Bilevel Models for Energy Pricing Problems

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Outline

- Bilevel Programming
- Demand Side Management
- Bilevel Energy Pricing Problems
- Numerical Results
- Conclusion
Hierarchical decision making process with opposite interests.

"Cake-Cutting Game"
Bilevel Programming

To deal with decision processes involving two decision-makers with a hierarchical structure:

- Two decision levels: a leader and a follower.
- The leaders sets his decision variables first. Then the follower reacts based on the choices of the leader.
- Each decision-maker controls a set of variables subject to constraints and seeks to optimize a given objective function.
- Objective functions usually are in conflict and decision-makers do not cooperate.

Related to Principal Agent Paradigm, Equilibrium constrained Mathematical Program, Stackelberg Game
Mathematical Formulation

\[
\begin{align*}
\max_{x} & \quad f(x) \\
\text{s.t.} & \quad x \in X
\end{align*}
\]

\[
\begin{align*}
\max_{x_1} & \quad f_1(x_1, x_2) \\
\text{s.t.} & \quad (x_1, x_2) \in X \\
& \quad x_2 \text{ solves } \min_{x_2} f_2(x_1, x_2) \\
\text{s.t.} & \quad (x_1, x_2) \in Y
\end{align*}
\]

The feasible region of the leader’s problem is implicitly determined by an optimization problem.

A subset of variables are constrained to be an optimal solution of another optimization problem parameterized by the remaining variables.
Pricing Problem

- The companies have to decide on prices of services to maximize revenue.
- Companies have the choice among several companies to minimize their costs.

Hierarchical decision making process with opposite interest.

\[
\max_p \quad p y \\
\text{s.t.} \\
(p, y) \in X \\
y \text{ solves } \min_y \quad C(p, y) \\
\text{s.t.} \\
(p, y) \in Y
\]

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A Bilevel Approach for Energy Pricing Problems
Main properties of bilevel programs

For fixed $x_1$, the second level problem is defined as:

$$x_2 \in P(x_1) = \arg \min_y \{ f_2(x_1, y) \text{ s.t. } (x_1, y) \in Y \}$$

Non unicity of the follower solution:

- **Optimistic case**: The leader is able to influence the second level decision maker so that the latter always selects $y$ to provide the best value of $f$.

- **Pessimistic case**: The leader behaves as thought the follower always selected the optimal decision which gives the worst value of his objective function.
Main properties of bilevel programs

Bilevel programs are intrinsically hard to solve.

- Leader’s domain is generally non convex and discontinuous

\[ IR = \{(x_1, x_2) \text{ s.t. } (x_1, x_2) \in X, x_2 \in P(x_1)\} \]

- \( x_2(x_1) \) is generally non differentiable
- Optimal solution non pareto-optimal
- Linear Linear bilevel programs:
  - Strongly NP-Hard
  - Checking strict or local optimality is Strongly NP-Hard
A Bilevel Approach for Energy Pricing Problems
Linear Bilevel Program

A Bilevel Approach for Energy Pricing Problems
Linear Bilevel Program

\[ S(\bar{x}_1) \]

\[ f_1 \]

\[ f_2 \]
Linear Bilevel Program

\[ f_1 \]
\[ f_2 \]

IR
Linear Bilevel Program

A Bilevel Approach for Energy Pricing Problems
Linear Bilevel Program

A Bilevel Approach for Energy Pricing Problems
Main properties of bilevel programs

Algorithms.

- General Case:
  - Penalty methods,
  - Descent methods
  - Metaheursitics
  - ...

- Linear bilevel programs
  - Branch and Bound
  - Extremal Points
  - Descent methods
  - Penalty methods
  - ...

To solve large size instances the structure of the problem has to be sharply exploited.
Applications

- Pricing problems on networks
- Network design problems
- Location problems
- Product bundling problems
- Security games
- Demand side management in the energy field
- ..
Bilevel energy Pricing Problems

Outline

▶ Demand Side Management
▶ Smart Grid
▶ Bilevel Pricing models
▶ Numerical Results
Demand Side Management

Motivation

- Demand for energy is largely uncontrollable and varies with time of day and season.
  - In UK, given the average demand across the year, the average utilization of the generation capacity is $\leq 55\%$.
  - Minimum demand in summer nights $\approx 30\%$ of the winter peak
- Energy is difficult to store in large quantities.
- Supply-demand balance failure $\rightarrow$ system instability
- Total capacity of installed generation must be $\geq$ max demand to ensure the security of supply.
Demand Side Management

Why Necessary?

- DSM: control and manipulate the demand to meet capacity constraints.
- DSM’s role: To improve the efficiency of operation and investment in the system.

Demand Side Management

Major DSM Techniques

- Direct load control, load limiters, load switching
- Commercial/industrial programs
- Demand bidding
- Time-of-use pricing
- Smart grid
  "The smart grid is expected to revolutionize electricity generation, transmission, and distribution allowing two-way flows for both electrical power and information. Moreover it can complement the current electric grid system by including renewable energy sources" (Bari et al.)
Demand Side Management

Smart Grid Technology

- Every customer is equipped with a device that can receive, process and transfer data: smart meter
- Smart meters communicate with each other → smart grid
- Meters are programmable according to the needs of the customer
- Smart grid receives prices from the supplier(s), demand from the customers and schedules the consumption

"Commitment to market based operation and deregulation of the electricity industry places consumers of electricity in the center of the decision-making process regarding the operation and future development of the system" - G. Strbac, 2008
Energy Pricing Problem

Bilevel Approach

Supplier(s)
- Maximize Revenue
- Minimize Peak
- Minimize Deviations

Hourly Prices
Incentives
Capacity

Smart Grid
- Schedule
- Maximize Utility Function

Demand
Time Windows

Customer1

Customer2

Customer n
Energy Pricing Problem

Properties

- Stackelberg game / Bilevel programming
- The supplier (upper level) and a group of customers connected to a smart grid (lower level).
- Prices from supplier and demand with time windows from customers are received by the smart grid.
- Time-of-use price control to minimize peak demand and maximize revenue.
- Demand-response to hourly changing prices to minimize cost and waiting time.
Residential Electricity Consumption

- Air Conditioning: 23%
- Refrigerators: 7%
- Water Heating: 9%
- Space Heating: 6%
- All Other Appliances and Lighting: 55%
### Objectives

- Leader maximizes \((\text{revenue} - \text{peak cost})\) by deciding on prices.
- Follower minimizes \((\text{billing cost} + \text{waiting cost})\) by deciding on the schedule of consumption.

### Assumptions

- A fixed upper bound for prices.
- Demand is fixed.
- All operations are preemptive.
- Every customer has a set of appliances with preferred time windows.
- All appliances have power consumption limits.
- One cycle has 24 time slots (hours).
Bilevel Models

- $H$: Set of time slots
- $p^h$: Price for time slot $h$
- $\Gamma$: Peak Load
- $\kappa$: Peak weight factor
- $N$: Set of customers
- $A_n$: Set of devices for customer $n$
- $\beta_{n,a}^{\text{max}}$: Power Limit of device $a$ for customer $n$
- $\chi^h_{n,a}$: Power level for each appliance $a$ in time slot $h$ for customer $n$
- $E_{n,a}$: Demand of customer $n$ for appliance $a$
- $[TW_{n,a}^b, TW_{n,a}^e]$ : Time Window of customer $n$ for appliance $a$
- $\lambda_n$: Delay sensitivity coefficient of customer $n$
- $C_{n,a}(h)$: Inconvenience cost for customer $n$ if appliance $a$ is at $h$

\[
C_{n,a}(h) := \lambda_n \times E_{n,a} \times \frac{(h - TW_{n,a}^b)}{(TW_{n,a}^e - TW_{n,a}^b)}
\]

$\forall n \in N, \forall a \in A_n, \forall h \in H.$
\[
\begin{align*}
\max_{\gamma} \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in H} p^h x^h_{n,a} - \kappa \Gamma \\
\text{s.t.} \quad \Gamma \geq \sum_{n \in N, a \in A_n} \sum_{h \in H} x^h_{n,a} \quad \forall h \in H \\
0 \leq p^h \leq p^{h}_{\text{max}} \quad \forall h \in H \\
\min_{x} \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in H} (p^h + C_{n,a}(h)) x^h_{n,a} \\
\text{s.t.} \quad x^h_{n,a} \leq \beta^\text{max}_{n,a} \quad \forall n \in N, \forall a \in A_n, \forall h \in H \\
\sum_{h \in H} x^h_{n,a} \geq E_{n,a} \quad \forall n \in N, \forall a \in A_n, \forall h \in H \\
x^h_{n,a} \geq 0 \quad \forall n \in N, \forall a \in A_n, \forall h \in H
\end{align*}
\]
Bilevel Model - Preemptive Monopolistic

Load Distribution Under Different Peak Parameters

Price Change Under Different Peak Parameters

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A Bilevel Approach for Energy Pricing Problems
Bilevel Model - Preemptive Competitive

\[
\begin{align*}
\max_{p, \Gamma} & \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in H} p^h x^h_{n,a} - \kappa \Gamma \\
\text{s.t.} & \quad \Gamma \geq \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in H} x^h_{n,a} \quad \forall h \\
& \quad 0 \leq p^h \leq p^h_{\max} \quad \forall h \in H \\
\min_{\bar{x}, \bar{x}} & \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in H} (p^h + C_{n,a}(h)) x^h_{n,a} \\
& + \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in H} (\bar{p}^h + C_{n,a}(h)) \bar{x}^h_{n,a} \\
\text{s.t.} & \quad x^h_{n,a} + \bar{x}^h_{n,a} \leq \beta_{n,a}^{\max} \quad \forall n \in N, \forall a \in A_n, \forall h \in H \\
& \quad \sum_{h \in H} (x^h_{n,a} + \bar{x}^h_{n,a}) \geq E_{n,a} \quad \forall n \in N, \forall a \in A_n \\
& \quad x^h_{n,a}, \bar{x}^h_{n,a} \geq 0 \quad \forall n \in N, \forall a \in A_n, \forall h \in H,
\end{align*}
\]
Exact Solution Method

Single Level formulation

- Optimality conditions (primal, dual and complementarity constraints) of the follower
- Complementary slackness constraints $\rightarrow$ linearized using binary variables
- Objective function is linearized using the follower’s dual objective function.
- Single level MIP
Exact Solution Method

Single Level formulation

Dual variables are $w_{n,a}^h$ and $v_{n,a}^h$

The dual constraints associated with $x_{n,a}^h$ are:

$$-w_{n,a}^h + v_{n,a}^h - p^h \leq C_{n,a}(h) \quad \forall n \in N, \forall a \in A_n, \forall h \in H$$

Complementarity slackness between $x_{n,a}^h$ and the dual constraints are

$$x_{n,a}^h(w_{n,a}^h - v_{n,a}^h + p^h + C_{n,a}(h)) = 0 \quad \forall n \in N, \forall a \in A_n, \forall h \in H$$

To linearize this constraints, consider binary variables $\psi_{n,a}^h$:

$$w_{n,a}^h - v_{n,a}^h + p^h + C_{n,a}(h) \leq M_1(1 - \psi_{n,a}^h)$$

$$x_{n,a}^h \leq M_1 \psi_{n,a}^h$$

$$\psi_{n,a}^h \in \{0, 1\}$$
Numerical Results

- Scenarios with 10 customers, 3 appliances for each customer (thus 30 appliances).
- Schedule horizon: 24 time slots.
- Peak weight ($\kappa$): 200, 400, 600, 800 and 1000.
- TWW 20% and 100% $\rightarrow$ the time window of each job is 20% or 100% larger than the minimum completion time (MCT).
- MCT is the minimum number of time slots required to meet demand ($E_{n,a}$) if we could use all devices at their maximum level.
Numerical Results

- For the experiments, CPLEX version 12.3 is used on a computer with 2.66 GHz Intel 283 Xeon CPU and 4 GB RAM, running under the Windows 7 operating system.
- Average value over 10 instances.
- The base case (BC) : solution where prices are equal to $p_{\text{max}}$ and appliances are scheduled to the preferred time slots.
- The user costs are split between electricity bill (EB) and inconvenience (EC).
- Percentages relative to the base case (BC).
## Numerical Results

### Costs for MP and CP; 20% TWW and 100% TWW instances

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>MP EB</th>
<th>MP IC</th>
<th>MP TC</th>
<th>CP EB</th>
<th>CP IC</th>
<th>CP TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>78.02</td>
<td>21.49</td>
<td>99.51</td>
<td>78.31</td>
<td>21.17</td>
<td>99.48</td>
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<tr>
<td>400</td>
<td>77.15</td>
<td>21.59</td>
<td>98.74</td>
<td>78.01</td>
<td>20.13</td>
<td>98.14</td>
</tr>
<tr>
<td>600</td>
<td>75.76</td>
<td>21.85</td>
<td>97.61</td>
<td>77.84</td>
<td>18.74</td>
<td>96.58</td>
</tr>
<tr>
<td>800</td>
<td>73.52</td>
<td>22.16</td>
<td>95.68</td>
<td>78.07</td>
<td>16.81</td>
<td>94.88</td>
</tr>
<tr>
<td>1000</td>
<td>71.50</td>
<td>22.48</td>
<td>93.99</td>
<td>77.63</td>
<td>16.28</td>
<td>93.91</td>
</tr>
<tr>
<td>Average</td>
<td>75.19</td>
<td>21.91</td>
<td>97.10</td>
<td>77.97</td>
<td>18.63</td>
<td>96.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>MP EB</th>
<th>MP IC</th>
<th>MP TC</th>
<th>CP EB</th>
<th>CP IC</th>
<th>CP TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>85.12</td>
<td>14.16</td>
<td>99.28</td>
<td>85.25</td>
<td>14.06</td>
<td>99.30</td>
</tr>
<tr>
<td>400</td>
<td>82.87</td>
<td>14.55</td>
<td>97.42</td>
<td>84.05</td>
<td>13.90</td>
<td>97.95</td>
</tr>
<tr>
<td>600</td>
<td>80.13</td>
<td>15.29</td>
<td>95.42</td>
<td>84.67</td>
<td>12.69</td>
<td>97.35</td>
</tr>
<tr>
<td>800</td>
<td>75.67</td>
<td>16.29</td>
<td>91.96</td>
<td>84.33</td>
<td>11.84</td>
<td>96.17</td>
</tr>
<tr>
<td>1000</td>
<td>74.95</td>
<td>16.44</td>
<td>91.39</td>
<td>83.70</td>
<td>11.45</td>
<td>95.15</td>
</tr>
<tr>
<td>Average</td>
<td>79.75</td>
<td>15.35</td>
<td>95.10</td>
<td>84.40</td>
<td>12.79</td>
<td>97.19</td>
</tr>
</tbody>
</table>
# Numerical Results

## Computation time and Gap for MP and CP with 20% TWW and 100% TWW

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>Av Comp Time</th>
<th>Av Gap</th>
<th># unsolved</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MP</td>
<td>CP</td>
<td>MP</td>
</tr>
<tr>
<td>200</td>
<td>1.10</td>
<td>1.20</td>
<td>0.00%</td>
</tr>
<tr>
<td>400</td>
<td>3.10</td>
<td>3.50</td>
<td>0.00%</td>
</tr>
<tr>
<td>600</td>
<td>6.80</td>
<td>13.10</td>
<td>0.00%</td>
</tr>
<tr>
<td>800</td>
<td>8.90</td>
<td>56.60</td>
<td>0.00%</td>
</tr>
<tr>
<td>1000</td>
<td>17.90</td>
<td>63.00</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>Av Comp Time</th>
<th>Av Gap</th>
<th># unsolved</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MP</td>
<td>CP</td>
<td>MP</td>
</tr>
<tr>
<td>200</td>
<td>28.10</td>
<td>321.10</td>
<td>0.00%</td>
</tr>
<tr>
<td>400</td>
<td>339.50</td>
<td>592.67</td>
<td>0.00%</td>
</tr>
<tr>
<td>600</td>
<td>2040.20</td>
<td>1232.88</td>
<td>0.00%</td>
</tr>
<tr>
<td>800</td>
<td>4666.40</td>
<td>2350.67</td>
<td>0.00%</td>
</tr>
<tr>
<td>1000</td>
<td>7707.00</td>
<td>2034.14</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

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A Bilevel Approach for Energy Pricing Problems
Numerical Results

(a) Peak Load for 20% TWW Instances

(b) Peak Load for 100% TWW Instances
Numerical Results

(c) Net Revenue for 20% TWW Instances

(d) Net Revenue for 100% TWW Instances
Numerical Results

(e) Load Distribution, MP

(f) Load Distribution, CP

(g) Leader Prices, MP

(h) Leader Prices, CP

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## Conclusion

### Our Work

- Innovative approach for DSM
- Explicitly integrated customer response into the optimization process of the supplier
- Efficient heuristic approach to provide good solutions

### What else is Possible?

- Stochastic approach (prices, demand)
- Renewable Energy Integration
- Robust modelling
THANK YOU FOR LISTENING
Q & A
Inverse Optimization

\[
\max_p \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in} p^h \tilde{x}_{n,a}^h
\]

s.t.

\[
\begin{align*}
0 \leq p^h &\leq p^h_{\text{max}} & \forall h \in H \\
-w_{n,a}^h + v_{n,a} - p^h & = C_{n,a}^h & \forall n \in N, \forall a \in A_n, \forall h \in \tilde{x}_{n,a}^h > 0 \\
-w_{n,a}^h + v_{n,a} - p^h & \leq C_{n,a}^h & \forall n \in N, \forall a \in A_n, \forall h \in \tilde{x}_{n,a}^h = 0 \\
w_{n,a}^h & = 0 & \forall n \in N, \forall a \in A_n, \forall h \in \tilde{x}_{n,a}^h < \beta_{n,a}^\text{max} \\
w_{n,a}^h & \geq 0 & \forall n \in N, \forall a \in A_n, \forall h \in \tilde{x}_{n,a}^h = \beta_{n,a}^\text{max} \\
v_{n,a} & = 0 & \forall n \in N, \forall a \in A_n \sum_{h \in} \tilde{x}_{n,a}^h > E_{n,a} \\
v_{n,a} & \geq 0 & \forall n \in N, \forall a \in A_n \sum_{h \in} \tilde{x}_{n,a}^h = E_{n,a}
\end{align*}
\]
Minimum Peak

\[
\begin{align*}
\min_{\Gamma, x} & \quad \Gamma \\
\text{s.t.} & \quad \Gamma \geq \sum_{n \in N} \sum_{a \in A_n} x_{n,a}^h & \forall h \in H \\
0 \leq x_{n,a}^h & \leq \beta_{n,a}^\text{max} & \forall n \in N, \forall a \in A_n, \forall h \in H \\
\sum_{h \in \mathcal{H}} x_{n,a}^h & \geq E_{n,a} & \forall n \in N, \forall a \in A_n
\end{align*}
\]
Fixed Peak

\[
\min_{p, \Gamma} \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in H} C_{n, a}^h x_{n, a}^h \\
\text{s.t. } \Gamma' = \sum_{n \in N} \sum_{a \in A_n} x_{n, a}^h \\
0 \leq x_{n, a}^h \leq \beta_{n, a}^{\text{max}} \\
\sum_{h \in H} x_{n, a}^h \geq E_{n, a} \\
\forall h \in H \\
\forall n \in N, \forall a \in A_n, \forall h \in H \\
\forall n \in N, \forall a \in A_n
\]