Two time scales stochastic dynamic optimization

Managing energy storage investment, aging and operation

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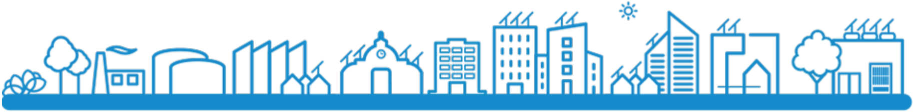
Optimization for subway stations

Paris subway stations consumption $=$ 40, 000 houses

Subway stations have unexploited energies that can be harnessed through electrical storage

We use stochastic optimization for short term control and long term aging and invesment management of batteries



École des Ponts ParisTech efficacity

Outline

1. Context: Electrical storage management in subway stations
   - Why electrical storage in subway stations?
   - Managing storage short term operations
   - Battery operation impacts long term aging!
2. Modeling: Management of batteries operation, aging and renewal
   - Two time scales management: investment/operation
   - Short term operation model
   - Long term renewals model
   - Two time scales stochastic optimization problem
3. Solving: Decomposition method and numerical results
   - Decomposition method
   - Numerical results

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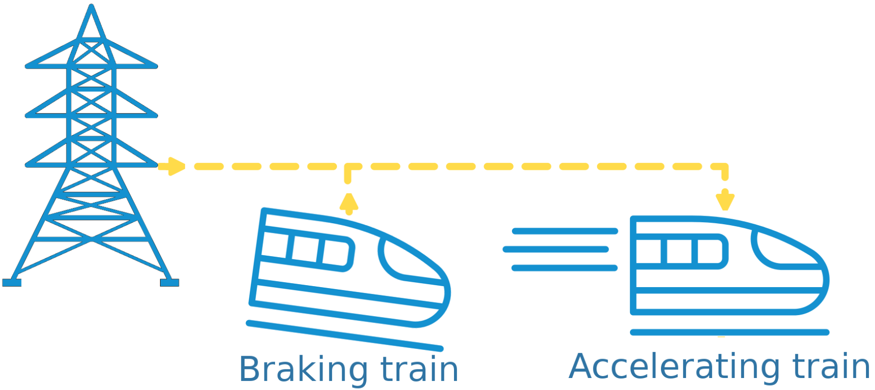
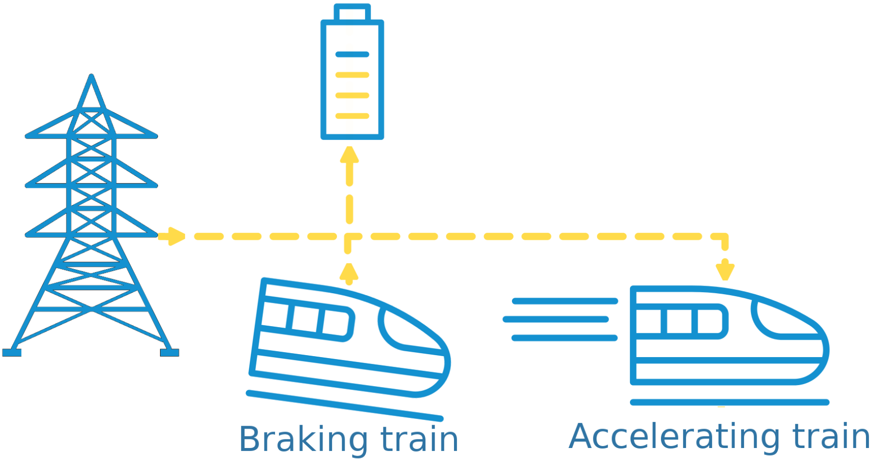
Why electrical storage in subway stations?

Subway stations have unexploited energy ressources

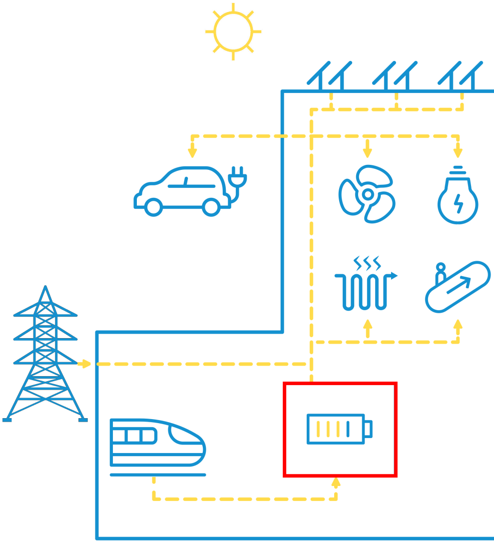


Energy recovery requires a buffer



Managing storage short term operations

Microgrid concept for subway stations



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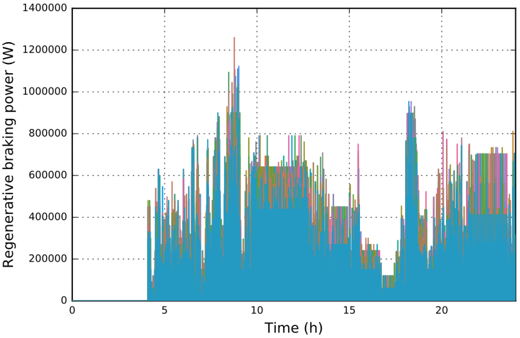
Stochastic optimization is relevant

Subways braking energy is unpredictible



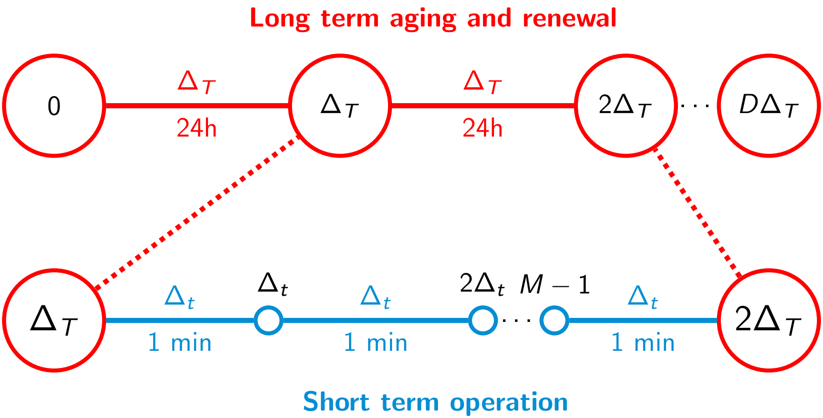
We can optimize battery operations using
Stochastic Dynamic Programming

Battery operation impacts long term aging!

Outline

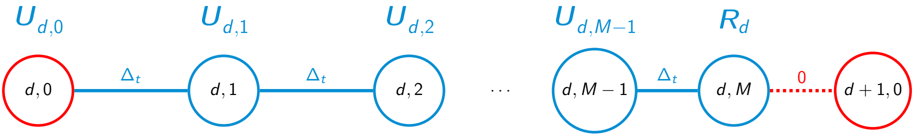
Two time scales management: investment/operation

Two time scales



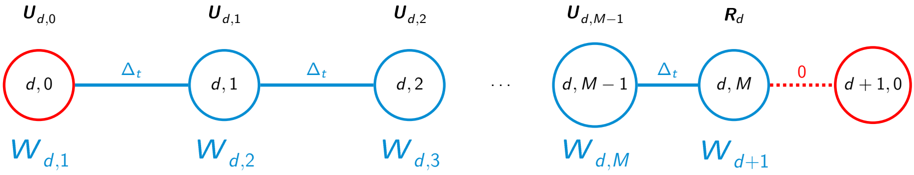
We make decisions every minutes m and every day d

- Day d, Minute m: How much energy $\boldsymbol{U}_{d,m}$ do I charge or discharge from my current battery with capacity $\boldsymbol{C}_d$?
- At the end of Day d should I buy a new battery with capacity $\boldsymbol{R}_d$?



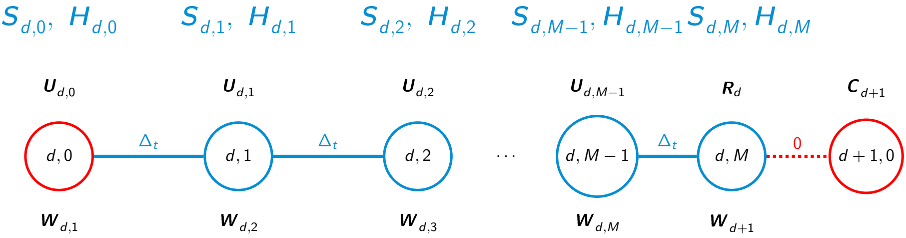
Uncertain events occur right after we made our decisions

- Day d, end of Minute m: we observe how much intermitent energy $\boldsymbol{W}_{d,m+1}$ we receive
- At the end of Day d we observe the batteries cost $\boldsymbol{W}_{d+1}$ on the market



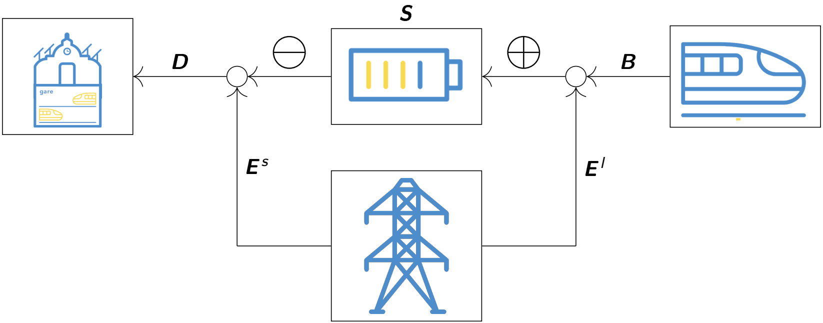
Decisions and uncertainty impact state variables

- Day d, end Minute m: decision $\boldsymbol{U}_{d,m}$ and realization $\boldsymbol{W}_{d,m+1}$ change our battery state of charge $\boldsymbol{S}_{d,m}$ to $\boldsymbol{S}_{d,m+1}$ and our battery state of health $\boldsymbol{H}_{d,m}$ to $\boldsymbol{H}_{d,m+1}$
- At the end of Day d decision $\boldsymbol{R}_d$ change our battery capacity $\boldsymbol{C}_d$ to $\boldsymbol{C}_{d+1}$



Short term operation model

Electrical network representation



Station node

- D: Demand station
- E^s: From grid to station
- $\ominus$: Discharge battery

Subways node

- B: Braking
- E^l: From grid to battery
- $\oplus$: Charge battery

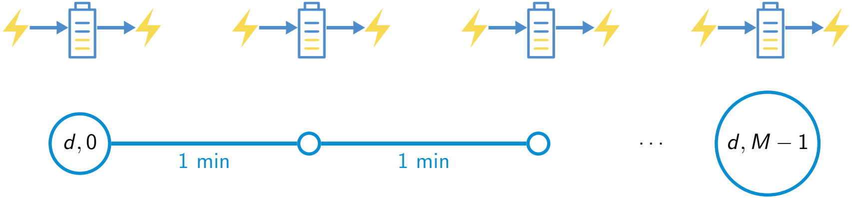
Battery state of charge dynamics

For a given charge/discharge strategy $\boldsymbol{U}$ over a day d:

$$\boldsymbol{S}_{d,m+1} = \boldsymbol{S}_{d,m} - \underbrace{\frac{1}{\rho_d}\boldsymbol{U}^-_{d,m}}_{\ominus} + \underbrace{\rho_c sat(\boldsymbol{S}_{d,m}, \boldsymbol{U}^+_{d,m}, \boldsymbol{B}_{d,m+1})}_{\oplus}$$

with

$$sat(x, u, b) = \min(\frac{S_{max} - x}{\rho_c}, \max(u, b))$$



Battery aging dynamics

For a given charge/discharge strategy $\boldsymbol{U}$ over a day d

$$\boldsymbol{H}_{d,m+1} = \boldsymbol{H}_{d,m} - \frac{1}{\rho_d}\boldsymbol{U}^{-}_{d,m} - \rho_c sat(\boldsymbol{S}_{d,m}, \boldsymbol{U}^{+}_{d,m}, \boldsymbol{B}_{d,m+1})$$

Every minute we save energy and money

If we have a battery on day d and minute m we save:

$$p^e_{d,m}\Big(\underbrace{\boldsymbol{E}^s_{d,m+1} + \boldsymbol{E}^l_{d,m+1} - \boldsymbol{D}_{d,m+1}}_{\text{Saved energy}}\Big)$$

$p^e_{d,m}$ is the cost of electricity on day d at minute m

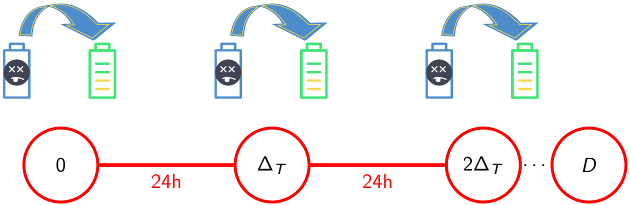
Summary of short term/Fast variables model

We call, at day d and minute m,

- fast state variables: $\boldsymbol{X}^f_{d,m} = \begin{pmatrix} \boldsymbol{S}_{d,m} \\ \boldsymbol{H}_{d,m} \end{pmatrix}$
- fast decision variables: $\boldsymbol{U}^f_{d,m} = \begin{pmatrix} \boldsymbol{U}^-_{d,m} \\ \boldsymbol{U}^+_{d,m} \end{pmatrix}$
- fast random variables: $\boldsymbol{W}^f_{d,m} = \begin{pmatrix} \boldsymbol{B}_{d,m} \\ \boldsymbol{D}_{d,m} \end{pmatrix}$
- fast cost function: $L^f_{d,m}(\boldsymbol{X}^f_{d,m}, \boldsymbol{U}^f_{d,m}, \boldsymbol{W}^f_{d,m+1})$
- fast dynamics: $\boldsymbol{X}^f_{d,m+1} = F^f_{d,m}(\boldsymbol{X}^f_{d,m}, \boldsymbol{U}^f_{d,m}, \boldsymbol{W}^f_{d,m+1})$

Long term renewals model

We decide our battery purchases at the end of each day



Should we replace our battery $\boldsymbol{C}_d$ by buying a new one $\boldsymbol{R}_d$ or not?

$$\boldsymbol{C}_{d+1} = \begin{cases} \boldsymbol{R}_d, \text{ if } \boldsymbol{R}_d > 0 \\ f(\boldsymbol{C}_d, \boldsymbol{H}_{d,M}), \text{ otherwise} \end{cases}$$

paying renewal cost $\boldsymbol{P}_d^b \boldsymbol{R}_d$ at uncertain market prices $\boldsymbol{P}_d^b$

Summary of long term/Slow variables model

We call, at day d,

- slow state variables: $\boldsymbol{X}^s_d = (\boldsymbol{C}_d)$
- slow decision variables: $\boldsymbol{U}^s_d = (\boldsymbol{R}_d)$
- slow random variables: $\boldsymbol{W}^s_d = (\boldsymbol{P}^b_d)$
- slow cost function: $L^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}) = \boldsymbol{P}^b_d \boldsymbol{R}_d$
- slow dynamics: $\boldsymbol{X}^s_{d+1} = F^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1})$

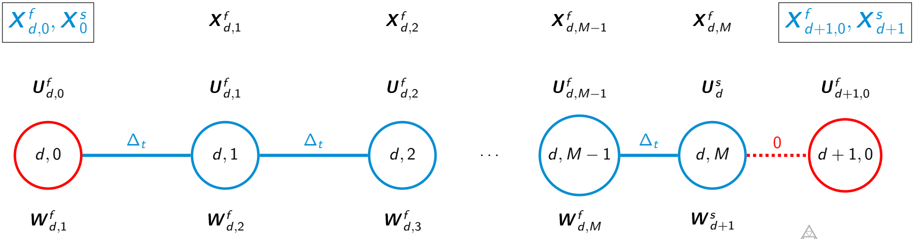
The link between time scales

The initial "fast state" at the begining of day d deduces from:

$$\boldsymbol{X}^f_{d,0} = \phi_d(\boldsymbol{X}^s_d, \boldsymbol{X}^f_{d-1,M})$$

The initial "slow state" at the begining of day $d+1$ deduces from all that happened the previous day:

$$\boldsymbol{X}^s_{d+1} = F^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:})$$



We formulate a two time scales stochastic optimization problem

We minimize fast and slow costs over the long term

$$\min_{\boldsymbol{X}^f, \boldsymbol{X}^s, \boldsymbol{U}^f, \boldsymbol{U}^s} \mathbb{E}\Big[\sum_{d=0}^{D-1}\Big(\sum_{m=0}^{M-1} L^f_{d,m}(\boldsymbol{X}^f_{d,m}, \boldsymbol{U}^f_{d,m}, \boldsymbol{W}^f_{d,m+1})\Big) + L^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:})\Big]$$

$$\boldsymbol{X}^f_{d,m+1} = F^f_{d,m}(\boldsymbol{X}^f_{d,m}, \boldsymbol{U}^f_{d,m}, \boldsymbol{W}^f_{d,m+1})$$

$$\boldsymbol{X}^f_{d,0} = \phi_d(\boldsymbol{X}^s_d, \boldsymbol{X}^f_{d-1,M})$$

$$\boldsymbol{X}^s_{d+1} = F^s_d(\boldsymbol{X}^s_d, \boldsymbol{U}^s_d, \boldsymbol{W}^s_{d+1}, \boldsymbol{X}^f_{d,0}, \boldsymbol{U}^f_{d,:}, \boldsymbol{W}^f_{d,:})$$

$$\boldsymbol{U}^f_{d,m} \preceq \mathcal{F}_{d,m}$$

$$\boldsymbol{U}^s_d \preceq \mathcal{F}_{d,M}$$

Stochastic optimal control reformulation

We call

$$\boldsymbol{X}_d = (\boldsymbol{X}^f_{d-1,M}, \boldsymbol{X}^s_d)$$
$$\boldsymbol{U}_d = (\boldsymbol{U}^f_{d,:}, \boldsymbol{U}^s_d)$$
$$\boldsymbol{W}_d = (\boldsymbol{W}^f_{d-1,:}, \boldsymbol{W}^s_d)$$

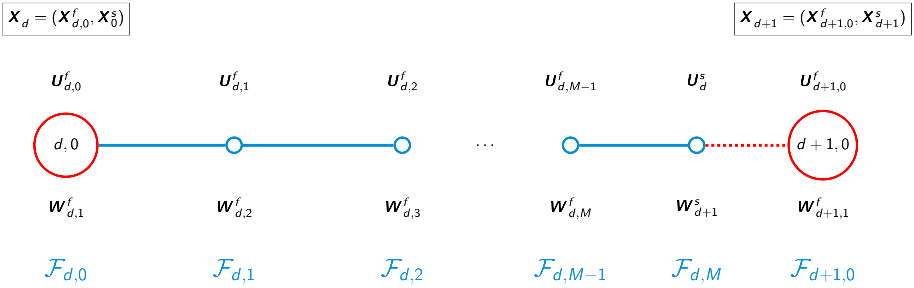
we can reformulate the problem as

$$\min_{\boldsymbol{X},\boldsymbol{U}} \mathbb{E}\Big[\sum_{d=0}^{D-1} L_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1})\Big]$$
$$\boldsymbol{X}_{d+1} = F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1})$$
$$\boldsymbol{U}^f_{d,m} \preceq \mathcal{F}_{d,m}$$
$$\boldsymbol{U}^s_d \preceq \mathcal{F}_{d,M}$$

where the non-anticipativity constraints are not standard

Information flow model

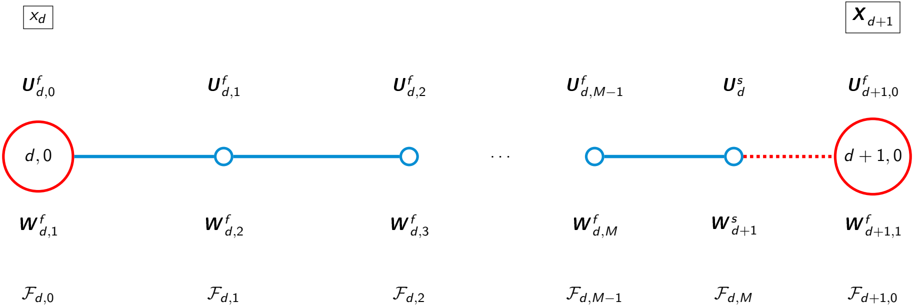
$$\mathcal{F}_{d,m} = \sigma\begin{pmatrix} \boldsymbol{W}^f_{d',m'},\ d'<d,\ m'\leq M+1 \\ \boldsymbol{W}^s_{d'},\ d'\leq d \\ \boldsymbol{W}^f_{d,m'},\ m'\leq m \end{pmatrix} = \sigma\begin{pmatrix} \text{previous days fast noises} \\ \text{previous days slow noises} \\ \text{current day previous minutes fast noises} \end{pmatrix}$$



We can write a dynamic programming equation

When the $\boldsymbol{W}_d$ are independent

$$V_d(x_d) = \min_{\boldsymbol{U}_d} \mathbb{E}\left[L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) + V_{d+1}(\boldsymbol{X}_{d+1})\right]$$



With value functions defined inductively

Every day d, we can define a value function that factorizes as function of the state $\boldsymbol{X}_d$ if the $\boldsymbol{W}_d$ are independent.

$$\begin{aligned} V_d(x_d) = \min_{\boldsymbol{X}_{d+1}, \boldsymbol{U}_d} \ & \mathbb{E}\Big[L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) + V_{d+1}(\boldsymbol{X}_{d+1})\Big] \\ \text{s.t } & \boldsymbol{X}_{d+1} = F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) \\ & \boldsymbol{U}^f_{d,m} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:m}) \\ & \boldsymbol{U}^s_d \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:M}) \\ & \boldsymbol{U}_d = (\boldsymbol{U}^f_{d,:}, \boldsymbol{U}^s_d) \\ & \boldsymbol{X}_d = x_d \end{aligned}$$

The value of the whole problem being: $V_0(x_0)$.

How to decompose the problem
into
a daily optimization problem
and
an intraday optimization problem?

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Let's split the min

$$V_d(x_d) = \min_{\boldsymbol{X}_{d+1}} \min_{\boldsymbol{U}_d} \mathbb{E}\left[L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) + V_{d+1}(\boldsymbol{X}_{d+1})\right]$$

$$\text{s.t } \boldsymbol{X}_{d+1} = F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1})$$

$$\boldsymbol{U}^f_{d,m} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:m})$$

$$\boldsymbol{U}^s_d \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:M})$$

$$\boldsymbol{U}_d = (\boldsymbol{U}^f_{d,:}, \boldsymbol{U}^s_d)$$

$$\boldsymbol{X}_d = x_d$$

We hide the fast decisions variables

Inside the value of the intraday control problem ϕ_d

with fixed initial state x_d

with fixed stochastic final state $\boldsymbol{X}_{d+1}$

$$V_d(x_d) = \min_{\boldsymbol{X}_{d+1}} \Big[\overbrace{\phi_d(x_d, [\boldsymbol{X}_{d+1}])}^{\text{intraday problem}} + \overbrace{\mathbb{E} V_{d+1}(\boldsymbol{X}_{d+1})}^{\text{next expected value}} \Big]$$

$$\text{s.t } \boldsymbol{X}_{d+1} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}_{d+1})$$

Significant difficulties remain

- Computing $\phi_d(x_d, [\boldsymbol{X}_{d+1}])$ for every $\boldsymbol{X}_{d+1}$ is very expensive!
- $\boldsymbol{X}_{d+1} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}_{d+1})$

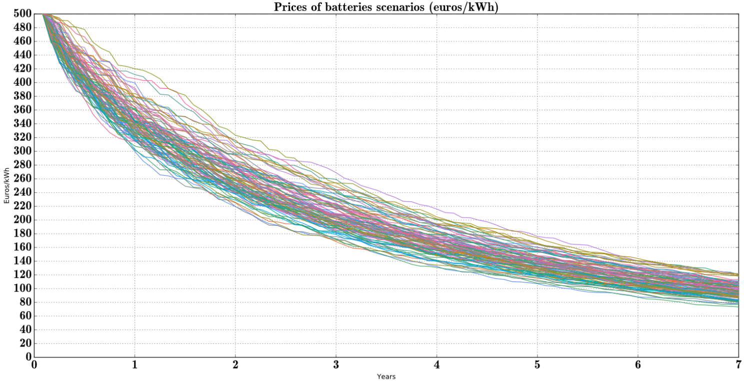
Then why is it interesting?

- We can solve the intraday problem ϕ_d with another method (DP, SDDP, SP, PH)
- We can exploit the problem periodicity ($\forall d,\ \phi_d = \phi_0$?)
- We can simplify measurability ($\boldsymbol{X}_{d+1} \preceq \sigma(\boldsymbol{X}_d)$)
- We can exploit value functions monotonicity (relax the coupling constraint $F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) \geq \boldsymbol{X}_{d+1}$) [2]

Numerical results

Synthetic price of batteries data

- Batteries cost stochastic model: synthetic scenarios that approximately coincide with market forecasts



Net Present Value

- 7 years horizon
- Yearly discount factor $=$ 0.95
- $10,000$ $\boldsymbol{C}^b$ scenarios to model randomness
- 1 buying/aging decision per month
- 1 charge/discharge decision every 15 min
- Constraint: having a battery everytime with at least one cycle a day

Objective: maximize expected discounted revenues over 7 years

Numerical method: Intraday DP + Extraday DP

We use DP for intraday decisions and another DP for end of the day decisions.

We exploit monotonicity (relax end of the day aging constraint), daily periodicity and we decide aging at the beginning of the day $\boldsymbol{X}_{d+1} \preceq \sigma(\boldsymbol{X}_d)$.

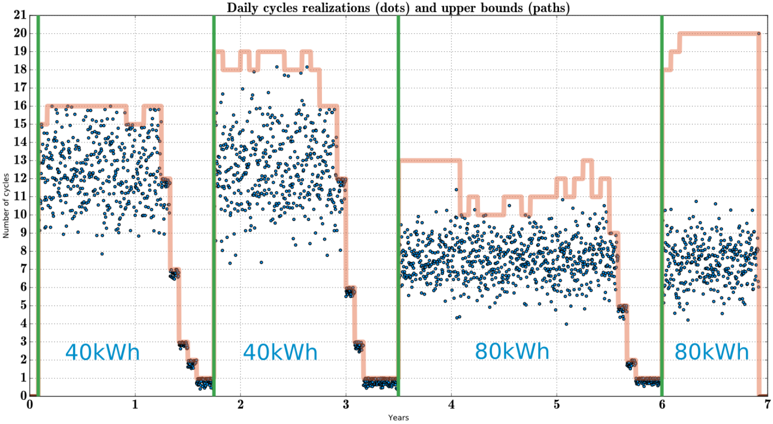
Results

	SDP	SDP + SDP
Offline comp. time	∞	1 min + 15 min
Simulation comp. time	?	[25s,30s]
Upper bound	?	+128k

In Julia with a Core I7, 1.7 Ghz, 8Go ram + 12Go swap SSD

1 simulation: cycles

NPV = 80,000 euros



Conclusion and ongoing work

Our study leads to the following conclusions:

- Controlling aging is relevant
- Our decomposition method provides encouraging results
- It can be used for aging aware intraday control
- We have to improve simplifications

We are now focusing on

- Improving risk modelling
- Improving batteries cost stochastic model
- Aging model with capacity degradation
- Dual decomposition of the coupling constraint

References

Pierre Haessig.
Dimensionnement et gestion d'un stockage d'énergie pour l'atténuation des incertitudes de production éolienne.
PhD thesis, Cachan, Ecole normale supérieure, 2014.

Benjamin Heymann, Pierre Martinon, and Frédéric Bonnans.
Long term aging : an adaptative weights dynamic programming algorithm.
working paper or preprint, July 2016.

Let's introduce an auxiliary variable

$$V_d(x_d) = \min_{\boldsymbol{Y}_{d+1}} \min_{\boldsymbol{X}_{d+1}} \min_{\boldsymbol{U}_d} \mathbb{E}\left[L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) + V_{d+1}(\boldsymbol{X}_{d+1})\right]$$

$$\begin{aligned}
\text{s.t } & \boldsymbol{X}_{d+1} = \boldsymbol{Y}_{d+1} \\
& F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) = \boldsymbol{Y}_{d+1} \\
& \boldsymbol{U}^f_{d,m} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:m}) \\
& \boldsymbol{U}^s_d \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:M}) \\
& \boldsymbol{U}_d = (\boldsymbol{U}^f_{d,:}, \boldsymbol{U}^s_d) \\
& \boldsymbol{X}_d = x_d \\
& \boldsymbol{Y}_{d+1} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}_{d+1})
\end{aligned}$$

Let's distribute the mins

$$
\begin{aligned}
V_d(x_d) = \min_{\boldsymbol{Y}_{d+1}} \; & \left[\min_{\boldsymbol{U}_d} \mathbb{E} \; L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) + \min_{\boldsymbol{X}_{d+1}} \mathbb{E} \; V_{d+1}(\boldsymbol{X}_{d+1}) \right] \\
\text{s.t } & \boldsymbol{X}_{d+1} = \boldsymbol{Y}_{d+1} \\
& F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) = \boldsymbol{Y}_{d+1} \\
& \boldsymbol{U}^f_{d,m} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:m}) \\
& \boldsymbol{U}^s_d \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:M}) \\
& \boldsymbol{U}_d = (\boldsymbol{U}^f_{d,:}, \boldsymbol{U}^s_d) \\
& \boldsymbol{X}_d = x_d \\
& \boldsymbol{Y}_{d+1} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}_{d+1})
\end{aligned}
$$

A first subproblem appears

For a given $\boldsymbol{Y}_{d+1} \in L^0(\Omega, \mathcal{F}, \mathbb{P})$, with $\sigma(\boldsymbol{Y}_{d+1}) \subset \sigma(\boldsymbol{X}_d, \boldsymbol{W}_{d+1})$,

$$
\begin{aligned}
\phi_d(x_d, [\boldsymbol{Y}_{d+1}]) = \min_{\boldsymbol{U}_d} \; & \mathbb{E}\; L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) \\
\text{s.t } & F_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_{d+1}) = \boldsymbol{Y}_{d+1} \\
& \boldsymbol{U}^f_{d,m} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:m}) \\
& \boldsymbol{U}^s_d \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}^f_{d,1:M}) \\
& \boldsymbol{U}_d = (\boldsymbol{U}^f_{d,:}, \boldsymbol{U}^s_d) \\
& \boldsymbol{X}_d = x_d
\end{aligned}
$$

We use the notation $f([\boldsymbol{W}])$ to emphasize that f's domain is $L^0(\Omega, \mathcal{F}, \mathbb{P})$.
This is the intraday problem with stochastic final state constraint!

Substitute in the dynamic programming equation

$$V_d(x_d) = \min_{\boldsymbol{Y}_{d+1}} \left[\phi_d(x_d, [\boldsymbol{Y}_{d+1}]) + \min_{\boldsymbol{X}_{d+1}} \mathbb{E}\ V_{d+1}(\boldsymbol{X}_{d+1})\right]$$

$$\text{s.t } \boldsymbol{X}_{d+1} = \boldsymbol{Y}_{d+1}$$

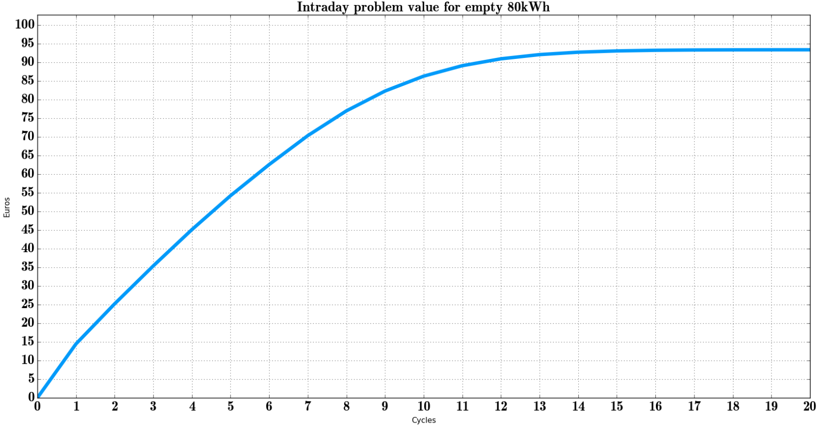
$$\boldsymbol{Y}_{d+1} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}_{d+1})$$

Finally let's eliminate this unecessary auxialiary variable

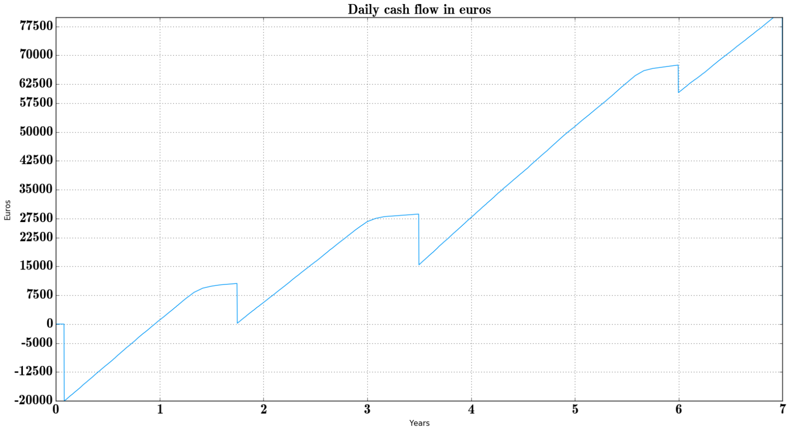
$$V_d(x_d) = \min_{\boldsymbol{X}_{d+1}} \left[\phi_d(x_d, [\boldsymbol{X}_{d+1}]) + \mathbb{E}\ V_{d+1}(\boldsymbol{X}_{d+1})\right]$$

$$\text{s.t } \boldsymbol{X}_{d+1} \preceq \sigma(\boldsymbol{X}_d, \boldsymbol{W}_{d+1})$$

Intraday value for empty 80 kWh battery



Cash flow


Daily cash flow in euros
77500
70000
62500
57500
50000
42500
35000
27500
22500
15000
7500
0
-5000
-12500
-20000
Euros
0
1
2
3
4
5
6
7
Years