Bilevel Models for Energy Pricing Problems

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Smart Energy and Stochastic Optimization
ENSTA ParisTech and Ecole des Ponts Paris Tech

Outline

Bilevel Programming

- Hierarchical decision making process with opposite interests.
- "Cake-Cutting Game"

Bilevel Programming

To deal with decision processes involving two decision-makers with a hierarchical structure:

- Two decision levels: a leader and a follower.
- The leaders sets his decision variables first. Then the folllower reacts based on the choices of the leader.
- Each decision-maker controls a set of variables subject to constraints and seeks to optimize a given objective function.
- Objective functions usually are in conflict and decision-makers do not cooperate.

Related to Principal Agent Paradigm, Equilibrium constrained Mathematical Program, Stackelberg Game

Mathematical Formulation

$$\max_{x} \; f(x)$$
$$\text{s.t.}$$
$$x \in X$$

$$\max_{x_1} \; f_1(x_1, x_2)$$
$$\text{s.t.}$$
$$(x_1, x_2) \in X$$
$$x_2 \text{ solves } \min_{x_2} \; f_2(x_1, x_2)$$
$$\text{s.t.}$$
$$(x_1, x_2) \in Y$$

The feasible region of the leader'sproblem is implicitly determined by an optimization problem.

A subset of variables are constrained to be an optimal solution of another optimization problem parameterized by the remaining variables

Pricing Problem

- The companies have to decide on prices of services to maximize revenue
- Companies have the choice among several companies to minimize their costs

Hierarchical decision making process with opposite interest.

$$
\begin{aligned}
\max_{p} \quad & p\ y \\
\text{s.t.} \quad & \\
& (p, y) \in X \\
& y \text{ solves } \min_{y} \quad C(p, y) \\
& \text{s.t.} \\
& (p, y) \in Y
\end{aligned}
$$

Main properties of bilevel programs

For fixed x_1, the second level problem is defined as:

$$x_2 \in P(x_1) = \arg\min_y \{ f_2(x_1, y) \ s.t. (x_1, y) \in Y \}$$

Non unicity of the follower solution:

- Optimistic case : The leader is able to influence the second level decision maker so that the latter always selects y to provide the best value of f.
- Pessimistic case : The leader behaves as thought the follower always selected the optimal decision which gives the worst value of his objective function

Main properties of bilevel programs

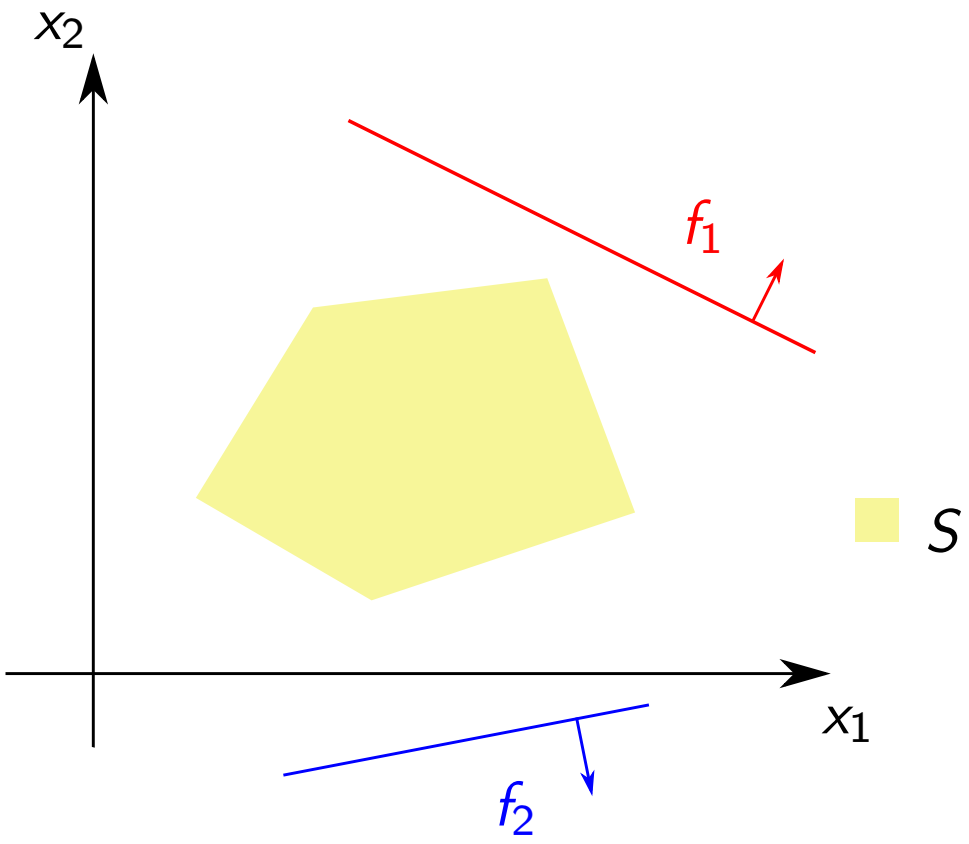
Bilevel programs are intrinsically hard to solve.

- Leader's domain is generally non convex and discontinuous

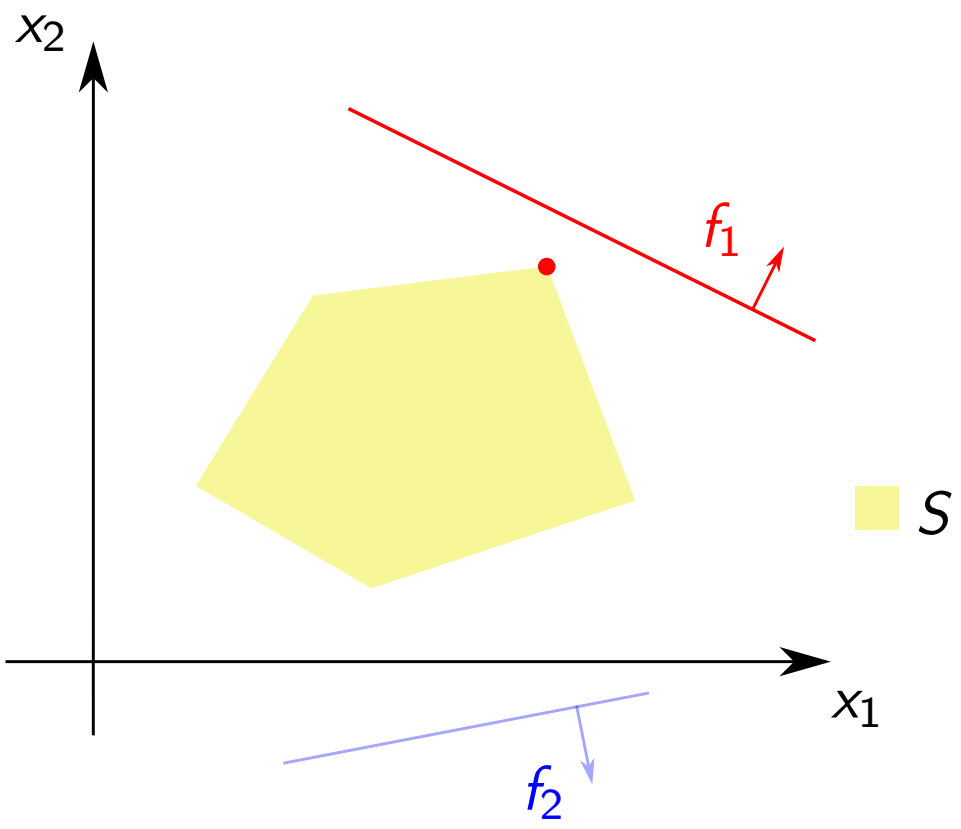
$$IR = \{(x_1, x_2) s.t. (x_1, x_2) \in X, x_2 \in P(x_1)\}$$

- $x_2(x_1)$ is generally non differentiable
- Optimal solution non pareto-optimal
- Linear Linear bilevel programs:
  - Strongly NP-Hard
  - Checking strict or local optimality is Strongly NP-Hard

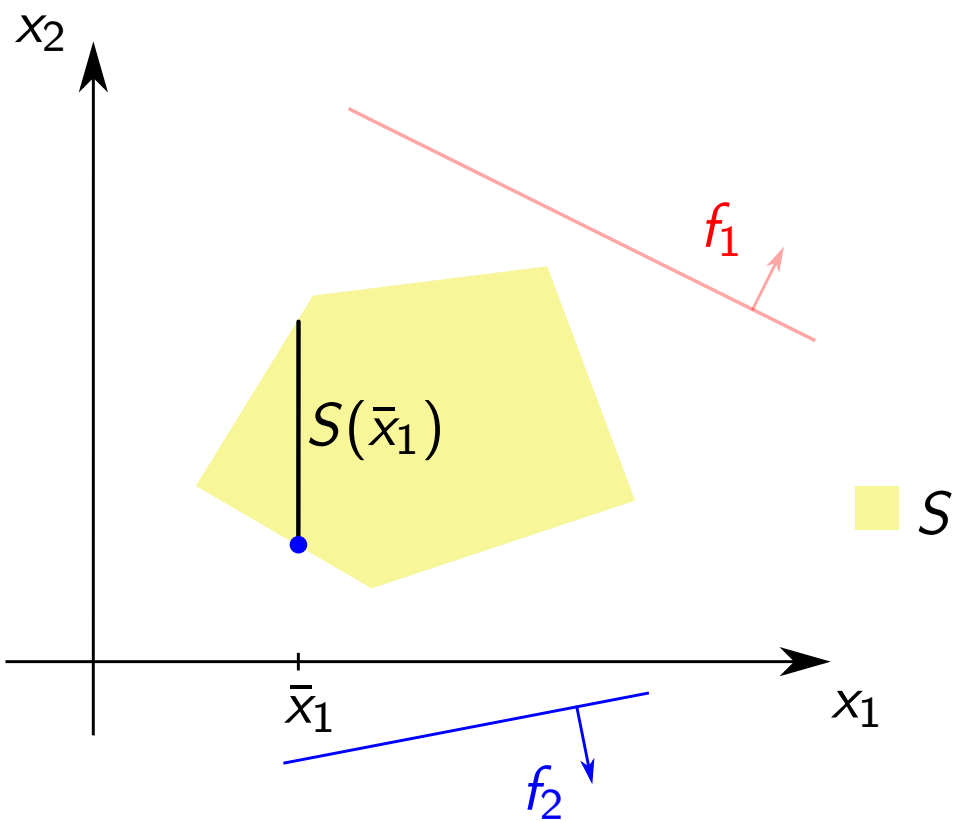
Linear Bilevel Program



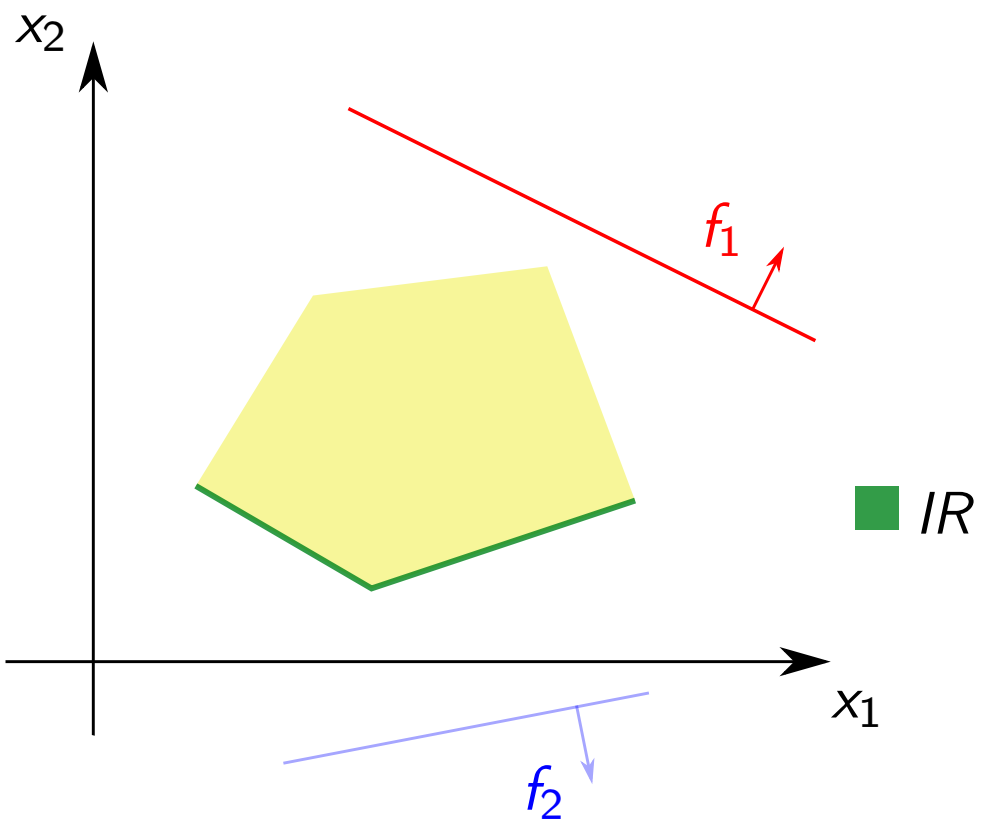
Linear Bilevel Program



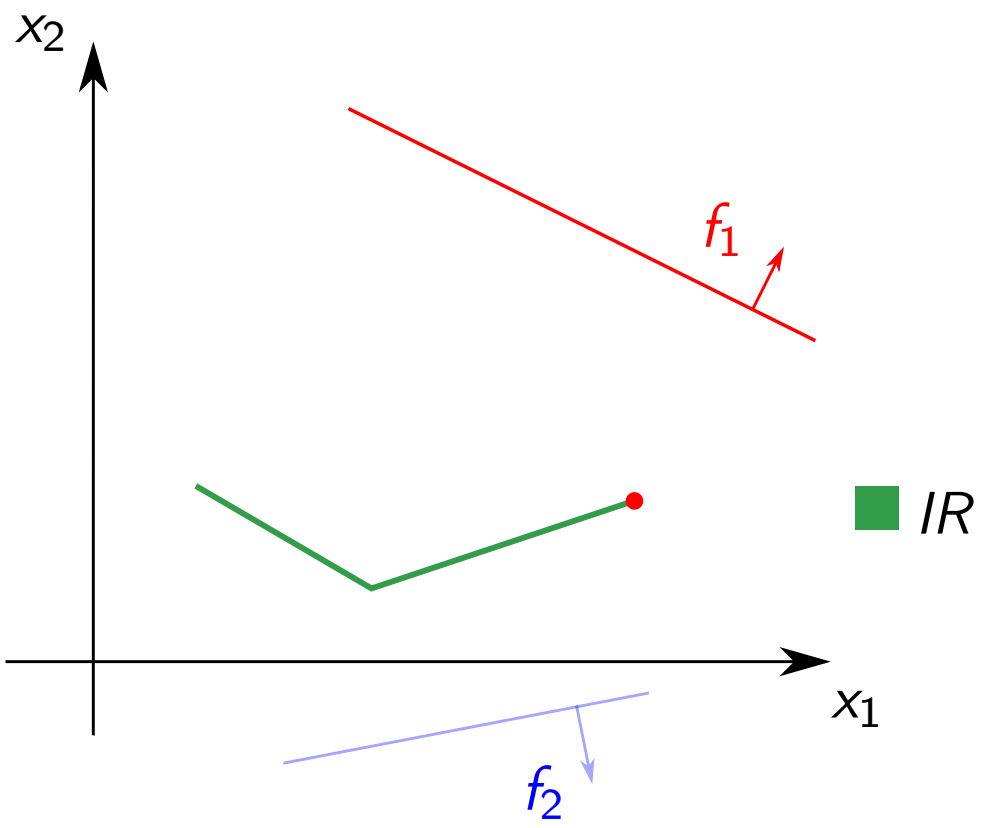
Linear Bilevel Program



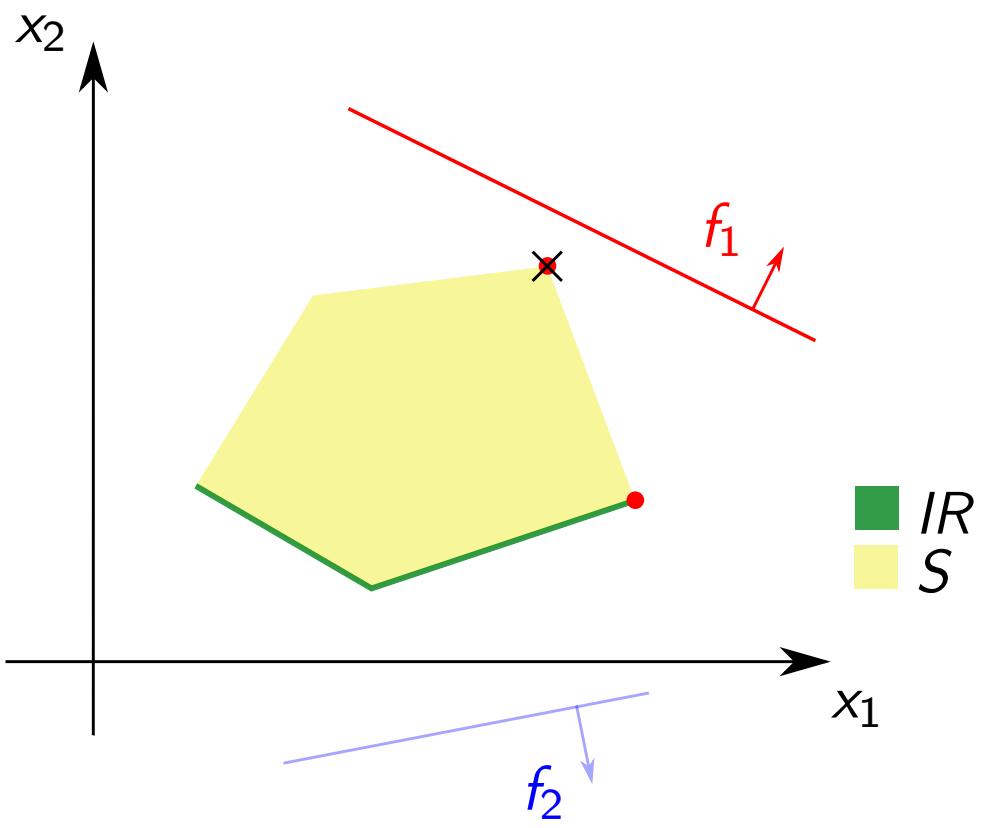
Linear Bilevel Program



Linear Bilevel Program



Linear Bilevel Program



Main properties of bilevel programs

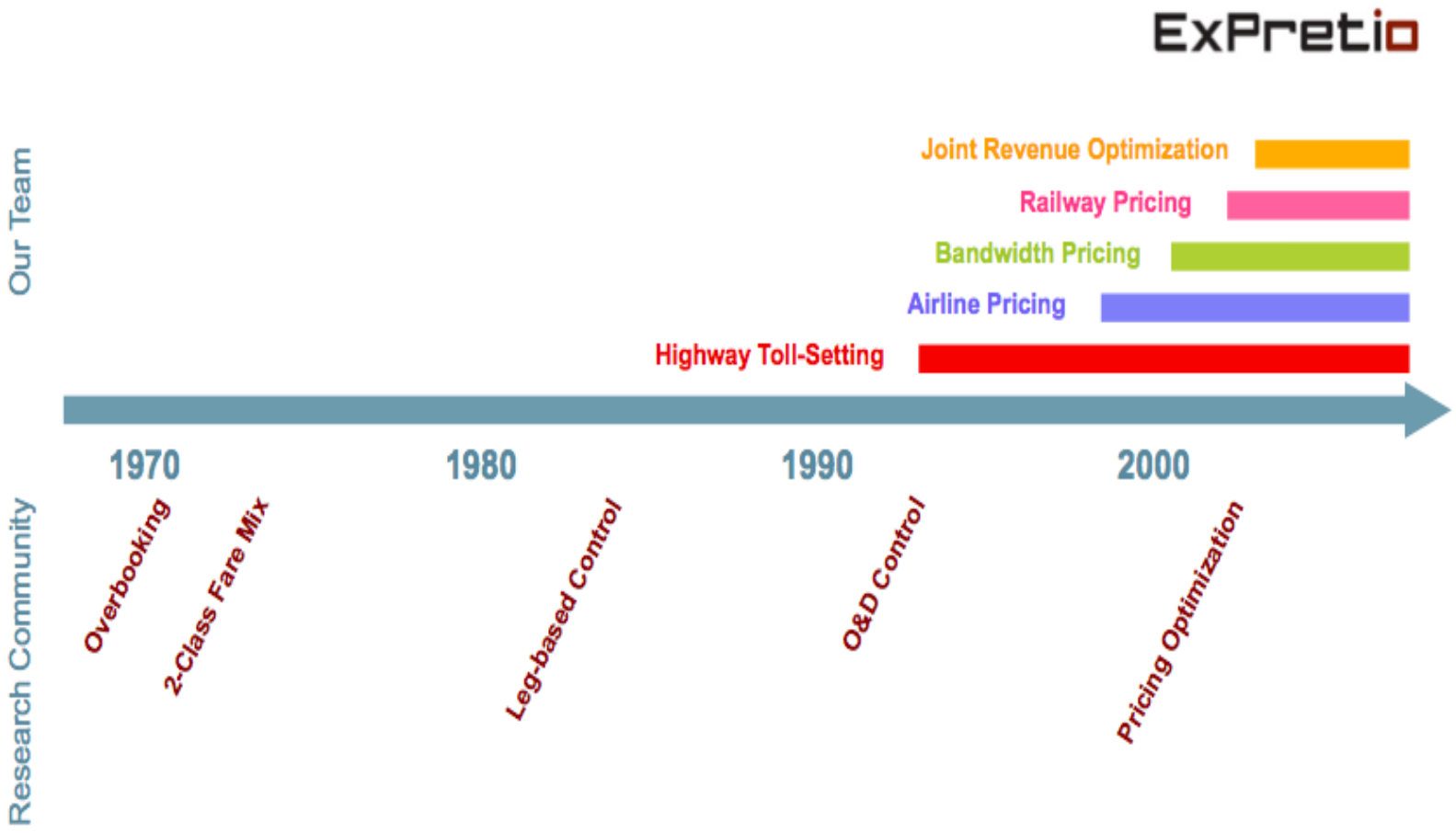
Algorithms.

- General Case:
  - Penalty methods,
  - Descent methods
  - Metaheursitics
  - ...
- Linear Linear bilevel programs
  - Branch and Bound
  - Extremal Points
  - Descent methods
  - Penalty methods
  - ...

To solve large size instances the structure of the problem has to be sharply exploited.

Applications

- Pricing problems on networks
- Network design problems
- Location problems
- Product bundling problems
- Security games
- Demand side management in the energy field
- ..


ExPretio
Our Team
Joint Revenue Optimization
Railway Pricing
Bandwidth Pricing
Airline Pricing
Highway Toll-Setting
1970
1980
1990
2000
Research Community
Overbooking
2-Class Fare Mix
Leg-based Control
O&D Control
Pricing Optimization

Bilevel energy Pricing Problems

Outline

- Demand Side Management
- Smart Grid
- Bilevel Pricing models
- Numerical Results

Demand Side Management

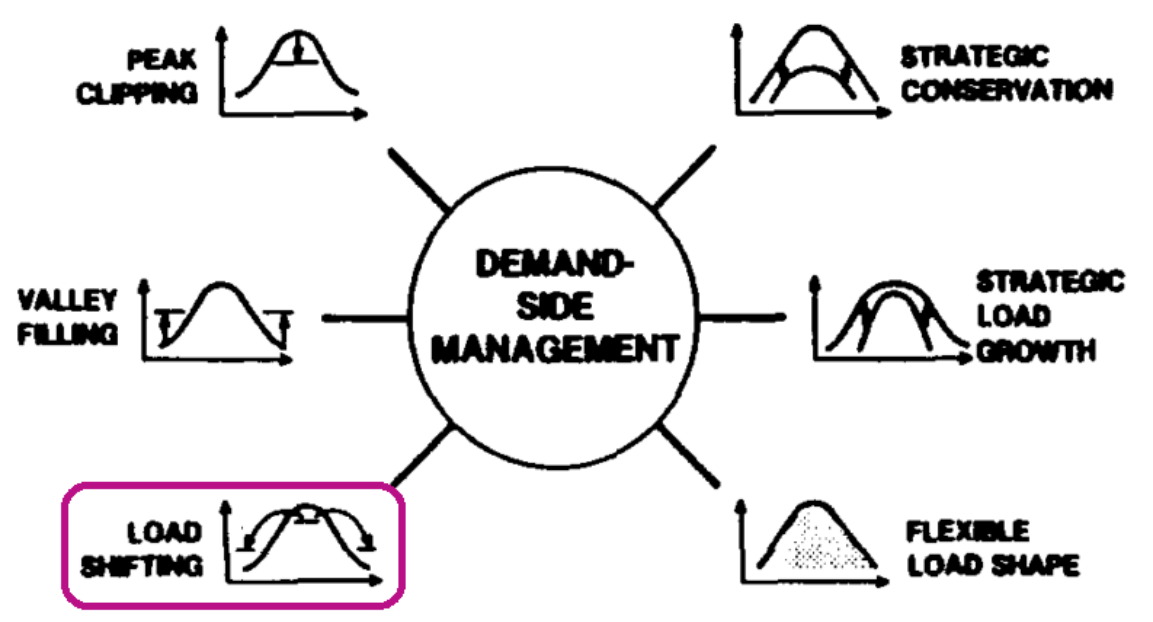
Motivation

- Demand for energy is largely uncontrollable and varies with time of day and season.
  - In UK, given the average demand across the year, the average utilization of the generation capacity is $\leq 55\%$.
  - Minimum demand in summer nights $\tilde{=}$ 30% of the winter peak
- Energy is difficult to store in large quantities.
- Supply-demand balance failure $\rightarrow$ system instability
- Total capacity of installed generation must be $\geq$ max demand to ensure the security of supply.

Demand Side Management

Why Necessary?

- DSM: **control** and **manipulate** the demand to meet capacity constraints.
- DSM's role: To improve the efficiency of operation and investment in the system.



[4]C.W. Gellings. The concept of demand side management for electric utilities. *Proceedings of the IEEE*, 73(10):14681470, 1985.

Demand Side Management

Major DSM Techniques

- Direct load control, load limiters, load switching
- Commercial/industrial programs
- Demand bidding
- Time-of-use pricing
- Smart grid
  "The smart grid is expected to revolutionize electricity generation, transmission, and distribution allowing two-way flows for both electrical power and information. Moreover it can complement the current electric grid system by including renewable energy sources" (Bari et al.)

Smart Grid Technology

- Every customer is equipped with a device that can receive, process and transfer data: smart meter
- Smart meters communicate with each other $\rightarrow$ smart grid
- Meters are programmable according to the needs of the customer
- Smart grid receives prices from the supplier(s), demand from the customers and schedules the consumption

Bari, Jiang, Saas, Jaekel. Challenges in the Smart Grid Applications: An overview. *International Journal of Distributed Sensor Networks*, 2014.
Siano. Demand response and smart grids a survey. *Renewable and Sustainable Energy Reviews*, 30, 2014.
Reddy, Veloso. Learned Behaviors of Multiple Autonomous Agents in Smart Grid Markets. *Proceedings of the 25 th AAAI Conference of Artificial Intelligence*, 2011.

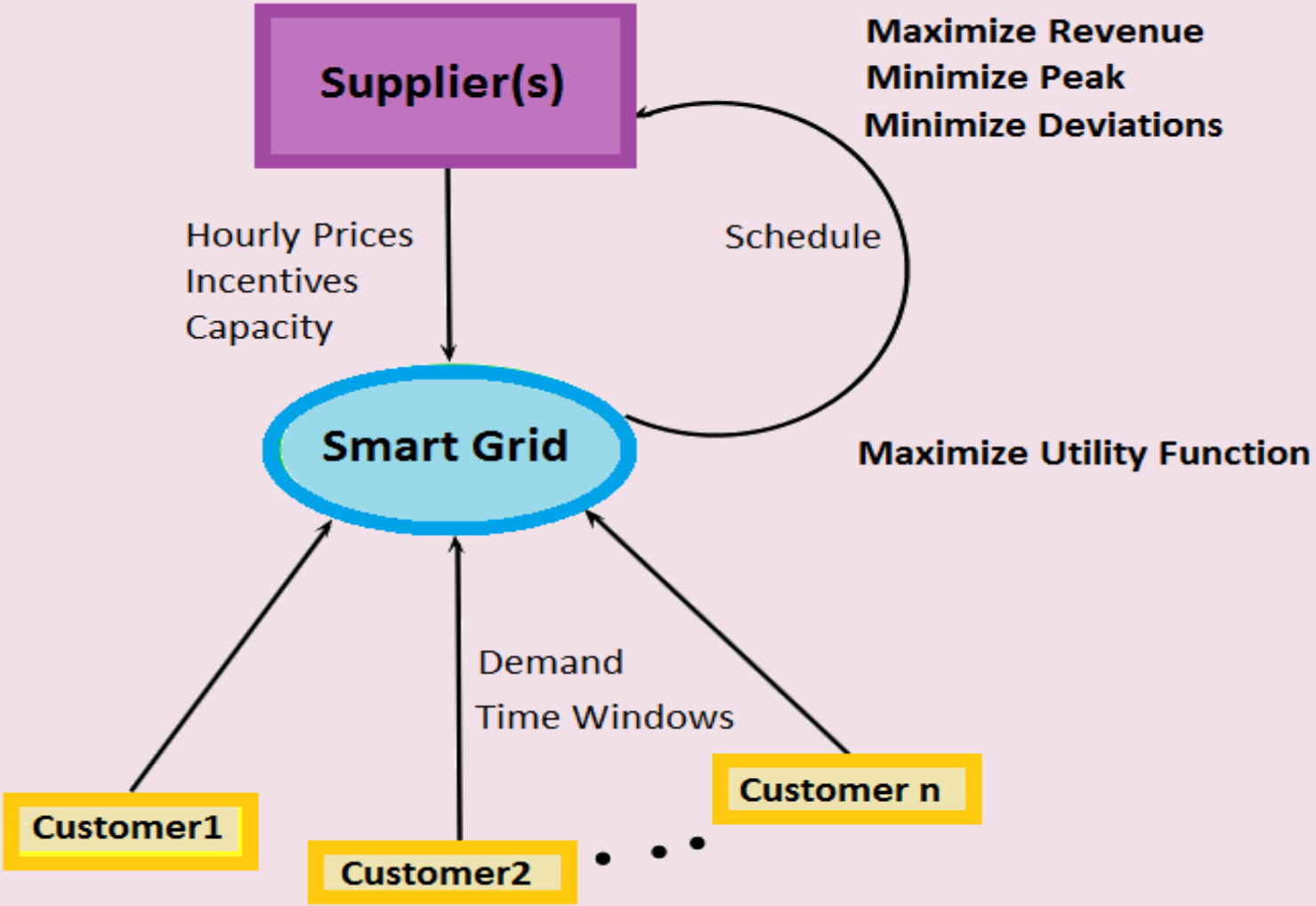
Demand Side Management

New Challenge

"Commitment to market based operation and deregulation of the electricity industry places consumers of electricity in the center of the decision-making process regarding the operation and future development of the system" -G. Strbac, 2008

Energy Pricing Problem

Bilevel Approach

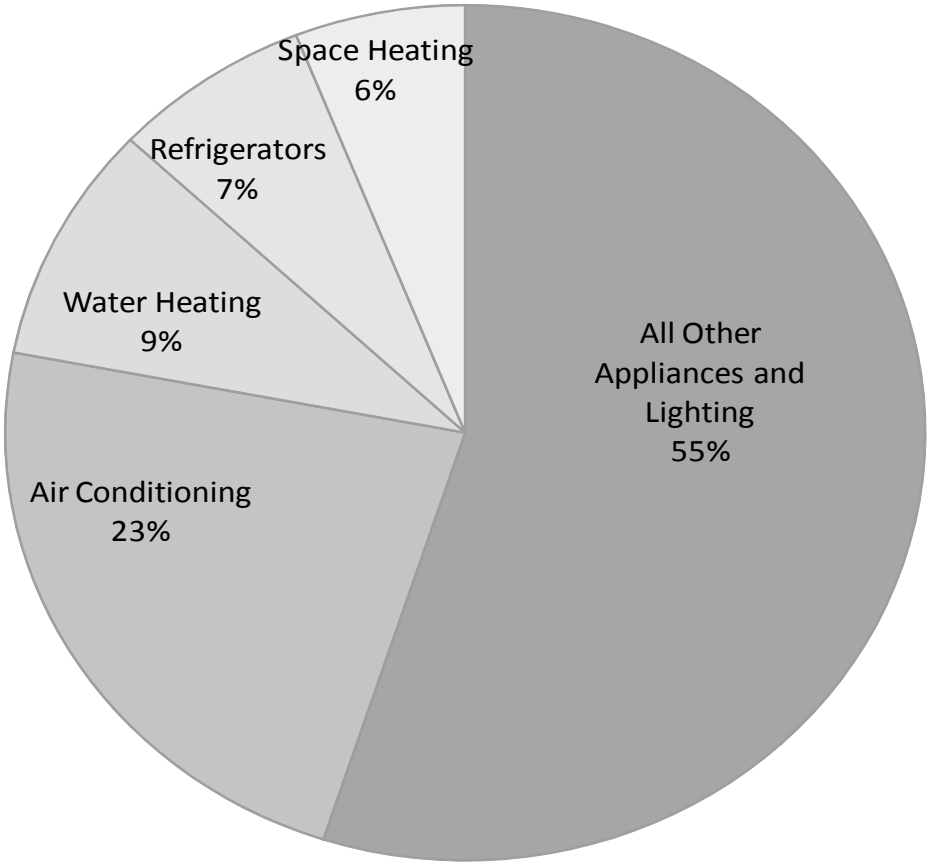


Energy Pricing Problem

Properties

- Stackelberg game / Bilevel programming
- The supplier (upper level) and a group of customers connected to a smart grid (lower level).
- Prices from supplier and demand with time windows from customers are received by the smart grid.
- Time-of-use price control to minimize peak demand and maximize revenue.
- Demand-response to hourly changing prices to minimize cost and waiting time.

Residential Electricity Consumption



Energy Pricing Problem

Objectives

- Leader maximizes (revenue - peak cost) by deciding on prices
- Follower minimizes (billing cost + waiting cost) by deciding on the schedule of consumption.

Assumptions

- A fixed upper bound for prices.
- Demand is fixed.
- All operations are preemptive.
- Every customer has a set of appliances with preferred time windows.
- All appliances have power consumption limits.
- One cycle has 24 time slots(hours) .

Bilevel Models

- H: Set of time slots
- p^h: Price for time slot h
- Γ: Peak Load
- κ: Peak weight factor
- N : Set of customers
- A_n: Set of devices for customer n
- $\beta^{\max}_{n,a}$: Power Limit of device a for customer n
- $x^h_{n,a}$: Power level for each appliance a in time slot h for customer n
- $E_{n,a}$: Demand of customer n for appliance a
- $[TW^b_{n,a}, TW^e_{n,a}]$: Time Window of customer n for appliance a
- λ_n : Delay sensitivity coefficient of customer n
- $C_{n,a}(h)$:Inconvenience cost for customer n if appliance a is at h

$$C_{n,a}(h) := \lambda_n \times E_{n,a} \times \frac{(h - TW^b_{n,a})}{(TW^e_{n,a} - TW^b_{n,a})}$$
$$\forall n \in N, \forall a \in A_n, \forall h \in H.$$

Bilevel Model - Preemptive Monopolistic

$$\max_{p,\Gamma} \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in H} p^h x^h_{n,a} - \kappa\Gamma$$

$$\text{s.t. } \Gamma \geq \sum_{\substack{n\in N, a\in A_n \\ h\in H}} x^h_{n,a} \qquad \forall h \in H$$

$$0 \leq p^h \leq p^h_{\max} \qquad \forall h \in H$$

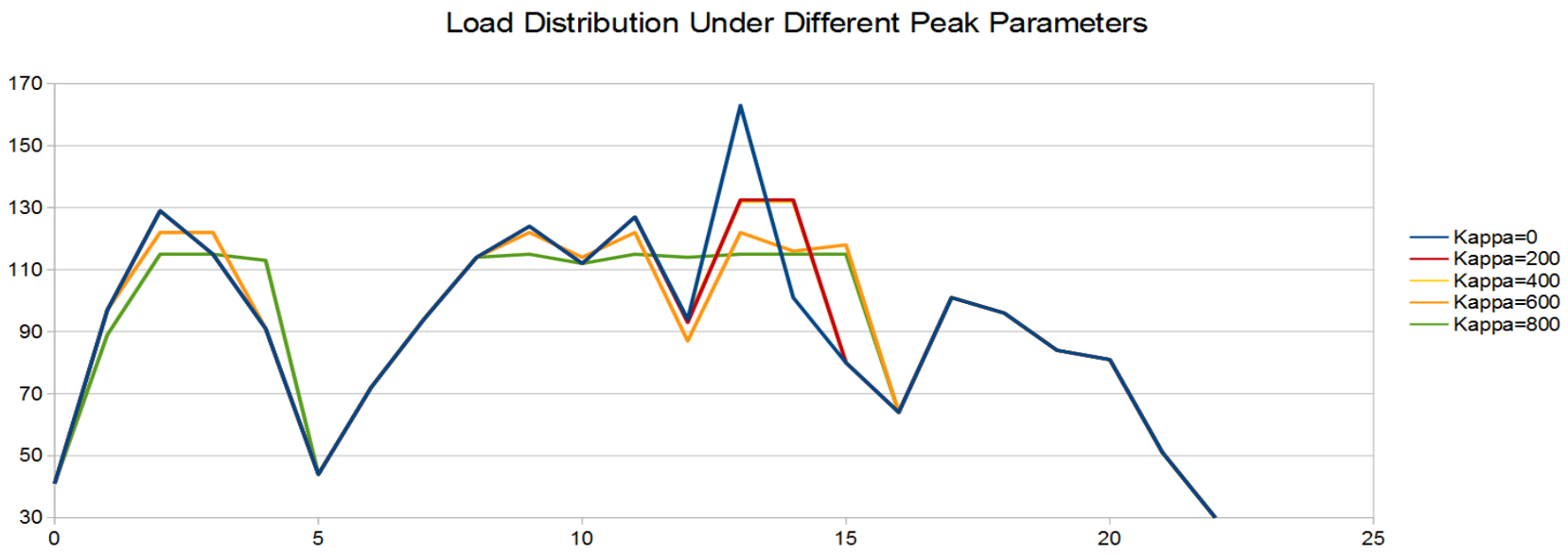
$$\min_{x} \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in H} (p^h + C_{n,a}(h)) x^h_{n,a}$$

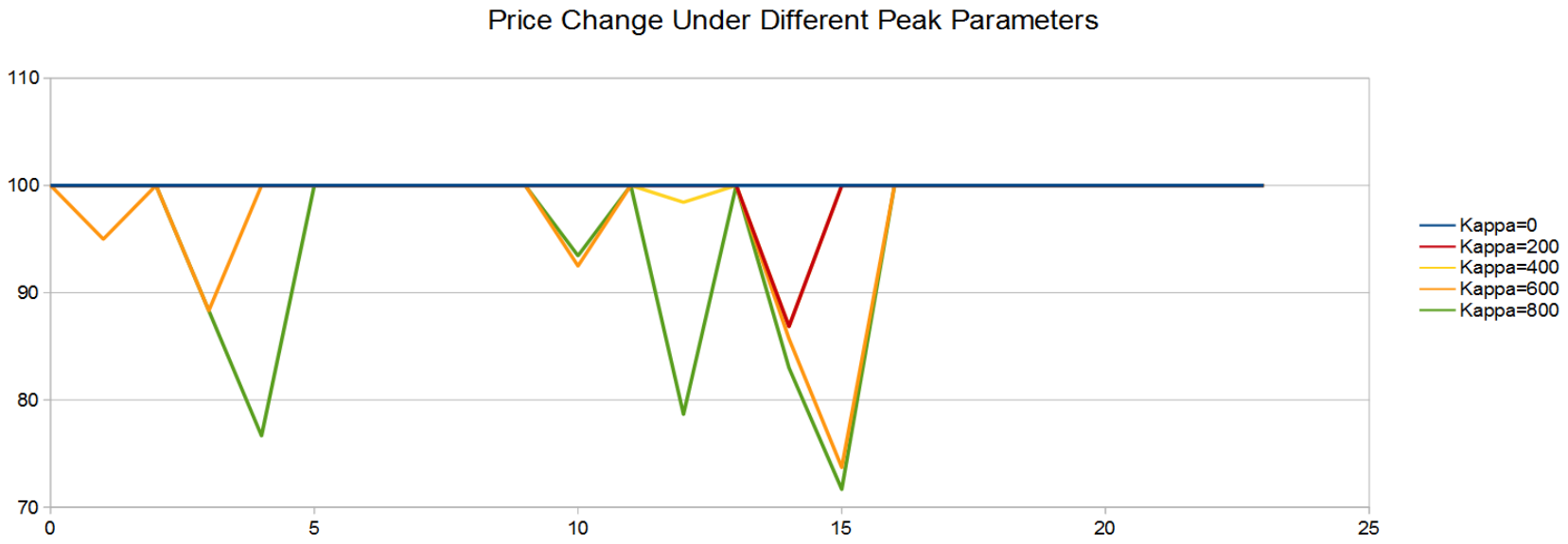
$$\text{s.t. } x^h_{n,a} \leq \beta^{\max}_{n,a} \qquad \forall n \in N, \forall a \in A_n, \forall h \in H$$

$$\sum_{h\in} x^h_{n,a} \geq E_{n,a} \qquad \forall n \in N, \forall a \in A_n,$$

$$x^h_{n,a} \geq 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in H$$

Bilevel Model - Preemptive Monopolistic





Bilevel Model - Preemptive Competitive

$$\max_{p,\Gamma} \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in H} p^h x^h_{n,a} - \kappa\Gamma$$

$$\text{s.t. } \Gamma \geq \sum_{\substack{n\in N, a\in A_n \\ h\in H}} x^h_{n,a} \qquad \forall h$$

$$0 \leq p^h \leq p^h_{\max} \qquad \forall h \in H$$

$$\min_{x,\bar{x}} \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in H} (p^h + C_{n,a}(h)) x^h_{n,a}$$

$$+ \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in H} (\bar{p}^h + C_{n,a}(h)) \bar{x}^h_{n,a}$$

$$\text{s.t. } x^h_{n,a} + \bar{x}^h_{n,a} \leq \beta^{\max}_{n,a} \qquad \forall n \in N, \forall a \in A_n, \forall h \in H$$

$$\sum_{h\in} (x^h_{n,a} + \bar{x}^h_{n,a}) \geq E_{n,a} \qquad \forall n \in N, \forall a \in A_n$$

$$x^h_{n,a}, \bar{x}^h_{n,a} \geq 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in H,$$

Exact Solution Method

Single Level formulation

- Optimality conditions (primal,dual and complementarity constraints) of the follower
- Complementary slackness constraints $\rightarrow$ linearized using binary variables
- Objective function is linearized using the follower's dual objective function.
- Single level MIP

Exact Solution Method

Single Level formulation

Dual variables are $w^h_{n,a}$ and $v_{n,a}$

The dual constraints associated with$x^h_{n,a}$ are:

$$-w^h_{n,a} + v_{n,a} - p^h \leq C_{n,a}(h) \quad \forall n \in N, \forall a \in A_n, \forall h \in H$$

Complementarity slackness between $x^h_{n,a}$ and the dual constraints are

$$x^h_{n,a}(w^h_{n,a} - v_{n,a} + p^h + C_{n,a}(h)) = 0 \ \ \forall n \in N, \forall a \in A_n, \forall h \in H$$

To linearize this constraints, consider binary variables $\psi^h_{n,a}$

$$w^h_{n,a} - v_{n,a} + p^h + C_{n,a}(h) \leq M_1(1 - \psi^h_{n,a})$$

$$x^h_{n,a} \leq M_1 \psi^h_{n,a}$$

$$\psi^h_{n,a} \in \{0, 1\}$$

Numerical Results

- Scenarios with 10 customers, 3 appliances for each customer (thus 30 appliances).
- Schedule horizon: 24 time slots.
- Peak weight (κ) : 200, 400, 600, 800 and 1000.
- TWW 20% and 100% $\rightarrow$ the time window of each job is 20% or 100% larger than the minimum completion time (MCT).
- MCT is the minimum number of time slots required to meet demand ($E_{n,a}$) if we could use all devices at their maximum level

Numerical Results

- For the experiments, CPLEX version 12.3 is used on a computer with 2.66 GHz Intel 283 Xeon CPU and 4 GB RAM, running under the Windows 7 operating system.
- Average value over 10 instances.
- The base case (BC) : solution where prices are equal to $p_{\max}$ and appliances are scheduled to the preferred time slots.
- The user costs are split between electricity bill (EB) and inconvenience (EC)
- Percentages relative to the base case (BC).

Numerical Results

Costs for MP and CP ; 20% TWW and 100% TWW instances

κ	MP EB	MP IC	MP TC	CP EB	CP IC	CP TC
200	78.02	21.49	99.51	78.31	21.17	99.48
400	77.15	21.59	98.74	78.01	20.13	98.14
600	75.76	21.85	97.61	77.84	18.74	96.58
800	73.52	22.16	95.68	78.07	16.81	94.88
1000	71.50	22.48	93.99	77.63	16.28	93.91
Average	75.19	21.91	97.10	77.97	18.63	96.60

κ	MP EB	MP IC	MP TC	CP EB	CP IC	CP TC
200	85.12	14.16	99.28	85.25	14.06	99.30
400	82.87	14.55	97.42	84.05	13.90	97.95
600	80.13	15.29	95.42	84.67	12.69	97.35
800	75.67	16.29	91.96	84.33	11.84	96.17
1000	74.95	16.44	91.39	83.70	11.45	95.15
Average	79.75	15.35	95.10	84.40	12.79	97.19

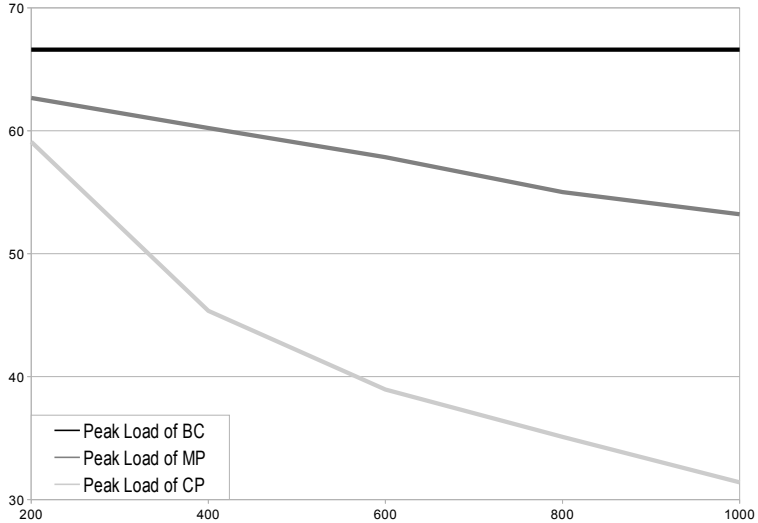
Numerical Results

Computation time and Gap for MP and CP with 20% TWW and 100% TWW

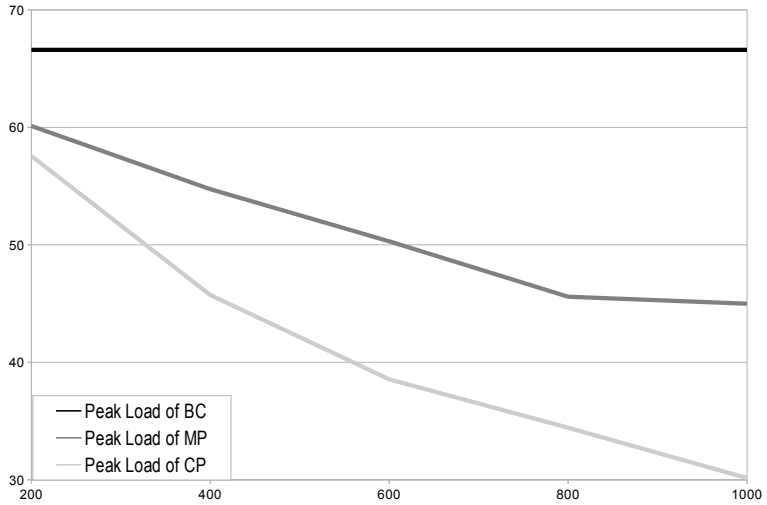
	Av Comp Time		Av Gap		# unsolved	
(κ)	MP	CP	MP	CP	MP	CP
200	1.10	1.20	0.00%	0.00%	0	0
400	3.10	3.50	0.00%	0.00%	0	0
600	6.80	13.10	0.00%	0.00%	0	0
800	8.90	56.60	0.00%	0.00%	0	0
1000	17.90	63.00	0.00%	0.00%	0	0

	Av Comp Time		Av Gap		# unsolved	
(κ)	MP	CP	MP	CP	MP	CP
200	28.10	321.10	0.00%	0.00%	0	0
400	339.50	592.67	0.00%	0.55%	0	1
600	2040.20	1232.88	0.00%	3.87%	0	2
800	4666.40	2350.67	0.00%	3.73%	0	4
1000	7707.00	2034.14	0.00%	6.04%	0	3

Numerical Results

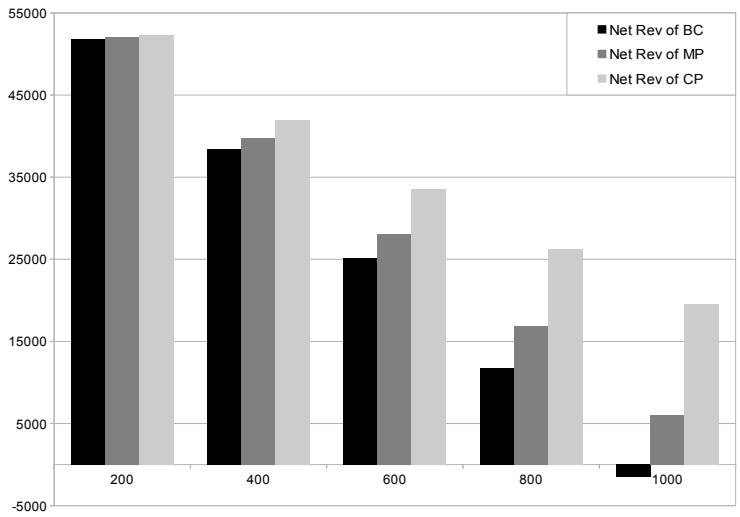


(a) Peak Load for 20% TWW Instances

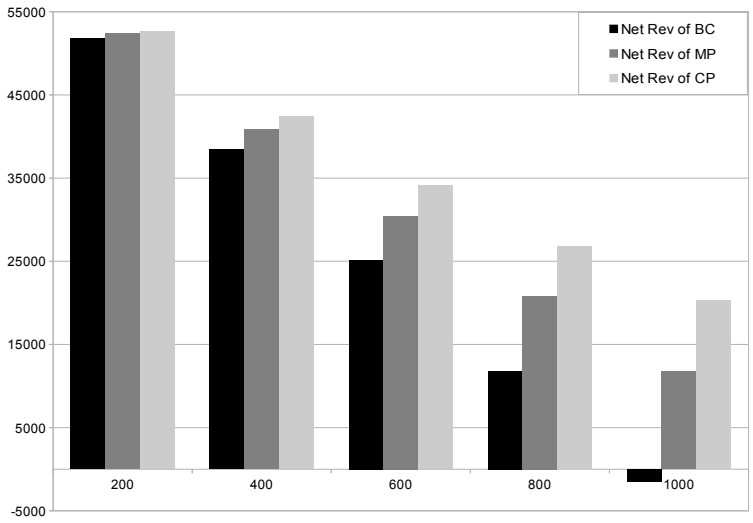


(b) Peak Load for 100% TWW Instances

Numerical Results

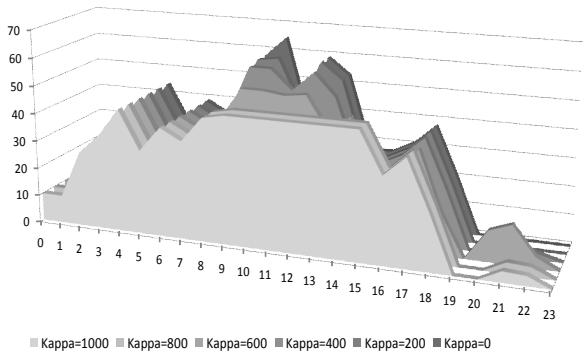


(c) Net Revenue for 20% TWW Instances

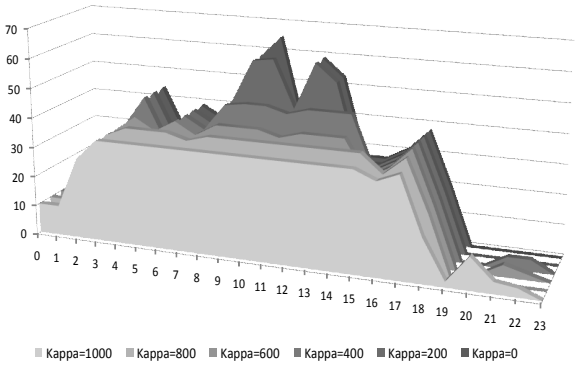


(d) Net Revenue for 100% TWW Instances

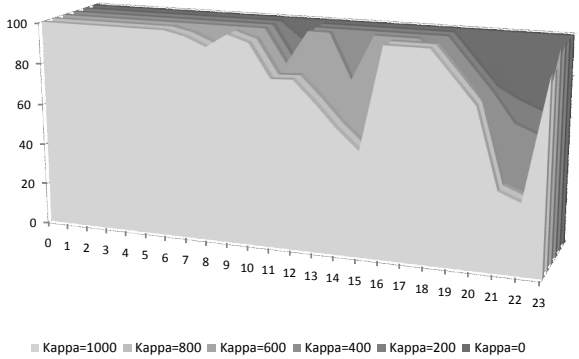
Numerical Results



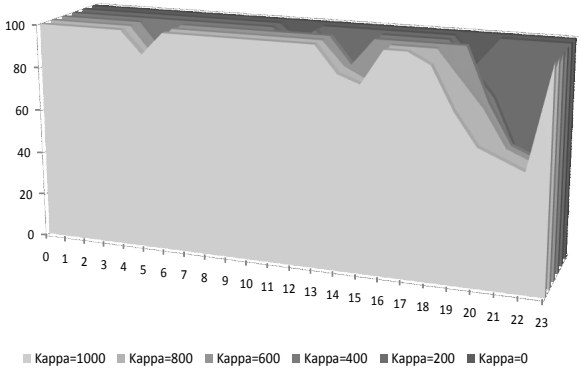
(e) Load Distribution, MP



(f) Load Distribution, CP



(g) Leader Prices, MP



(h) Leader Prices, CP

Conclusion

Our Work

- Innovative approach for DSM
- Explicitly integrated customer response into the optimization process of the supplier
- Efficient heuristic approach to provide good solutions

What else is Possible?

- Stochastic approach (prices, demand)
- Renewable Energy Integration
- Robust modelling

THANK YOU FOR LISTENING
Q & A

Inverse Optimization

$$\max_{p} \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in} p^h \tilde{x}^h_{n,a}$$

s.t.

$$\begin{aligned}
& 0 \leq p^h \leq p^h_{\max} && \forall h \in H \\
& -w^h_{n,a} + v_{n,a} - p^h = C^h_{n,a} && \forall n \in N, \forall a \in A_n, \forall h \in \tilde{x}^h_{n,a} > 0 \\
& -w^h_{n,a} + v_{n,a} - p^h \leq C^h_{n,a} && \forall n \in N, \forall a \in A_n, \forall h \in \tilde{x}^h_{n,a} = 0 \\
& w^h_{n,a} = 0 && \forall n \in N, \forall a \in A_n, \forall h \in \tilde{x}^h_{n,a} < \beta^{\max}_{n,a} \\
& w^h_{n,a} \geq 0 && \forall n \in N, \forall a \in A_n, \forall h \in \tilde{x}^h_{n,a} = \beta^{\max}_{n,a} \\
& v_{n,a} = 0 && \forall n \in N, \forall a \in A_n \sum_{h \in} \tilde{x}^h_{n,a} > E_{n,a} \\
& v_{n,a} \geq 0 && \forall n \in N, \forall a \in A_n \sum_{h \in} \tilde{x}^h_{n,a} = E_{n,a}.
\end{aligned}$$

Minimum Peak

$$
\begin{aligned}
\min_{\Gamma, x} \quad & \Gamma \\
\text{s.t.} \quad & \Gamma \geq \sum_{n \in N} \sum_{a \in A_n} x^h_{n,a} && \forall h \in H \\
& 0 \leq x^h_{n,a} \leq \beta^{max}_{n,a} && \forall n \in N, \forall a \in A_n, \forall h \in \\
& \sum_{h \in} x^h_{n,a} \geq E_{n,a} && \forall n \in N, \forall a \in A_n
\end{aligned}
$$

Fixed Peak

$$\min_{p,\Gamma} \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in} C^h_{n,a} x^h_{n,a}$$

$$\text{s.t. } \Gamma' = \sum_{n\in N} \sum_{a\in A_n} x^h_{n,a} \qquad \forall h \in H$$

$$0 \leq x^h_{n,a} \leq \beta^{max}_{n,a} \qquad \forall n \in N, \forall a \in A_n, \forall h \in$$

$$\sum_{h\in} x^h_{n,a} \geq E_{n,a} \qquad \forall n \in N, \forall a \in A_n$$

Bibliography