Modelling wholesale prices in hydro-dominated electricity systems using stochastic optimization[1]

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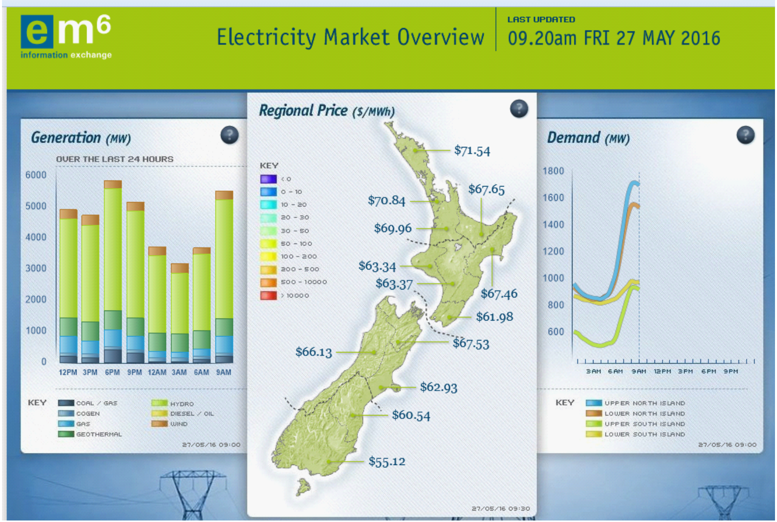
[1]SESO Workshop, Paris, May 30, 2016

New Zealand electricity prices last Friday

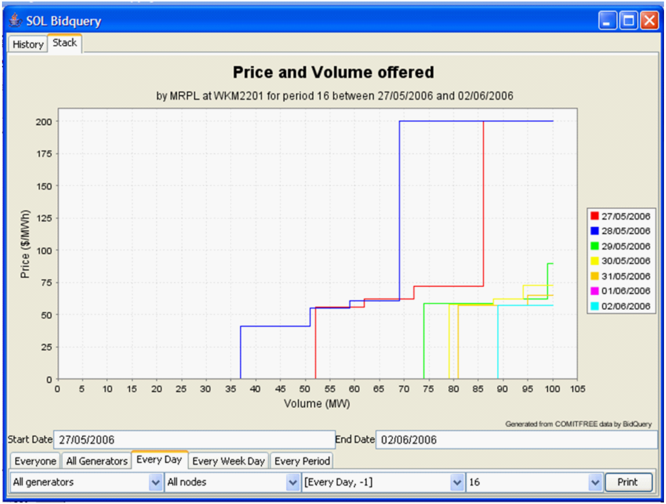


New Zealand electricity prices last Friday.

How does electricity market work?

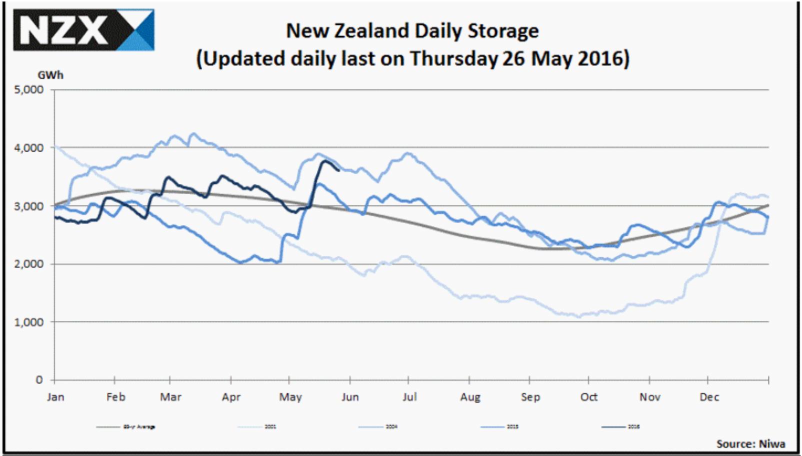
- Every trading period (30 minutes), generators submit to the ISO piecewise constant supply functions with at most 5 steps. These are locked in at gate closure. Generators also supply indicative offers for future periods.
- The ISO solves a single period economic dispatch model to compute dispatch and prices (dual variables) for 250 nodes. The ISO also computes a sequence of provisional dispatches and prices for future trading periods using indicateive offers and forecast demand, and makes the provisional prices and dispatches public.
- The generators plan the next set of offers to make based on observed dispatch, price, and the observed provisional outcomes.
- In theory, perfectly competitive generators will offer supply functions that approximate their marginal cost of production.

What do supply functions look like



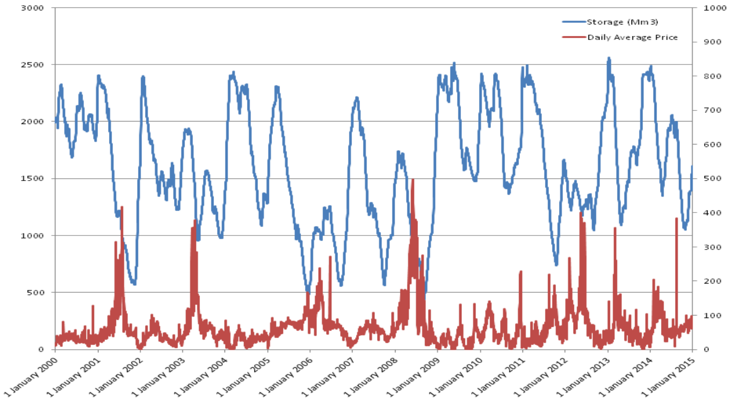
Energy offers from hydro generator at 8am on consecutive days in 2006.

Reservoir storage (GWh)



Total Storage:	3613.69 GWh	Total Storage as Percentage of Average:	122%

New Zealand electricity prices and reservoir levels



New Zealand electricity prices and reservoir levels over last 15 years.

What is this talk about?

- Can stochastic programming tell us what prices will be in the future?
- Can stochastic programming tell us what prices ought to be?
- Are market designs efficient?
- Are prices competitive?
- Should we (and if so how should we?) design markets to account for stochasticity?
- I outline some of the models we have developed at EPOC to help answer these questions.

An aside: perfect competition and workable competition

- Perfectly competitive partial equilibrium optimizes a social planning problem...

  ...so in principle we can find an equilibrium by solving a suitable optimization model.

- Perfect competition in electricity markets does not exist, so regulators aim for workable competition. Nevertheless, perfectly competitive models are very useful

  as benchmarks;
  as indicators of market inefficiencies.

Summary

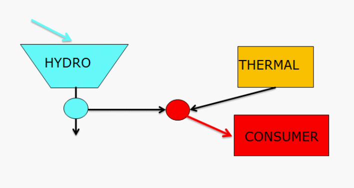
Summary

Single period pool market

$$\text{DSP: min} \quad \sum_{j\in\mathcal{T}} f_j(v_j) - \sum_{c\in\mathcal{C}} c_c(d_c)$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{H}} g_i(u_i) + \sum_{j\in\mathcal{T}} v_j \geq \sum_{c\in\mathcal{C}} d_c, \qquad [p]$$

$$u \in \mathcal{U}, \quad v \in \mathcal{V}.$$



u	*hydro* water flow rate
v	*thermal* generation
d_c	*demand*

Social plan = risk neutral perfectly competitive equilibrium

To minimize Lagrangian for DSP with Lagrange multiplier p we solve each agent problem separately.

$$\begin{aligned} \text{HP}(i)\text{:}\max \quad & p g_i(u_i) \\ \text{s.t.} \quad & u_i \in \mathcal{U}_i. \end{aligned}$$

$$\begin{aligned} \text{TP}(j)\text{: } \max \quad & p v_j - f_j(v_j) \\ \text{s.t.} \quad & v_j \in \mathcal{V}_j. \end{aligned}$$

$$\text{CP}(c)\text{: } \max \quad c_c(d_c) - p d_c.$$

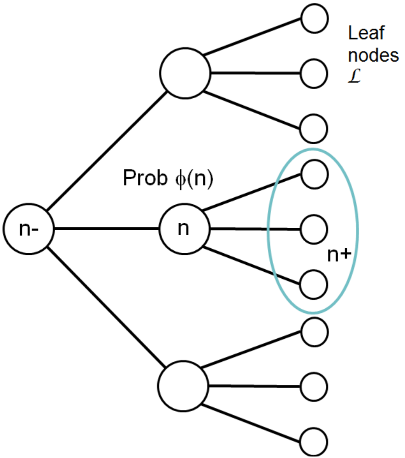
Social plan = perfectly competitive equilibrium

This defines a perfectly competitive equilibrium defined by the individual optimality conditions and market clearing condition.

$$
\begin{aligned}
\text{CE:} \quad & u_i \in \arg\max \mathrm{HP}(i), \\
& v_j \in \arg\max \mathrm{TP}(j), \\
& d_c \in \arg\max \mathrm{CP}(c), \\
& 0 \leq \textstyle\sum_{i \in \mathcal{H}} g_i(u_i) + \sum_{j \in \mathcal{T}} v_j - \sum_{c \in \mathcal{C}} d_c \perp p \geq 0.
\end{aligned}
$$

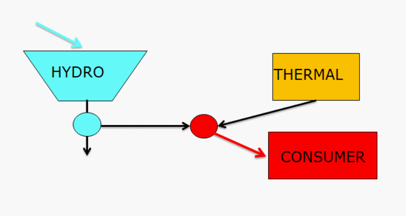
Solutions to CE can be computed in GAMS/EMP as a MOPEC (Ferris, Dirkse, Jagla, Meeraus, 2013) but easier to solve DSP when they give the same answer.

Uncertain inflows: consider a scenario tree



Each node n spans a period (week) and corresponds to a realization $\omega(n)$ of reservoir inflows in that period.

Social plan minimizes total expected system disbenefit



$$\text{SSP: min} \quad \sum_{n\in\mathcal{N}} \phi(n) \left(\sum_{j\in\mathcal{T}} f_j(v_j(n)) - \sum_{c\in\mathcal{C}} c_c(d_c(n))\right) + \sum_{n\in\mathcal{L}} \phi(n) \sum_{i\in\mathcal{H}} Q_i(x_i(n))$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{H}} g_i(u_i(n)) + \sum_{j\in\mathcal{T}} v_j(n) \geq \sum_{c\in\mathcal{C}} d_c(n), \qquad n \in \mathcal{N},$$

$$x_i(n) = x_i(n-) - u_i(n) - s_i(n) + \omega_i(n), \qquad i \in \mathcal{H}, n \in \mathcal{N},$$

$$u(n) \in \mathcal{U}, \quad v(n) \in \mathcal{V}, \quad x(n) \in \mathcal{X}, \quad s(n) \in \mathcal{S}.$$

Social plan = risk neutral perfectly competitive equilibrium

To minimize Lagrangian for social plan with Lagrange multipliers $\phi(n)p(n)$ we solve each agent problem separately.

$$
\begin{array}{lll}
\mathrm{HP}(i):\max & \sum_{n\in\mathcal{N}}\phi(n)p(n)g_i(u_i(n)) - \sum_{n\in\mathcal{L}}\phi(n)Q_i(x_i(n)) & \\
\text{s.t.} & x_i(n) = x_i(n-) - u_i(n) - s_i(n) + \omega_i(n), & n\in\mathcal{N}, \\
 & u_i(n)\in\mathcal{U}_i, \quad x_i(n)\in\mathcal{X}_i, \quad s_i(n)\in\mathcal{S}_i. &
\end{array}
$$

$$
\begin{array}{ll}
\mathrm{TP}(j):\ \max & \sum_{n\in\mathcal{N}}\phi(n)(p(n)v_j(n) - f_j(v_j(n))) \\
\text{s.t.} & v_j(n)\in\mathcal{V}_j.
\end{array}
$$

$$
\mathrm{CP}(c):\ \max \quad \sum_{n\in\mathcal{N}}\phi(n)\left(c_c(d_c(n)) - p(n)d_c(n)\right).
$$

Social plan = perfectly competitive equilibrium

This defines a perfectly competitive equilibrium defined by the individual optimality conditions and market clearing condition.

$$\text{CE:} \quad u_i, x_i, s_i \in \arg\max \mathrm{HP}(i),$$

$$v_j(n) \in \arg\max \mathrm{TP}(j),$$

$$d_c(n) \in \arg\max \mathrm{CP}(c),$$

$$0 \leq \textstyle\sum_{i\in\mathcal{H}} g_i(u_i(n)) + \sum_{j\in\mathcal{T}} v_j(n) - \sum_{c\in\mathcal{C}} d_c(n) \perp p(n) \geq 0.$$

Potential incompleteness of the hydro model

Our model above was derived assuming a single hydro agent. It assumes

- all hydro generating stations operated by a single agent;
- a single future value function $Q_i(x)$ for this agent/social planner.

With competing hydro agents, for separability we will require

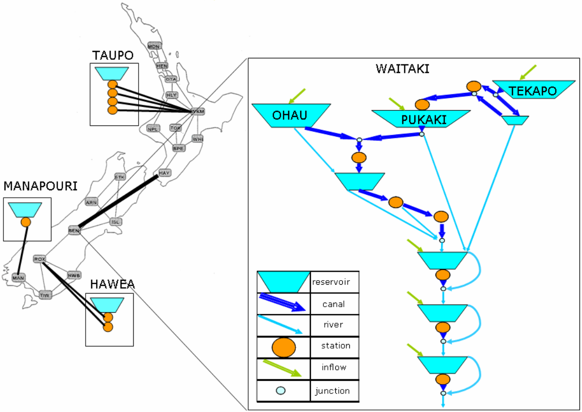
- a future value function for the social planner that is the sum of individual hydro agent's values (more about this later) or a decision horizon long enough to discount the dependence at $n \in \mathcal{L}$ away;
- prices to enable efficient transfer of water between competing agents on a river chain (Lino et al, 2003)

Summary

EPOC optimization models

vSPD	250 node DC-Load flow model of the New Zealand wholesale electricity market. This is a GAMS version of SPD, the dispatch system used by the ISO. Given the same inputs, it yields identical dispatch and prices.
Clairvoyant	48-period dynamic model of a single day's operations of the New Zealand wholesale electricity market including river chains. Energy dispatch can anticipate later decisions.
DOASA	SDDP model of the New Zealand electricity system, using an aggregated transmission network.

The main hydro catchments in New Zealand



Approximate network representation of New Zealand electricity network showing main hydro-electricity generators.

Models for studying daily inefficiency

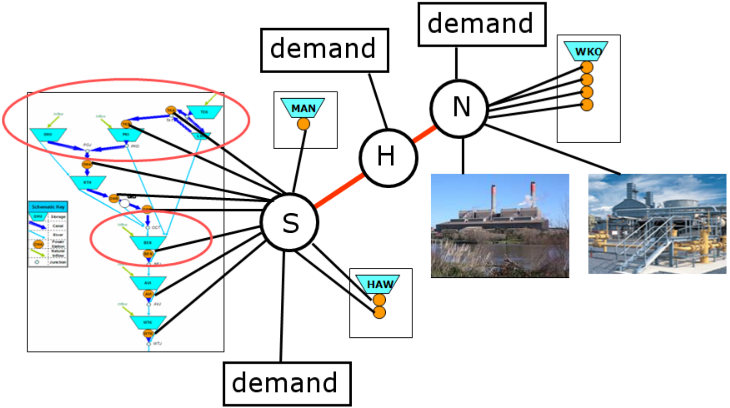
Historical	What actually happened.
Clairvoyant	Solve 48-period dynamic model of a single day's operations operations assuming perfect foresight of what demand actually happened.
StackvSPD	Agents update next period offers by solving their own river-chain optimization with forecast prices, submit to SPD, and roll forward one trading period.
Rolling Central	Rolling horizon version of clairvoyant model using forecast demand to dispatch all plant in current period.

Results of simulations in 2009

	Historical	Clairvoyant	Stack vSPD	Rolling Central
Fuel Cost	\$ 1,837,724.095	\$ 1,631,029.74	\$ 1,818,094.29	\$ 1,636,496.61
Infeasibility Cost	\$ 9,586.61	\$ 151.34	\$ 151.34	\$ 151.34
Total Cost	\$ 1,847,310.70	\$ 1,631,181.07	\$ 1,818,245.63	\$ 1,636,647.94

Breakdown of costs (NZ \$) for Historical, Clairvoyant, Stack vSPD, and Rolling Central models. Cost displayed is average daily cost for the months of February 2009 and June 2009. [Source: N.Porter, 2014]

The DOASA model



New Zealand model has seven state variables corresponding to seven storage reservoirs.

Shortage Costs

Energy deficit in any stage is met by load shedding at an increasing shortage cost in three tranches. This is equivalent to having three dummy thermal plant at each location with capacities equal to 5% of load, 5% of load and 90% of load, for each load sector, and costs as follows

	Up to 5%	Up to 10%	VOLL	North Is	South Is
Industrial	\$1,000	\$2,000	\$10,000	0.34	0.58
Commercial	\$2,000	\$4,000	\$10,000	0.27	0.15
Residential	\$2,000	\$4,000	\$10,000	0.39	0.27

Load reduction costs (NZD/MWh) and proportions of load that is industrial, commercial, and residential in each island.

Rolling horizon simulation

Solve DOASA to compute a least-cost policy for a social planner, and simulate this policy in Clairvoyant using end conditions for each day that come from DOASA cutting planes. In our model, we simulate the policy obtained for 4 weeks and then re-solve DOASA to compute an updated policy. We call this policy the counterfactual.

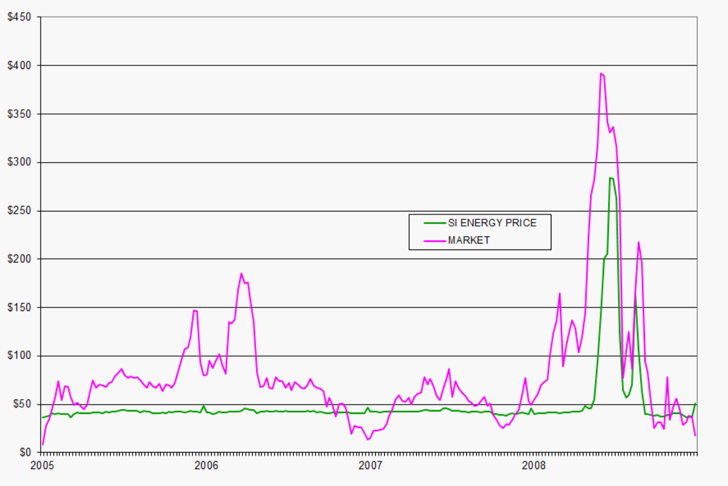
Modelling assumptions and caveats

- No spinning reserve;
- No extra costs for SRMC apart from fuel, and no fuel take-or-pay contracts or supply constraints;
- No snowmelt model or coal stockpiles;
- No contracting;
- Outages modelled using POCP database;
- 300 cuts per solve.

South Island storage

Figure: Simulated and actual South Island storage trajectories in market (pink) and counterfactual (green) 2005-2008.

South Island prices



South Island weekly average prices in market (pink) and counterfactual (green)

Summary

Dual representation of coherent risk measures
(Artzner et al, 1999, Shapiro & Ruszczynski, 2006)

A coherent risk measure of a random disbenefit Z can be expressed as

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z]$$

where $\mathcal{D}$ is a convex set of probability measures called the risk set.

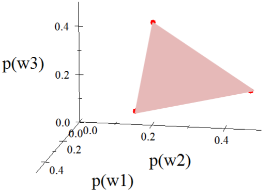
Example: three outcomes

Consider possible disbenefit outcomes

$$Z(\omega_1) < Z(\omega_2) < Z(\omega_3)$$

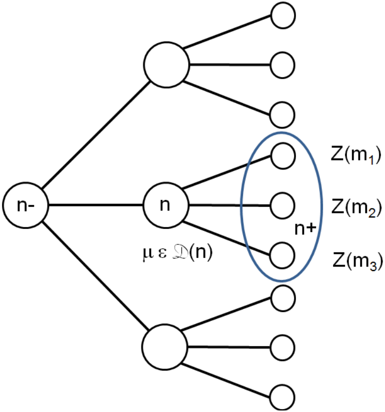
Let the risk set

$$\mathcal{D}=\text{conv}\{(\frac{1}{2},\frac{1}{4},\frac{1}{4}),(\frac{1}{4},\frac{1}{2},\frac{1}{4}),(\frac{1}{4},\frac{1}{4},\frac{1}{2})\}$$



$$\rho(Z) = \max_{\mu\in\mathcal{D}} \mathbb{E}_\mu[Z] = \frac{1}{4}Z(\omega_1) + \frac{1}{4}Z(\omega_2) + \frac{1}{2}Z(\omega_3)$$

Scenario trees and risk measures



Each node m corresponds to a realization $\omega(m)$ of reservoir inflows and disbenefit $Z(m)$ in that period.

Dynamic risk measures
(Epstein &Schneider, 2003, Artzner et al 2007, Ruszczynski, 2010)

Consider a random sequence of disbenefits $Z(n)$ corresponding to the nodes of the scenario tree. Each node $n \in \mathcal{N} \setminus \mathcal{L}$ in the scenario tree is endowed with a risk set $\mathcal{D}(n)$. The dynamic risk measure we will use is constructed recursively as follows. For every leaf node we set the risk-adjusted disbenefit

$$\rho(n) = Z(n)$$

and for every other node we set

$$\rho(n) = Z(n) + \max_{\mu \in \mathcal{D}(n)} \sum_{m \in n+} \mu(m)\rho(m).$$

Dynamic optimization under risk

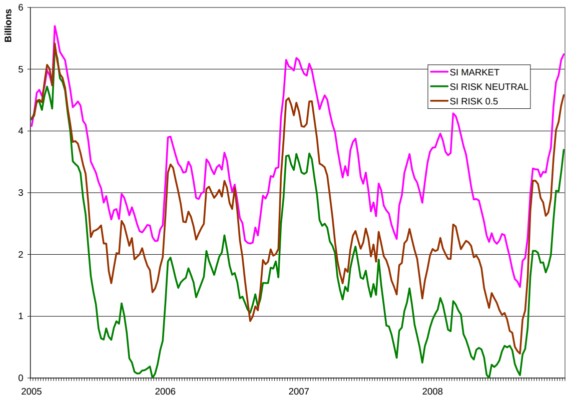
Suppose each node $n \in \mathcal{N} \setminus \mathcal{L}$ in the scenario tree has risk set $\mathcal{D}(n)$. We seek a policy (actions $u_i(n)$, $x_i(n)$, $s_i(n)$, $v_j(n)$, $d_c(n)$) giving disbenefits $Z(n)$ that minimize risk-adjusted disbenefit $\rho(1)$, where

$$\rho(n) = Z(n), \quad n \in \mathcal{L},$$

and for every other node we set

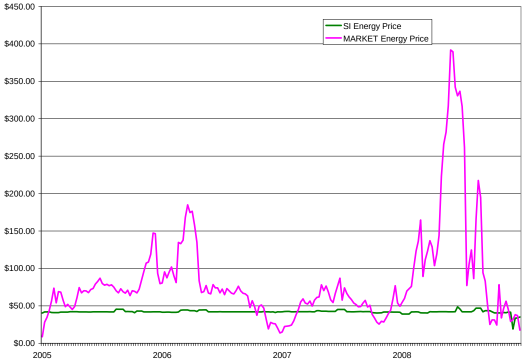
$$\rho(n) = Z(n) + \max_{\mu \in \mathcal{D}(n)} \sum_{m \in n+} \mu(m)\rho(m).$$

Risk-averse storage trajectories



South Island storage trajectories for varying levels of risk aversion.

Risk-averse average prices



Weekly average South Island prices from risk averse model with $\lambda = 0.5$ (green) compared with historical Benmore prices (pink).

The change in fuel cost

	Annual thermal fuel cost ($M)		
	MARKET	λ=0	λ=0.5
2005	451.79	349.27	377.99
2006	490.99	444.62	432.03
2007	492.51	441.56	447.70
2008	508.49	435.27	424.19

Annual fuel cost for different levels of risk aversion. The risk neutral solution ($\lambda = 0$) incurs load shedding cost of $95M in 2008. The risk-averse solution ($\lambda = 0.5$) incurs no load shedding.

Recall dynamic risk measure

For agent $a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}$ consider a random sequence of disbenefits $Z_a(n)$ defined for each node of the scenario tree. Each agent a at each node $n \in \mathcal{N} \setminus \mathcal{L}$ in the scenario tree is endowed with her own risk set $\mathcal{D}_a(n)$. The dynamic risk measure we will use for agent a is constructed recursively as follows. For every leaf node we set

$$\rho_a(n) = Z_a(n)$$

and for every other node we set

$$\rho_a(n) = Z_a(n) + \max_{\mu \in \mathcal{D}_a(n)} \sum_{m \in n+} \mu(m)\rho_a(m).$$

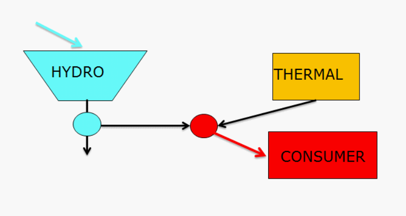
Dynamic risked competitive equilibrium
(Philpott, Ferris, Wets, 2016)

Consider a set of agents $a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}$ and stochastic process of inflows for each $a \in \mathcal{H}$ defined by a scenario tree with nodes $n \in \mathcal{N}$ and leaves $\mathcal{L}$. A dynamic risked equilibrium is a stochastic process of energy prices $\{p(n) \mid n \in \mathcal{N}\}$ in the scenario tree, and for each agent a, a stochastic process of production/consumption decisions $\{x_a(n) \mid n \in \mathcal{N}\}$, with the property that

$$0 \leq \sum_{a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}} x_a(n) \perp p(n) \geq 0, \; n \in \mathcal{N}$$

and $x_a(\cdot)$ is a solution to the risk-averse optimization problem where agent a minimizes $\rho_a(1)$ evaluated using prices $\{p_n \mid n \in \mathcal{N}\}$ and their individual risk sets $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$.

Example: three agents, two periods, 5 inflow scenarios



$$
\begin{aligned}
f(v) &= v^2, \\
g(u) &= 1.5u - 0.015u^2, \\
Q(x) &= -10\log(0.05x + 0.005), \qquad x(0) = 10, \\
c(d) &= 40d - 2d^2, \\
\omega(1) &= 2, \quad \omega(m) = 0, 2, 4, 6, 8 \text{ with equal probability}, \\
\mathcal{D}_a &= \text{conv}\{(0.36, 0.16, 0.16, 0.16, 0.16), (0.16, 0.36, 0.16, 0.16, 0.16), \\
&\quad (0.16, 0.16, 0.36, 0.16, 0.16), (0.16, 0.16, 0.16, 0.36, 0.16), \\
&\quad (0.16, 0.16, 0.16, 0.16, 0.36)\}.
\end{aligned}
$$

Example: risk neutral equilibrium

stage	ω_m	price	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0	2	2.316	5.851	1.158				
1	0	4.516	4.622	2.258	6.439	23.906	14.902	45.248
1	2	2.806	5.575	1.403	3.309	21.167	30.441	54.916
1	4	1.840	6.121	0.920	2.187	19.218	39.534	60.939
1	6	1.313	6.423	0.656	1.771	18.637	44.601	65.009
1	8	1.004	6.600	0.502	1.593	18.807	47.599	67.999

Table: Risk neutral equilibrium.

Example: risk averse equilibrium

stage	ω_m	price	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
					2.646	18.988	32.323	56.038
0	2	2.156	5.942	1.078	-	-	-	-
1	0	4.614	4.568	2.307	6.485	22.930	15.539	44.954
1	2	2.865	5.541	1.432	3.214	20.232	31.396	54.842
1	4	1.872	6.103	0.936	2.039	18.214	40.733	60.985
1	6	1.331	6.412	0.665	1.605	17.584	45.931	65.120
1	8	1.015	6.594	0.508	1.420	17.732	48.995	68.147

Table: Risk averse equilibrium. Red cells show the worst-case welfare values for each agent and system. Blue cells are risk adjusted welfare for each agent and system.

Example: risk averse social plan

stage	ω_m	price	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
					3.070	22.029	29.168	56.166
0	2	2.652	5.661	1.326	-	-	-	-
1	0	4.316	4.733	2.158	6.415	25.818	13.529	45.762
1	2	2.687	5.642	1.343	3.562	23.024	28.398	54.985
1	4	1.776	6.158	0.888	2.547	21.222	36.996	60.764
1	6	1.277	6.444	0.638	2.165	20.739	41.800	64.704
1	8	0.982	6.613	0.491	1.999	20.955	44.665	67.618

Table: Risk averse social plan using common risk set. Red cells show the worst-case welfare values for each agent and system. Blue cells are risk adjusted welfare for each agent and system.

Example in practice

Genesis to close last two coal-fired power units at Huntly

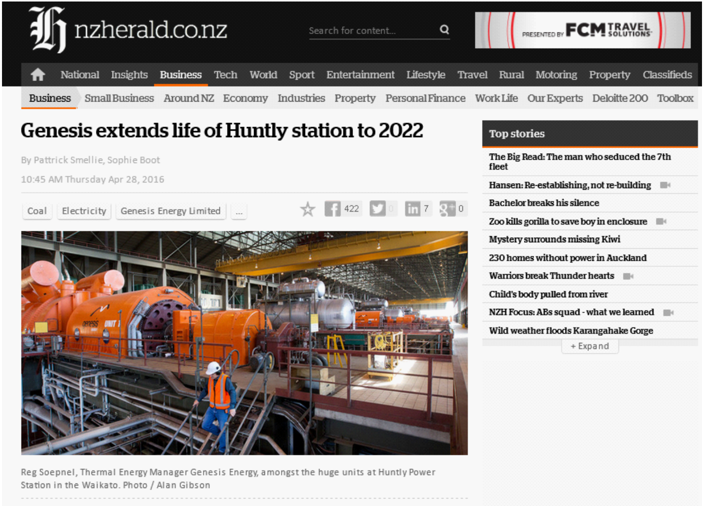
JAMES WEIR
Last updated 17:29, August 6 2015

RORY O'SULLIVAN/Stuff.co.nz

Prime Minister John Key is unsurprised by the news that Genesis Energy are to close the two remaining coal-fired units at Huntly power station.

Stuff.co.nz , August 6, 2015

Example in practice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Genesis extends life of Huntly station to 2022

By Pattrick Smellie, Sophie Boot

10:45 AM Thursday Apr 28, 2016

Coal | Electricity | Genesis Energy Limited | ...

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Reg Soepnel, Thermal Energy Manager Genesis Energy, amongst the huge units at Huntly Power Station in the Waikato. Photo / Alan Gibson

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NZ Herald, April 28, 2016

Contracts enable risk to be traded

Suppose we introduce contracts for differences. A single contract for differences written at strike price f pays the holder $p(m) - f$ in scenario m. Agent a settles q_a (typically positive for consumers and negative for generators) of these contracts at time 0 which pays her $(p(m) - f)q_a$ in scenario m at time 1. The market for contracts must clear, so

$$\sum_a q_a = 0.$$

Example: risk averse equilibrium with contracts

stage	ω_m	price	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
					3.953	23.045	29.168	56.166
0	2	2.652	5.661	1.326	-	-	-	-
1	0	4.316	4.733	2.158	3.133	22.713	19.916	45.762
1	2	2.687	5.642	1.343	3.451	22.919	28.615	54.985
1	4	1.776	6.158	0.888	4.206	22.792	33.766	60.764
1	6	1.277	6.444	0.638	4.797	23.228	36.679	64.704
1	8	0.982	6.613	0.491	5.204	23.986	38.428	67.618

Table: Risk-averse competitive equilibrium with contracts. Red cells show the worst-case welfare values for each agent and system. Blue cells are risk adjusted welfare for each agent and system.

Example: contracts settled in equilibrium

stage	ω_m	price	contract (T)	contract (H)	contract (C)
0	2	2.629	-1.946	-1.840	3.786
1	0	4.316	-3.283	-3.104	6.387
1	2	2.687	-0.112	-0.106	0.218
1	4	1.776	1.660	1.570	-3.230
1	6	1.277	2.632	2.489	-5.121
1	8	0.982	3.206	3.031	-6.237

Table: Traded contracts (red) and net contract receipts of the three agents in equilibrium.

Example: risk-averse social plan

stage	ω_m	price	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
					3.070	22.029	29.168	56.166
0	2	2.652	5.661	1.326	-	-	-	-
1	0	4.316	4.733	2.158	6.415	25.818	13.529	45.762
1	2	2.687	5.642	1.343	3.562	23.024	28.398	54.985
1	4	1.776	6.158	0.888	2.547	21.222	36.996	60.764
1	6	1.277	6.444	0.638	2.165	20.739	41.800	64.704
1	8	0.982	6.613	0.491	1.999	20.955	44.665	67.618

Table: Risk-averse social planning solution using a common risk set. Red cells show the worst-case welfare values for each agent and system. Blue cells are risk adjusted welfare for each agent and system. Adding receipts from contracts gives risked equilibrium with contracts.

Example: risk averse equilibrium with contracts

stage	ω_m	price	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
					3.953	23.045	29.168	56.166
0	2	2.652	5.661	1.326	-	-	-	-
1	0	4.316	4.733	2.158	3.133	22.713	19.916	45.762
1	2	2.687	5.642	1.343	3.451	22.919	28.615	54.985
1	4	1.776	6.158	0.888	4.206	22.792	33.766	60.764
1	6	1.277	6.444	0.638	4.797	23.228	36.679	64.704
1	8	0.982	6.613	0.491	5.204	23.986	38.428	67.618

Table: Risk-averse competitive equilibrium with contracts. Red cells show the worst-case welfare values for each agent and system. Blue cells are risk adjusted welfare for each agent and system.

A more general result for dynamic risked equilibrium
(Philpott, Ferris, Wets, 2016)

Suppose the risk set in node $n \in \mathcal{N} \setminus \mathcal{L}$ of each agent a is different, i.e. $\mathcal{D}_a(n), \quad a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}$. What risk set $\mathcal{D}_0(n)$ should the system use to make an optimal risk-averse social plan correspond to the competitive equilibrium? (They all used the same risk set in the above example).

Theorem

(Heath and Ku 2004, Ralph and Smeers, 2011) If there is a rich enough set of contracts and

$$\cap_{a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}} \mathcal{D}_a(n) \neq \emptyset, \quad n \in \mathcal{N} \setminus \mathcal{L},$$

then in equilibrium all agents and the system use risk sets

$$\mathcal{D}_0(n) = \cap_{a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}} \mathcal{D}_a(n), \quad n \in \mathcal{N} \setminus \mathcal{L}.$$

Arrow-Debreu securities complete the risk market

Arrow-Debreu securities are contracts that charge a price $\mu(m)$ in node $n \in \mathcal{N}$, to receive a payment of 1 in node $m \in n+$. These form a complete market for risk in each $n \in \mathcal{N} \setminus \mathcal{L}$ (i.e. contracts traded in node n span the $|n+|$ payoff outcomes).

Let $\{x_a(n) \mid n \in \mathcal{N}, a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}\}$ be a solution to the risk-averse social planning problem with risk sets $\mathcal{D}_0(n) \neq \emptyset$. Suppose this gives prices $\{p(n) \mid n \in \mathcal{N}\}$. These prices and quantities form a dynamic risked equilibrium in which agents trade risk i.e. agent a minimizes $\rho_a(1)$ with a policy defined by $x_a(\cdot)$ together with a policy of trading Arrow-Debreu securities at each node n.

Conclusions

- Competitive equilibria need not be welfare maximizing. Suboptimality in many of our examples is not shown to be an artifact of imperfect competition, but of incompleteness in the market design.
- Including trade in specific instruments in the equilibrium model completes the market, and recovers the social optimum.
- The extent to which we complete the market will depend on transaction costs.

Short-run efficiency comparisons

	Annual generator spot market revenue ($M)		
	MARKET	COUNTERFACTUAL	DIFFERENCE
2005	2918.60	1413.58	1414.76
2006	2881.05	1450.79	1453.52
2007	1883.51	1443.49	1448.82
2008	4065.19	1859.97	1373.22

	Annual thermal fuel cost ($M)		
	MARKET	COUNTERFACTUAL	DIFFERENCE
2005	451.79	382.33	69.46
2006	490.99	442.94	48.05
2007	492.51	433.89	58.62
2008	508.49	435.29	73.20

	Annual generator short-term rents ($M)		
	MARKET	COUNTERFACTUAL	DIFFERENCE
2005	2466.81	1031.25	1435.55
2006	2390.06	1007.84	1382.22
2007	1391.01	1009.60	381.41
2008	3556.71	1424.69	2132.02

Annual productive efficiency losses and generator rents (in 2008 NZD) for market compared with counterfactual.

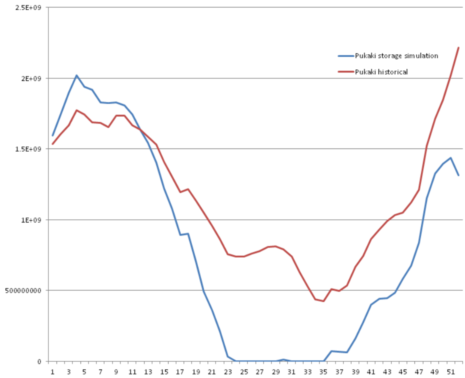
Are counterfactual prices really that low in 2008?

- The counterfactual water values are lower than the market yet the South Island storage in market is higher.
- Market South Island weekly average price is \$282 in Week 20 of 2008.
- Counterfactual South Island weekly average price is \$56 in Week 20 of 2008.
- Test water values at Pukaki by solving more accurately starting from historical market reservoir levels in Week 1 of 2008.

Storage for policy with fuel costs

Prices for policy with fuel costs

Storage for policy with marked-up thermal costs



Prices for policy with marked-up thermal costs