for Energy Asset Management and Hedging in Commodity Markets, 2012.

R. Kiesel and M. Kustermann
Structural Models for Coupled Electricity Markets
http ://papers.ssrn.com/sol3/papers.cfm ?abstract$_i d =$ 2501352, *workingpaper*2015.

R. Füss, S. Mahringer, M. Prokopczuk
Electricity Spot and Derivatives Pricing when Markets are Interconnected
https ://ideas.repec.org/p/usg/sfwpfi/201323.html, working paper.

Propositions 1/2

Proposition

Set $g(\boldsymbol{V}_t) = \sum_{i=0}^{k-1} C_t^{A,i} - D_t^A$.
$\boldsymbol{V}_t \in \mathcal{C}_{A,+}$ *is equivalent to the following inequalities :*

1. $0 \leq g(\boldsymbol{V}_t) < \bar{E}$
2. $D_t^B - g(\boldsymbol{V}_t) \in I_t^{B,l}$
3. $f^A\left(S_t^{A,k-1}, \bar{C}_t^A, D_t^A + g(\boldsymbol{V}_t)\right) \leq f^B\left(S_t^{B,l}, \bar{C}_t^B, D_t^B - g(\boldsymbol{V}_t)\right)$
4. $f^A\left(S_t^{A,k}, \bar{C}_t^A, D_t^A + g(\boldsymbol{V}_t)\right) \geq f^B\left(S_t^{B,l}, \bar{C}_t^B, D_t^B - g(\boldsymbol{V}_t)\right)$

Proposition

Set $g(\boldsymbol{V}_t) = D_t^A - \sum_{i=0}^{k-1} C_t^{A,i}$.
$\boldsymbol{V}_t \in \mathcal{C}_{A,-}$ *is equivalent to the following inequalities :*

1. $\underline{E} < g(\boldsymbol{V}_t) \leq 0$
2. $D_t^B - g(\boldsymbol{V}_t) \in I_t^{B,l}$
3. $f^B\left(S_t^{B,l}, \bar{C}_t^B, D_t^B - g(\boldsymbol{V}_t)\right) < f^A\left(S_t^{A,k}, \bar{C}_t^A, D_t^A + g(\boldsymbol{V}_t)\right)$

Propositions 2/2

Proposition

Set $g(\boldsymbol{V}_t) = \sum_{i=0}^{k-1} C_t^{B,i} - D_t^B$.
$\boldsymbol{V}_t \in \mathcal{C}_{B,+}$ *is equivalent to the following inequalities :*

1. $0 \leq g(\boldsymbol{V}_t) < \bar{E}$
2. $D_t^A + g(\boldsymbol{V}_t) \in I_t^{A,k}$
3. $f^B\left(S_t^{B,l-1}, \bar{C}_t^B, D_t^B - g(\boldsymbol{V}_t)\right) < f^A\left(S_t^{A,k}, \bar{C}_t^A, D_t^A + g(\boldsymbol{V}_t)\right)$
4. $f^B\left(S_t^{B,l}, \bar{C}_t^B, D_t^B - g(\boldsymbol{V}_t)\right) \geq f^A\left(S_t^{A,k}, \bar{C}_t^A, D_t^A + g(\boldsymbol{V}_t)\right)$

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Set $g(\boldsymbol{V}_t) = D_t^B - \sum_{i=0}^{k-1} C_t^{B,i}$.
$\boldsymbol{V}_t \in \mathcal{C}_{B,-}$ *is equivalent to the following inequalities :*

1. $\underline{E} < g(\boldsymbol{V}_t) \leq 0$
2. $D_t^A + g(\boldsymbol{V}_t) \in I_t^{A,k}$
3. $f^A\left(S_t^{A,k}, \bar{C}_t^A, D_t^A + g(\boldsymbol{V}_t)\right) < f^B\left(S_t^{B,l}, \bar{C}_t^B, D_t^B - g(\boldsymbol{V}_t)\right)$

2015 capacities for France and Germany

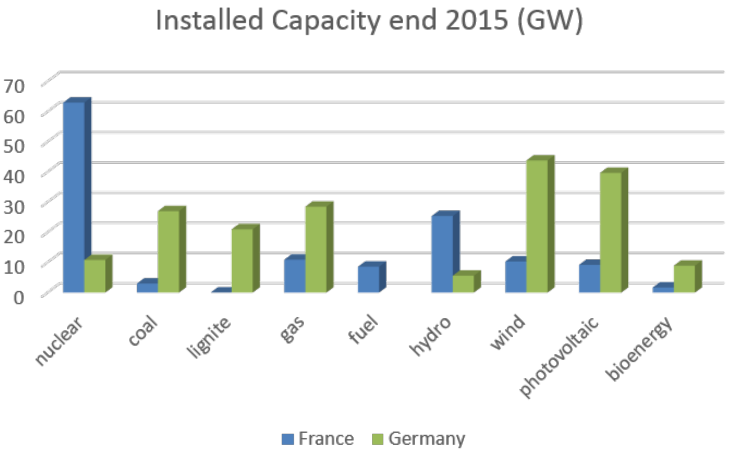

FIGURE – YAH prices

Link between interconnection flows and renewable production

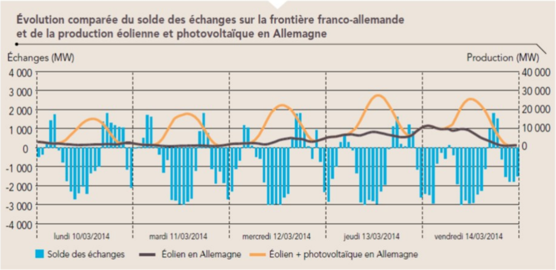

FIGURE – Example of flows between France and Germany, source RTE

2013 price convergence in CWE region

France and German prices converged around 60 % of time in 2013

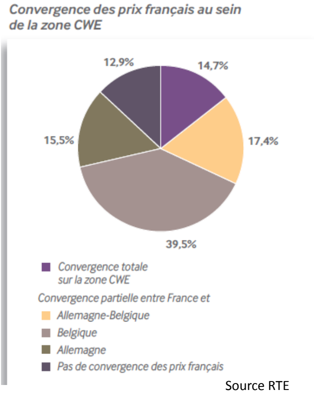

FIGURE – 2013 price convergence