# Mechanism Design for Wholesale Electricity Markets

with piece-wise linear costs and quadratic externalities

Benjamin Heymann[1]
Alejandro Jofré [2]

[1]INRIA and Ecole polytechnique, France

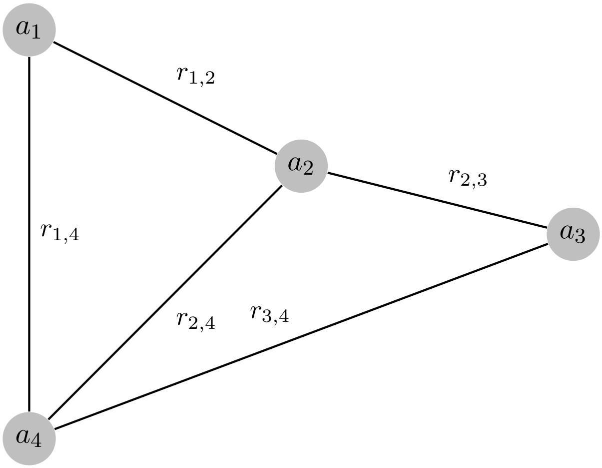
[2]CMM and Universidad de Chile

SESO, May 30th 2016

# My three objectives in this talk

- Present the market model.
- Convince you that in this model producers exercise market power.
- Show you how we can deal with this market power using mechanism design. The final formulation we get is surprisingly simple.

# The market



# The actors of the market

(a) The ISO minimizes supply cost s.t. supply meet demand

(b) a producer wants to make some profit

(c) another producer competes with (b)

Figure : Some actors of the market

# The auction rules

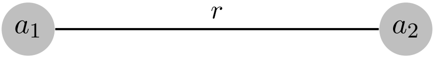
**The auction**

- Each producer $i$ bids a marginal price $b_i$
- The ISO decides the production allocation $q_i(b)$ so that supply $\geq$ demand and total cost is minimized
- Producers get paid $x$ according to their bid and the production they had to supply individually (pay as bid market): $x_i(b) = b_i q_i(b)$
- Electricity can travel through the lines, but quadratic loss

♣

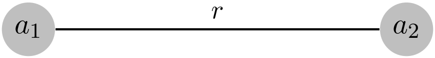
# The ISO problem

# The binodal example




- Demand $= d$ at each node.
- Assume the bids are $b_1$ and $b_2$.
- The operator has to decide the production levels $q_1$ and $q_2$.
- The operator can send $h_{12}$ of electricity from 1 to 2 (lose $rh_{12}^2$).
- The operator can send $h_{21}$ of electricity from 2 to 1(lose $rh_{21}^2$).
- At optimality, $h_{12}h_{21} = 0$.
- At optimality electricity will flow from the cheapest node to the most expensive.

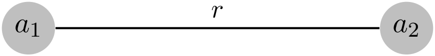
# The binodal example




**The ISO solves**

$$
\begin{aligned}
&\underset{q,h}{\text{minimize}}\ b_1 q_1 + b_2 q_2\\
&\text{subject to:}\\
&q_1 - h_{12} + h_{21} \geq \frac{r}{2}[h_{12}^2 + h_{21}^2] + d\\
&q_2 - h_{21} + h_{12} \geq \frac{r}{2}[h_{12}^2 + h_{21}^2] + d\\
&q_i, h_i \geq 0 \text{ for } i = 1, 2
\end{aligned}
$$

# The binodal example




**The ISO solves**

$$\underset{q,h}{\text{minimize}} \quad b_1 \, q_1 + b_2 \, q_2$$

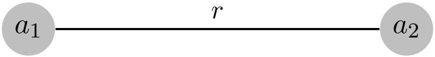
subject to:

$$q_1 - h_{12} + h_{21} \geq \frac{r}{2}[h_{12}^2 + h_{21}^2] + d$$

$$q_2 - h_{21} + h_{12} \geq \frac{r}{2}[h_{12}^2 + h_{21}^2] + d$$

$$q_i, h_i \geq 0 \text{ for } i = 1, 2$$

# The binodal example




**The ISO solves**

$$\underset{q,h}{\text{minimize}}\ b_1 q_1 + b_2 q_2$$

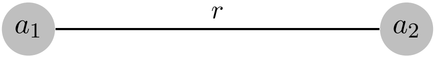
subject to:

$$q_1 - h_{12} + h_{21} \geq \frac{r}{2}[{h_{12}}^2 + {h_{21}}^2] + d$$

$$q_2 - h_{21} + h_{12} \geq \frac{r}{2}[{h_{12}}^2 + {h_{21}}^2] + d$$

$$q_i, h_i \geq 0 \text{ for } i = 1, 2$$

# The binodal example




**The ISO solves**

$$\underset{q,h}{\text{minimize}}\ b_1 q_1 + b_2 q_2$$

subject to:

$$q_1 - h_{12} + h_{21} \geq \frac{r}{2}[h_{12}^2 + h_{21}^2] + d$$

$$q_2 - h_{21} + h_{12} \geq \frac{r}{2}[h_{12}^2 + h_{21}^2] + d$$

$$q_i, h_i \geq 0 \text{ for } i = 1, 2$$

# More generally, the ISO solves

**$ISO(b)$**

$$
\begin{aligned}
\underset{(q,h)}{\text{minimize}} \quad & \sum_{i\in\{Agents\}} \sum_{j\in\{Slopes\}} q_i^j b_i^j \\
\text{subject to} \quad & \sum_{j\in\{Slopes\}} q_i^j + \sum_{i'\in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i \\
& h_{i,i'} \geq 0 \\
& q_i^j \geq 0 \\
& q_i^j \leq \bar{q}.
\end{aligned}
$$

# More generally, the ISO solves

$ISO(b)$

$$\begin{aligned}
\underset{(q,h)}{\text{minimize}} \quad & \sum_{i \in \{Agents\}} \sum_{j \in \{Slopes\}} q_i^j b_i^j \\
\text{subject to} \quad & \sum_{j \in \{Slopes\}} q_i^j + \sum_{i' \in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i \\
& h_{i,i'} \geq 0 \\
& q_i^j \geq 0 \\
& q_i^j \leq \bar{q}.
\end{aligned}$$

In this talk I won't discuss the general ISO(b) resolution, but we will get some insights from the binodal linear case (and I will be happy to answer any related questions at the end of my presentation).

# Solution of ISO(b) for the linear binodal setting

If $q_1$ and $q_2$ are positive, then $q_1 = F(b_1, b_2)$ and $q_2 = F(b_2, b_1)$, where

$$F(\lambda_1, \lambda_2) = d + \frac{1}{2r}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)^2 - \frac{1}{r}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right).$$

# Solution of ISO(b) for the linear binodal setting

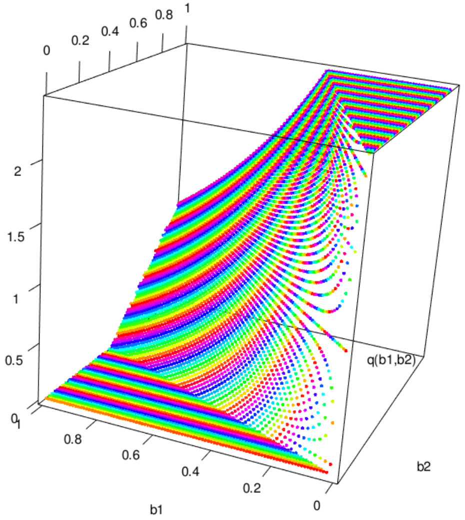
$$F(\lambda_1, \lambda_2) = d + \frac{1}{2r}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)^2 - \frac{1}{r}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right) \quad \tilde{q} = 2\left[\frac{1 - \sqrt{1 - 2dr}}{r}\right]$$

Then

$$q_i(b_i, b_{-i}) = \begin{cases} F(b_i, b_{-i}) & \text{if } F(b_i, b_{-i}) \geq 0 \text{ and } F(b_{-i}, b_i) \geq 0 \\ \tilde{q} & \text{if } F(b_{-i}, b_i) < 0 \text{ and } F(b_i, b_{-i}) \geq 0 \\ 0 & \text{if } F(b_i, b_{-i}) < 0 \text{ and } F(b_{-i}, b_i) \geq 0 \end{cases}$$

And the payments are $x_i(b) = b_i q_i(b_i, b_{-i})$ (pay-as-bid).

# Plot of the solution with respect to $b_1$ and $b_2$



What if I cut the line? $(r = +\infty)$

# What if I cut the line? ($r = +\infty$)

$\rightarrow$Then I get two monopolies that I need to regulate

# Market Power

- Assume each producer has a linear production cost of slope $c$
- Assume that this is common knowledge
- Profit for a producer 1: $(b_1 - c)F(b_1, b_2)$
- Profit for a producer 2: $(b_2 - c)F(b_2, b_1)$
- We get the Nash equilibrium strategies:

**Market power**

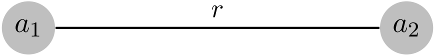
$$b^* = \frac{c}{1 - 2dr} > c$$

What if $r = 0$ ?

What if $r = 0$ ? $\rightarrow$ bids$= c$, earns 0.

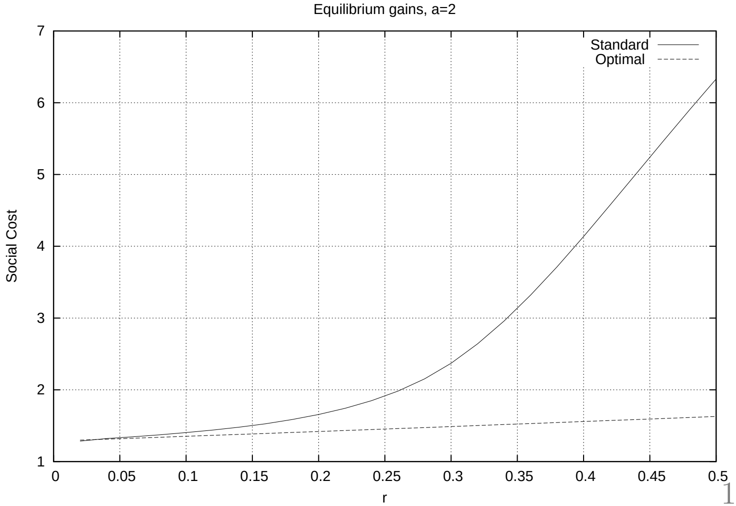
# Mechanism Design

# Just to recap, why are we doing this?

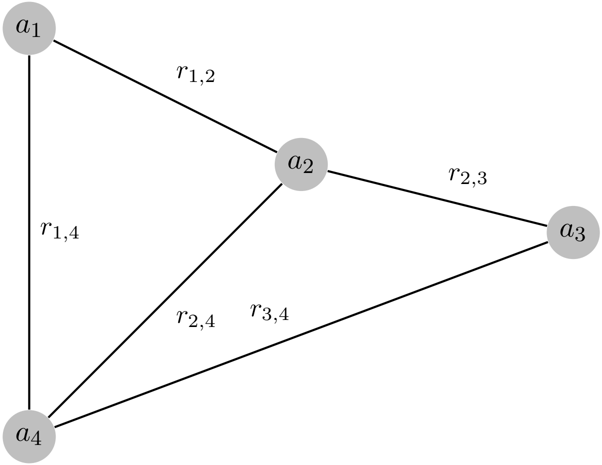


# To minimize tax money overspending !!!

Social cost: Optimal Mechanism (doted line) vs Standard Setting



# The market



# The New Mechanism

- Producers bid
- (before) They receive production allocation and payments (from the ISO allocation problem) ($x_i(b) = b_i q_i(b)$)
- (in the next slides) They receive production allocation and payments (BUT a new rule $(q_i(b), x_i(b))$ )

What if the ISO knew the $c_i$ ?

## The Setting

- At each node $i$:
  - There is a fixed demand for electricity $d_i$
  - There is an electricity producer whose production cost is piecewise-linear of slopes $(c_i^1, \ldots, c_i^N)$ such that for a production level between $j\bar{q}$ and $(j+1)\bar{q}$, the marginal cost is $c_i^j$
  - $c_i$ is unknown to the ISO and the other producers, but they have a prior (probability distribution) $f_i(c_i)$ on it.
  - This probability distribution corresponds to some common knowledge on $i$.
  - From a technical perspective, $f_i$ needs to satisfy some properties to make the forthcoming construction possible.
- The nodes are connected by edges
- When a quantity $h_{i,j}$ of electricity is sent from node $i$ to node $j$ we lose, $r_{i,j}h_{i,j}^2$ in the process
- Objective for the operator (ISO): To produce enough electricity to meet demand while minimizing the total cost

♣

**Theorem (Revelation Principle)**

*To any Bayesian Nash equilibrium of a game of incomplete information, there exists a payoff-equivalent direct revelation mechanism that has an equilibrium where the players truthfully report their types.*

**Theorem (Revelation Principle)**

*"We can limit our search of new $(x, q)$ to those such that the mechanism is truthful"*

**Theorem (Revelation Principle)**

*"We can limit our search of new $(x, q)$ to those such that the mechanism is truthful"*

- Idea: change the ISO behaviors
- Tool: the revelation principle
- So: we perform an optimization over the truthful direct mechanisms

# Construction of the Mechanism

# The optimization problem

## Problem

$$\underset{(q,x,h)}{minimize} \sum_{i \in \{Producers\}} Average\ Payment\ for\ producer\ i$$

$$subject\ to$$

$$Supply\ is\ greater\ than\ demand.$$

$$It\text{'}s\ optimal\ for\ all\ producers\ to\ tell\ the\ truth.$$

$$Every\ producer\ wants\ to\ participate\ in\ the\ market.$$

# The Optimization Problem

**Problem**

$$\underset{(q,x,h)}{minimize} \sum_{i \in \{Producers\}} \mathbb{E} x_i(c)$$

*subject to*

*supply greater than demand*

*It's optimal for all producers to tell the truth*

*Every producer wants to participate in the market.*

# The optimization problem

## Problem

$$\underset{(q,x,h)}{minimize} \sum_{i \in \{Producers\}} \mathbb{E}x_i(c)$$

*subject to*

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h_{i,i'}^2(c) + h_{i',i}^2(c)}{2} r_{i,i'} \geq d_i$$

*It's optimal for all producers to tell the truth*

*The producer want to participate in the market.*

# The optimization problem

## Problem

$$\underset{(q,x,h)}{minimize} \sum_{i \in \{Producers\}} \mathbb{E} x_i(c)$$

$$subject\ to\ (c, c')$$

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h^2_{i,i'}(c) + h^2_{i',i}(c)}{2} r_{i,i'} \geq d_i$$

$$U_i(c_i, c_i) \geq U_i(c_i, c'_i)$$

*The producer want to participate in the market.*

# The optimization problem

## Problem

$$
\underset{(q,x,h)}{minimize} \sum_{i \in \{Producers\}} \mathbb{E} x_i(c)
$$

$$
subject\ to\ for\ all\ (c, c')
$$

$$
q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h^2_{i,i'}(c) + h^2_{i',i}(c)}{2} r_{i,i'} \geq d_i
$$

$$
U_i(c_i, c_i) \geq U_i(c_i, c'_i)
$$

$$
U_i(c_i, c_i) \geq 0.
$$

# The optimization problem

## Problem

$$\underset{(q,x,h)}{minimize} \sum_{i \in \{Producers\}} \mathbb{E} x_i(c)$$

$$subject\ to\ for\ all\ (c, c')$$

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h^2_{i,i'}(c) + h^2_{i',i}(c)}{2} r_{i,i'} \geq d_i$$

$$U_i(c_i, c_i) \geq U_i(c_i, c'_i) \quad HARD$$

$$U_i(c_i, c_i) \geq 0. \quad HARD$$

# The Reformulation

$$\underset{(q,h)}{\text{minimize}} \quad \mathbb{E} \sum_{i \in \{Producers\}} \sum_{j \in \{Slopes\}} q_i^j(c)\hat{c}_i^j$$

subject to for all $c$

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h^2_{i,i'}(c) + h^2_{i',i}(c)}{2} r_{i,i'} \geq d_i$$

And set $x_i(c) = \sum_{j \in \{Slopes\}} q_i^j(c)c_i^j + \int_{c_i^j}^{\bar{c}_i^j} q_i^j(t; c_{-i})dt$.

Where $\hat{c}_i^j = (c_i^j + K_i^j(c_i^j))$ can be computed from the data.

# The ISO allocation problem

**$ISO(b)$**

$$
\begin{aligned}
\underset{(q,h)}{\text{minimize}} \quad & \sum_{i \in \{Agents\}} \sum_{j \in \{Slopes\}} q_i^j b_i^j \\
\text{subject to} \quad & \sum_{j \in \{Slopes\}} q_i^j + \sum_{i' \in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i \\
& h_{i,i'} \geq 0 \\
& q_i^j \geq 0 \\
& q_i^j \leq \bar{q}.
\end{aligned}
$$

# The Reformulation

$$\underset{(q,h)}{\text{minimize}} \quad \mathbb{E} \sum_{i \in \{Producers\}} \sum_{j \in \{Slopes\}} q_i^j(c)\hat{c}_i^j$$

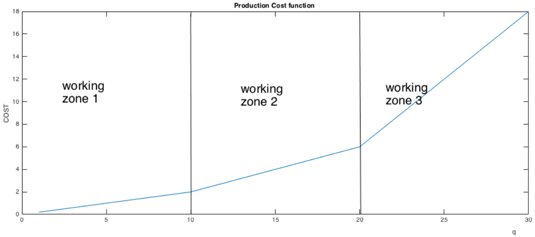
subject to for all $c$

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h_{i,i'}^2(c) + h_{i',i}^2(c)}{2} r_{i,i'} \geq d_i$$

And set $x_i(c) = \sum_{j \in \{Slopes\}} q_i^j(c) c_i^j + \int_{c_i^j}^{\bar{c}_i^j} q_i^j(t; c_{-i}) dt$.

Where $\hat{c}_i^j = (c_i^j + K_i^j(c_i^j))$ can be computed from the data.

# The assumptions on $f_i$




- NON OVERLAPPING WORKING ZONES: $\boldsymbol{C}_i = [c_i^{1-}, c_i^{1+}] \times \ldots \times [c_i^{N-}, c_i^{N+}]$
- DISCERNABILITY ASSUMPTION: the virtual cost $J_{i,j}(c_i^j) = c_i^j + K_i^j(c_i^j)$ is increasing in $j$
- MONOTONE LIKELIHOOD RATIO PROPERTY: $c_i^j \to c_i^j + K_j^i(c_i^j)$ is increasing in $c_i^j$

# Other aspects

- Robustness
- Allocation algorithm and fixed point
- Bayesian game equilibrium computation and fictitious play ODE
- Generalizable to other networks

**Conclusion**

I presented a mechanism to deal with market power in power markets.

# Bibliography


Juan F Escobar and Alejandro Jofré.
Monopolistic competition in electricity networks with resistance losses.
*Economic theory*, 44(1):101–121, 2010.

Nicolas Figueroa, Alejandro Jofré, and Benjamin Heymann.
Cost-minimizing regulations for a wholesale electricity market.
*Energy Journal*, 2016 (submitted).

Benjamin Heymann and Alejandro Jofré.
Mechanism design and allocation algorithms for network markets with piece-wise linear costs and quadratic externalities.
2016.

Benjamin Heymann and Alejandro Jofré.
Mechanism design and auctions for electricity network.
2016.

Jean-Jacques Laffont and David Martimort.
*The theory of incentives: the principal-agent model.*
Princeton university press, 2009.

Roger B Myerson.
Optimal auction design.
*Mathematics of operations research*, 6(1):58–73, 1981.

# Some definitions

- A *direct mechanism* is a triple $(q, x, h) \in (\mathcal{Q}, \mathcal{X}, \mathbb{H})$
- BAYESIAN NASH EQUILIBRIUM= strategy profile such that each player maximizes his expected payoff given his belief about the other players' types and given the strategies played by the other players.
- $K_i^j(c_i^j) = \int_{c_i^{j-}}^{c_i^j} f_i^j(s)ds / f_i^j(c_i^j)$
- $Q_i^j(c_i) = \mathbb{E}_{-i} \min((q_i(c_i, c_{-i}) - (j-1)\bar{q})^+, \bar{q})$
- $X_i(c_i) = \mathbb{E}_{-i} x_i(c_i, c_{-i})$
- $U_i(c_i, c_i') = X_i(c_i') - \sum_{j \in J} c_i^j Q_i^j(c_i')$