Stochastic HUC

via

Multi-level Scenario Trees

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joint with

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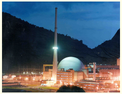
Energy Optimization

Optimal management of a power mix, given that

– Power can be generated by different technologies



– The production of electricity needs to be **coordinated**.

Short-term coordination: unit commitment

- Optimal scheduling (next day) of generation units **coupled** by system-wide constraints
- Declined in many different versions
  - Bilateral or centralized market frameworks
  - System with hydro/thermal/nuclear utilities
  - Intermittent sources (sun and wind)
- Uncertain intermittent and run-of-river generation
- Other sources of uncertainty
  - energy demand
  - unit availability
  - energy prices
- A large-scale stochastic nonlinear problem with 0-1 variables

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- Declined in many different versions
  - Bilateral or centralized market frameworks
  - System with hydro/thermal/nuclear utilities
  - **Intermittent sources (sun and wind) renewable is nice, but ...**
- Uncertain intermittent and run-of-river generation
- (Most common) sources of uncertainty
  - renewable generation (water inflows, wind, sun),
  - energy demand
  - unit availability
  - energy prices
- A large-scale **stochastic nonlinear** problem with 0-1 variables

Impact of intermittent sources

- Wind is unpredictable
  - Intra-hour variability
- Batteries provide have scalable and flexible storage systems:
  - dynamics to charge/discharge
- Demand-side management:
  - to smooth rapid woltage swings, when customers go on and off the grid massively (sunset!)

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To reflect these features, the UC mathematical optimization model is
mixed 0-1, stochastic, with nonlinear relations

Our HUC formulation: pieces of the puzzle

- Where is the nonconvexity
- How we represent uncertainty
- Which decomposition method we put in place
- Tricks to make it work

Results and comments

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- Where is the nonconvexity
  - **hydro-production function**
- How we represent uncertainty
  - **multi-level tree with a fan**
- Which decomposition method we put in place
  - **Benders'-like**
- Tricks to make it work
  - **Lower convex-hull**
  - **Shrewd bundle stabilization**
  - **IPOPT with good starting point**

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Results and comments on **simulation**

Productivity of hydro-units is nonconvex

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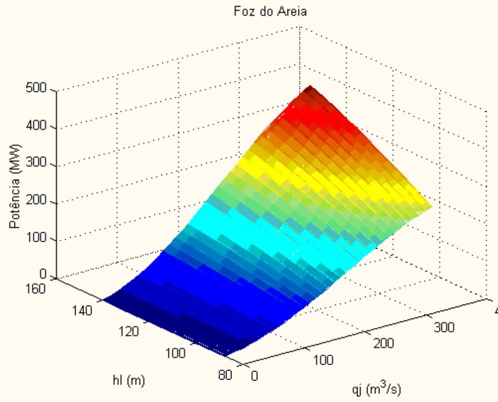
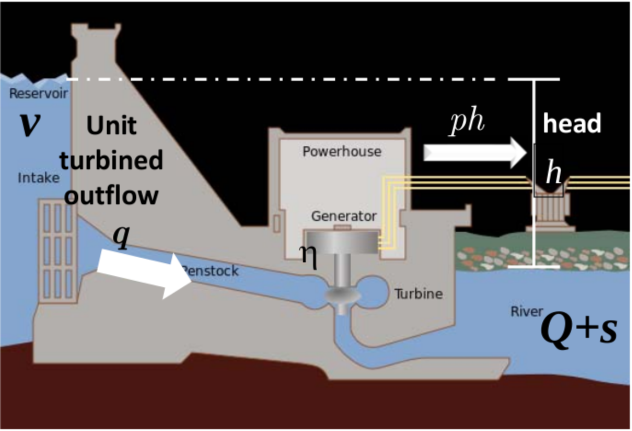
Generally the hydro-production function is nonconvex



The hydro-production function



- ph: unit output power (MW)
- $\eta(\cdot)$: unit efficiency

$$ph = 0{,}00981 \cdot \eta(q^2, h^2) \cdot h \cdot q \;\Rightarrow\; h = f(v^4, Q^4, s^2, q^2)$$

$$\Rightarrow\; ph = f(v^{12}, Q^{12}, s^{12}, q^7) \quad \text{(worst case)}$$

The hydro-production function

- **Huge reservoirs in short-term horizon**

$$ph = f(Q^{12}, s^{12}, q^{7}) \quad \text{or} \quad ph = f(Q^{12}, q^{7})$$

- **Run of river plants**

$$ph = f(v^{12}, Q^{12}, s^{12}, q^{7}) \quad \text{or} \quad ph = (v^{12}, Q^{12}, q^{7})$$

- **Polynomials that represents forebay and tailrace levels can be very different**

$$h = f(v, Q, s, q^{2}) \Rightarrow ph = f(v^{3}, Q^{3}, s^{3}, q^{7}) \quad \text{(easy cases)}$$

$$ph = f(Q^{3}, s^{3}, q^{7}),\ ph = f(Q^{3}, q^{7}),\ ph = f(v^{3}, Q^{3}, q^{7})$$

The hydro-production function

Hydro-generated energy *ph* is a polynomial of

- reservoir volume v
- volume of water going through the considered turbine q
- volume of water going through all turbines Q
- spillage s

Given an operational vector $y \supset (ph, v, q, Q, s)$, the relation

$$ph = f(v, q, Q, s)$$

is represented by

$$\mathtt{hp}(y) = 0$$

Dealing with Uncertainty

- [master] **Strategic level sets units on/off (every 8h)**
- [slave] **Operational level defines the generation for the commitment given by the strategic level, for each considered scenario**

  Ramp and reservoirs balance constraints

  are dynamic, cannot be split!
- **this work:** multi-level scenario tree for HUC

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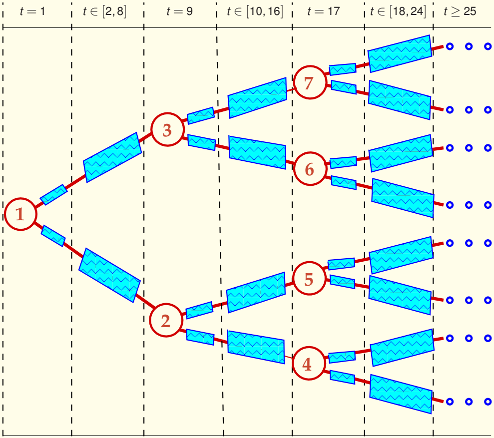
new • Multi-period scenario fan for continuous variables **(operational)**

new • Benders-like decomposition **à la bundle**

new • Sustainable scenario selection (simulation)

Multi-level Scenario Trees: usual approach

For multistage trees with 2 type of variables



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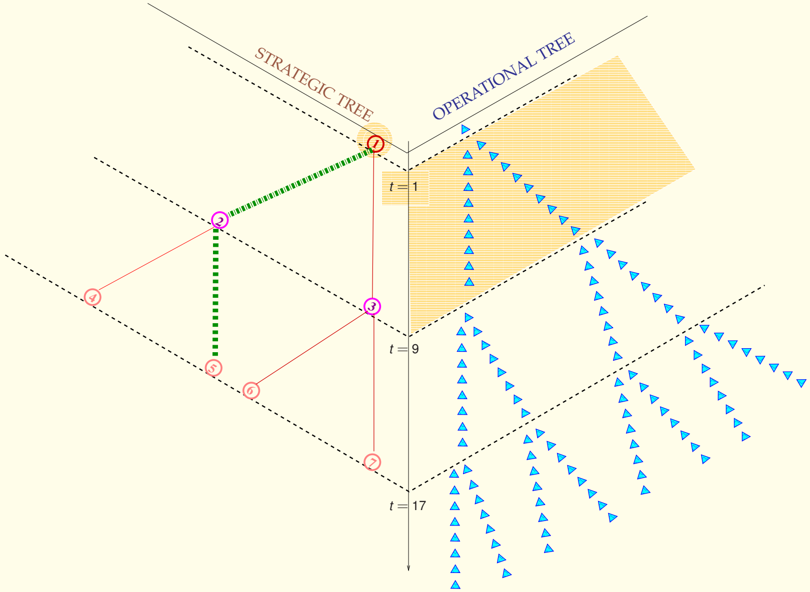

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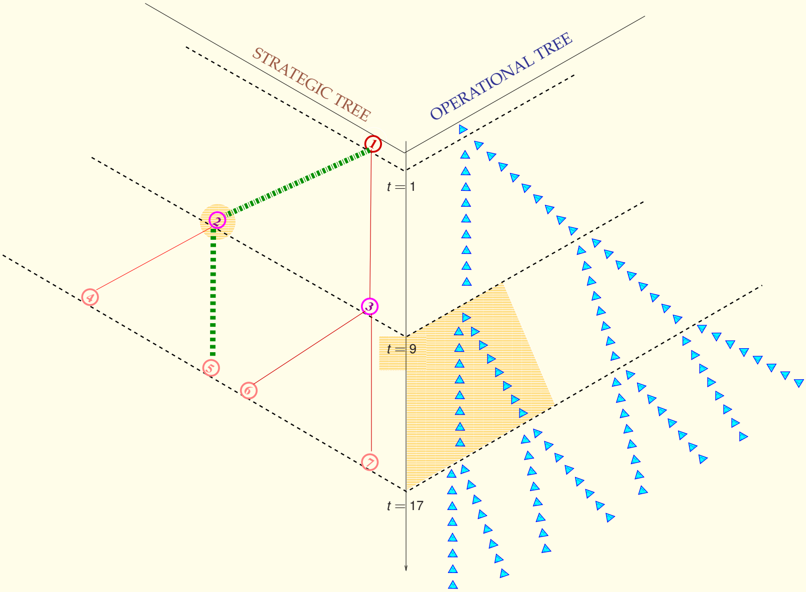

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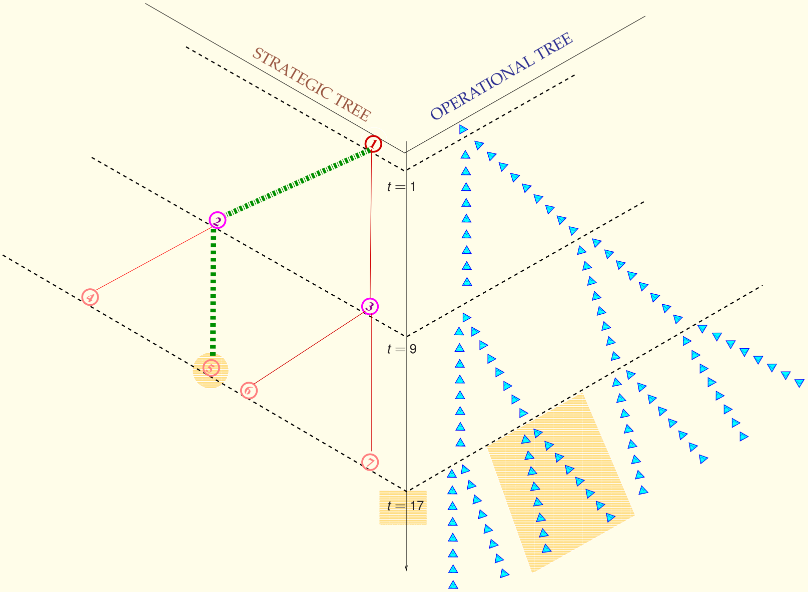

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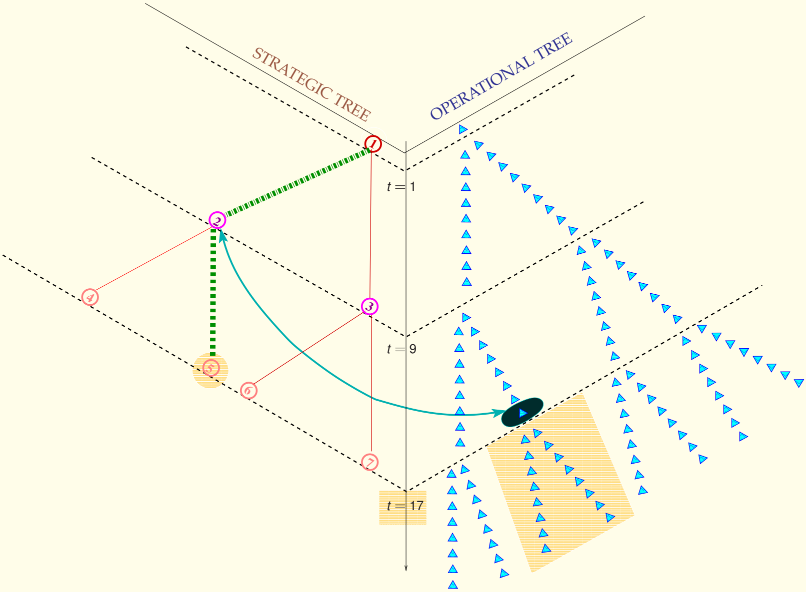
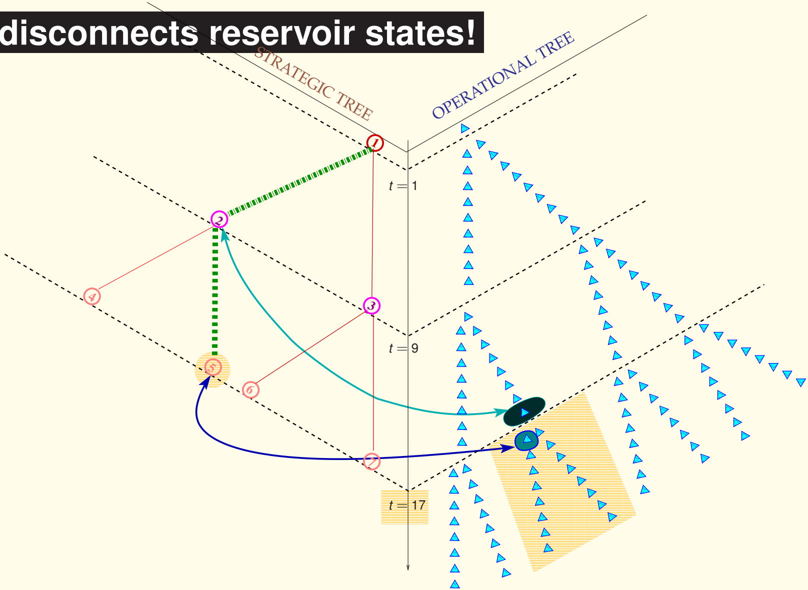
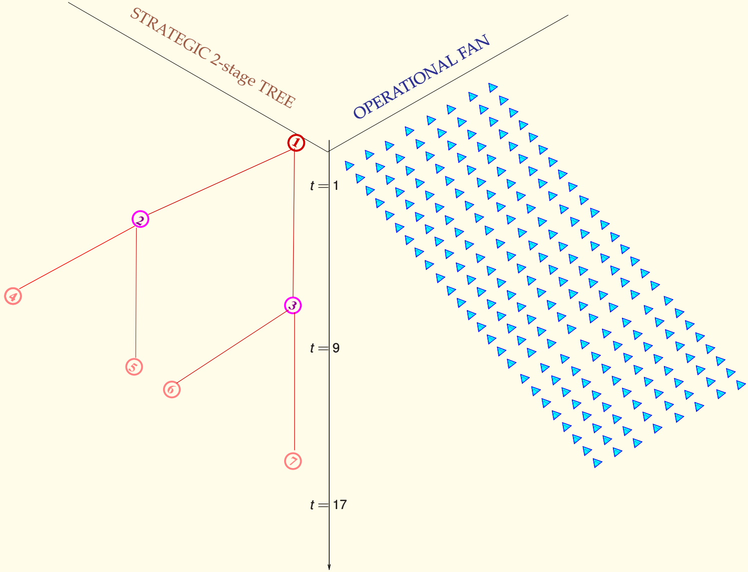


Multi-level Scenario Trees: usual approach

disconnects reservoir states!



Multi-level Scenario Trees: our approach



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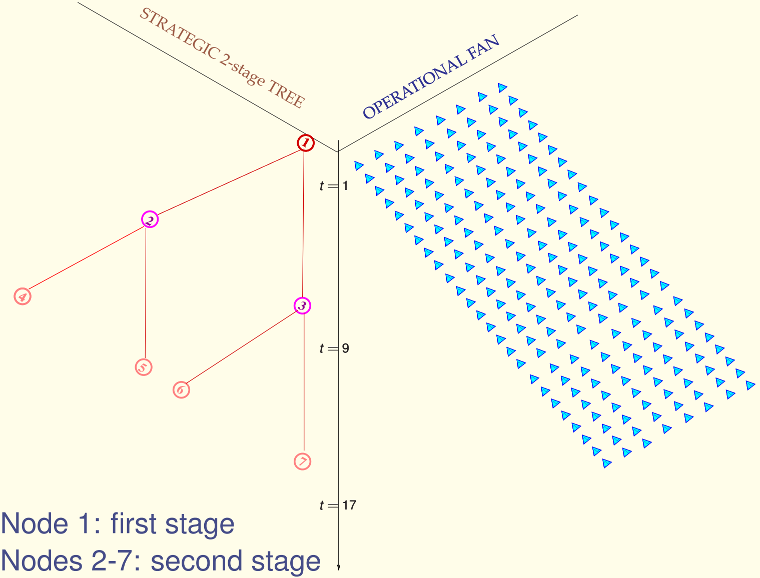
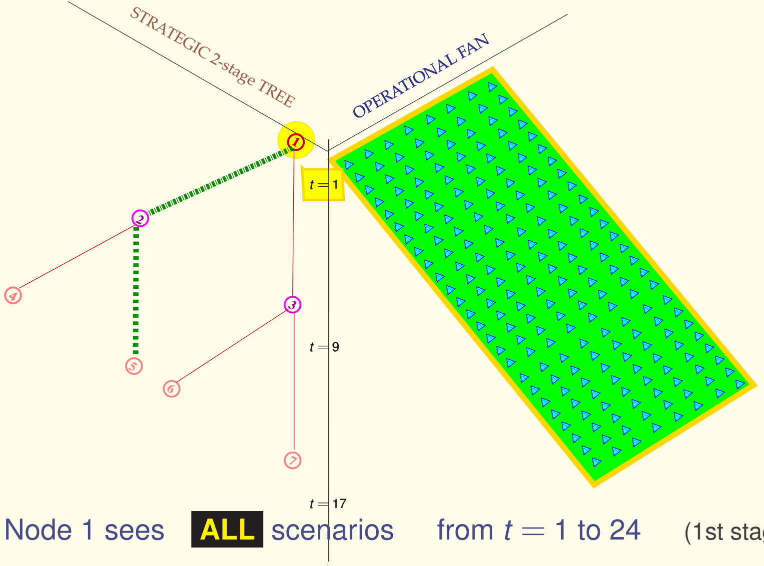


Node 1: first stage

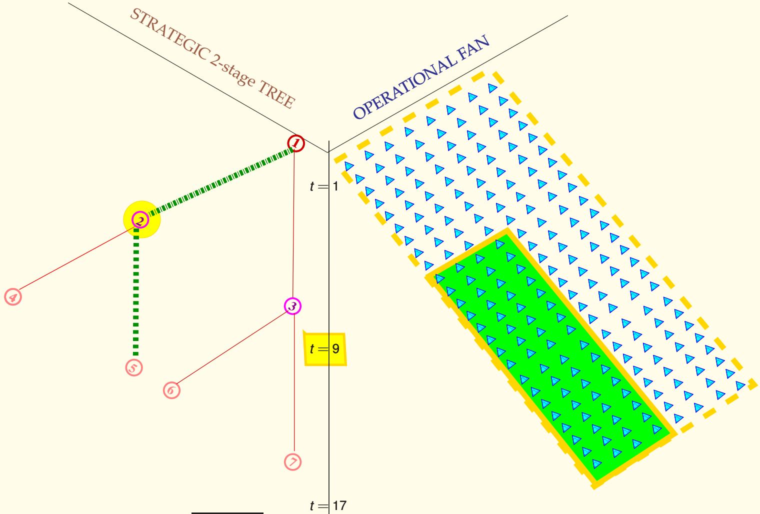
Nodes 2-7: second stage

Multi-level Scenario Trees: our approach



Node 1 sees **ALL** scenarios from $t = 1$ to 24 (1st stage)

Multi-level Scenario Trees: our approach



Node 1 sees **ALL** scenarios from $t = 1$ to 24 (1st stage)

Node 2 sees **FOUR** scenarios from $t = 9$ to 24 (2nd stage)

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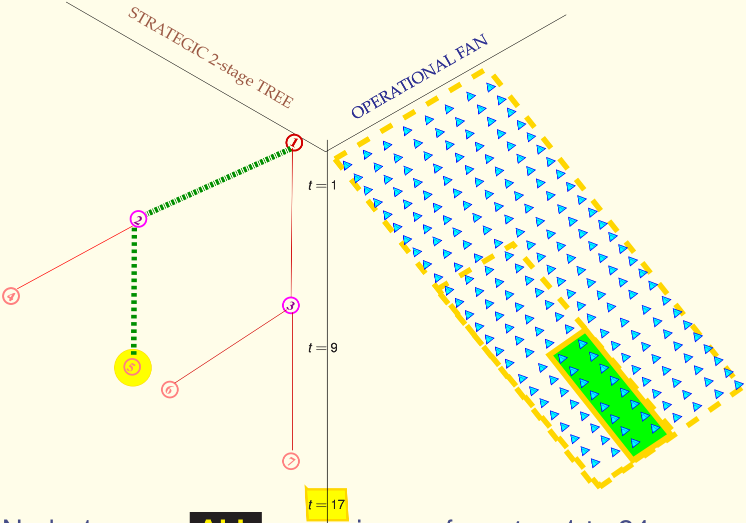

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Node 5 sees **TWO** scenarios from $t = 17$ to 24 (2nd stage)

Mathematical Formulation no uncertainty for now

$$
\begin{cases}
\min & \langle c, x \rangle + f(y) \\
\text{s.t.} & x \in \{0,1\}, y \geq 0 \\
 & Ax = a \\
 & By = b \\
 & x\, y_{low} \leq y \leq x\, y^{up} \\
 & \mathrm{hp}(y) = 0
\end{cases}
$$

- $Ax = a$: start-up, shut-down
- $By = b$: water balance, ramp , flow limits, demand
- $x\, y_{low} \leq y \leq x\, y^{up}$: generation only if switched on
- $\mathrm{hp}(y) = 0$: hydro-production function

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- h can be nonconvex

Mathematical Formulation if uncertainty

$$\left\{\begin{array}{ll} \min & \mathbb{E}_s\left[\langle c_s, x_s\rangle + \mathbb{E}_{o\in O(s)}\left(f_s^o(y_s^o)\right)\right] \\ \text{s.t.} & x_s \in \{0,1\}\,, y_s^o \geq 0 \\ & A_s x_s = a_s \\ & B_s^o y_s^o = b_s^o \\ & x_s\, y_{low} \leq y_s^o \leq x_s\, y^{up} \\ & \mathrm{hp}(y_s^o) = 0 \end{array}\right.$$

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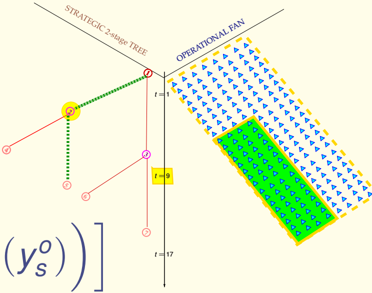
Mathematical Formulation if uncertainty



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Mathematical Formulation

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- $Ax = a$: start-up, shut-down
- $By = b$: water balance, ramp , flow limits, demand
- $x\, y_{low} \leq y \leq x\, y^{up}$: Coupling $\Rightarrow$ Benders' Decomposition
- $\mathrm{hp}(y) = 0$: hydro-production function

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Mathematical Formulation: Benders decomposition

$$
\begin{cases}
\min & \langle c, x \rangle + \mathbb{V}_k(x) \\
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water balance
ramp , flow limits
demand

- $\mathbb{V}_k$ is a piecewise linear approximation of the value-function $\mathbb{V}$, computed by the slaves (feasibility+optimality cuts)

Mathematical Formulation: Benders decomposition

$$\begin{cases} \min & \langle c, x \rangle + \mathbb{V}_k(x) \\ \text{s.t.} & x \in \{0,1\} \\ & Ax = a \end{cases}$$

where

$$\mathbb{V}(x^k) = \begin{cases} \min & f(y) \\ \text{s.t.} & y \geq 0 \\ & By = b \\ & x^k y_{low} \leq y \leq x^k y^{up} \\ & \mathrm{hp}(y) = 0 \end{cases}$$

- $\mathbb{V}_k$ is a piecewise linear approximation

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- Convergence under P1 assumption, **convex** hp (Geoffrion 1972)

Benders decomposition

MASTER

$$\begin{cases} \min & \langle c, x\rangle + \mathbf{W}_k(x) \\ \text{s.t.} & x \in \{0,1\} \\ & Ax = a \end{cases}$$

SLAVE(S)

$$\mathbb{W}(x_k) = \begin{cases} \min & f(y) \\ \text{s.t.} & \{y \geq 0 : By = b\} \\ & x_k\, y_{low} \leq y \leq x_k\, y^{up} \\ & conv\, \mathtt{hp}(y) \leq 0 \end{cases}$$

- SLAVE gives a *cut* $\langle s_k, \cdot\rangle + r_k$, computed using $\mathbb{W}(x_k)$ and a multiplier for the coupling constraints.
- Cutting-plane model $\mathbf{W}_k$ (no feasibility cuts in HUC):

$$\text{MASTER} \begin{cases} \min & \langle c, x\rangle + \alpha \\ \text{s.t.} & x \in \{0,1\} \cap \{x : Ax = a\} \\ & \alpha \geq \langle s_i, x\rangle + r_i\,, \qquad i \in I_k \subset \{1:k\}\,. \end{cases}$$

- If kth-MASTER solution is denoted by x_{k+1}, Benders stops when $\Delta_k := v_k^{up} - v_k^{low} \leq tol$, where

$$v_k^{up} := \min_{i \in I_k}\{\langle c, x_i\rangle + \mathbb{W}(x_i)\} \quad \text{and} \quad v_k^{low} := \max_{i \in I_k}\{\langle c, x_{i+1}\rangle + \mathbf{W}_i(x_{i+1})\}\,.$$

Mathematical Formulation: Benders decomposition

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- Convergence under P1 assumption, convex hp (Geoffrion 1972)
- For nonconvex hp, X. Li, A. Tomasgard, and P.I. Barton, JOTA (2011)
  - Benders with convex $\mathbb{W}$ gives a lower bound if $\boxed{conv(\mathrm{hp}) \leq \mathrm{hp}}$

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  - Upper bound from solving the NLP computing $\mathbb{V}(x_k)$ (with hp)

Creating a **lower** convex hull $conv(\mathrm{hp}) \le \mathrm{hp}$

- $\mathrm{hp}(y) = 0$ represents the relation $ph = f(v, q, Q, s)$

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For a sample $S := \{y_1, \ldots, y_m\} \subset B$, let
$\mathcal{P}_2(y, A, b, c) := \langle y, Ay \rangle + \langle b, y \rangle + c$

$$\Longrightarrow F(v, q, Q, s) := \mathcal{P}_2(y, A^*, b^*, c^*)$$

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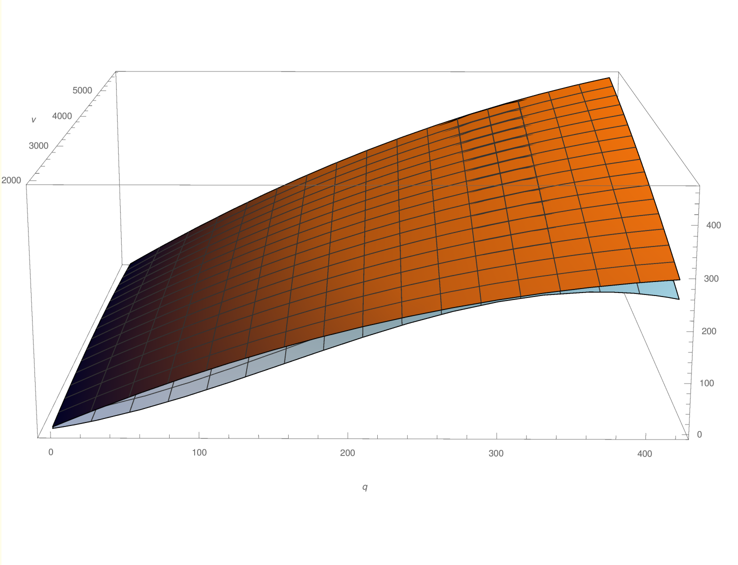
where (A^*, b^*, c^*) solves

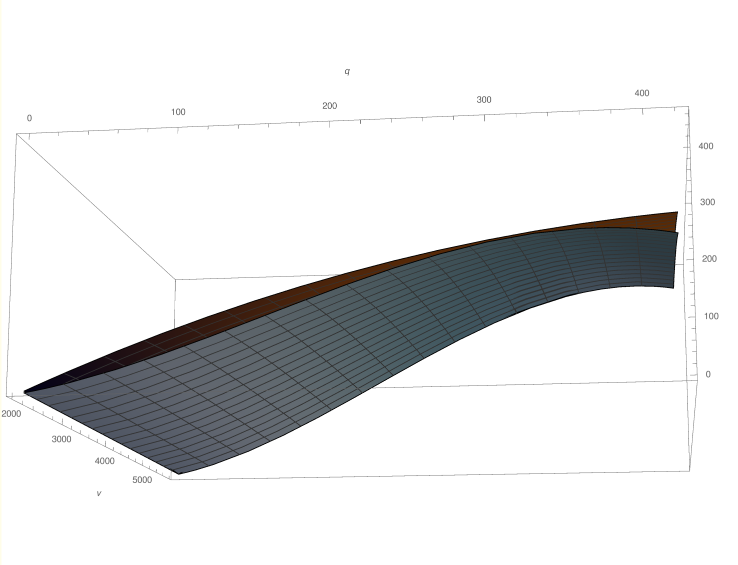
$$\begin{cases} \underset{A,b,c}{\text{minimize}} & \sum_{i \in S} \mathcal{P}_2(y^i, A, b, c) \\ \text{subject to} & ph^i \leq \mathcal{P}_2(y^i, A, b, c), i \in S \quad \text{(recall } ph \subset y) \\ & A \in \mathbb{R}^{4\times 4}, b \in \mathbb{R}^4, c \in \mathbb{R}, \\ & A \text{ negative semidefinite.} \end{cases}$$

Creating a **lower** convex hull



sample with 160.000 y^i (SDPT3 $\approx$ 1h)

Creating a lower convex hull



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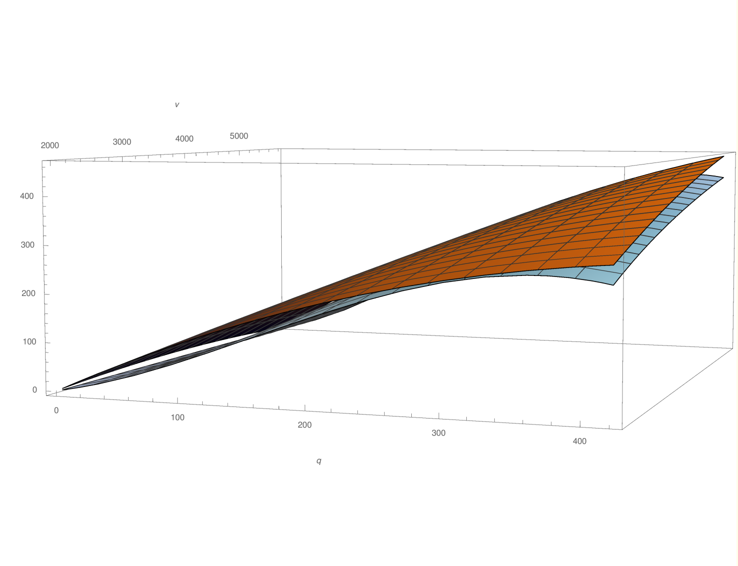
Creating a lower convex hull



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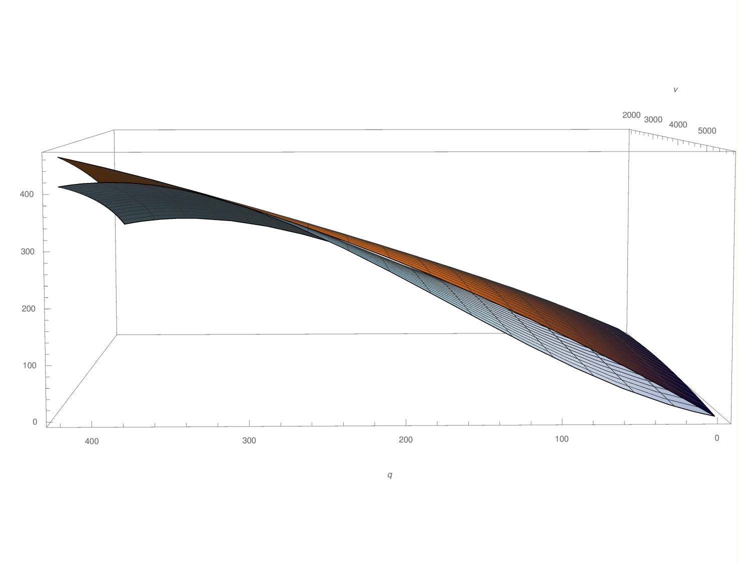
Creating a **lower** convex hull



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Mathematical Formulation: Benders decomposition

$$\left\{\begin{array}{ll} \min & \langle c, x\rangle + \mathbf{W}_k(x) \\ \text{s.t.} & x \in \{0,1\} \\ & Ax = a \end{array}\right.$$

where

$$\mathbb{W}(x_k) = \left\{\begin{array}{ll} \min & f(y) \\ \text{s.t.} & y \geq 0 \\ & By = b \\ & x_k\, y_{low} \leq y \leq x_k\, y^{up} \\ & \boxed{conv(\mathrm{hp})(y) \leq 0} \end{array}\right.$$

- Convergence under P1 assumption, convex $\mathtt{hp}$ (Geoffrion 1972)
- For nonconvex $\mathtt{hp}$, X. Li, A. Tomasgard, and P.I. Barton, JOTA (2011)
  - Benders with convex $\mathbb{W}$ gives a lower bound if $conv(\mathtt{hp}) \leq \mathtt{hp}$
  - Upper bound from solving the NLP computing $\mathbb{V}(x_k)$ (with $\mathtt{hp}$)

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Great idea . . . but

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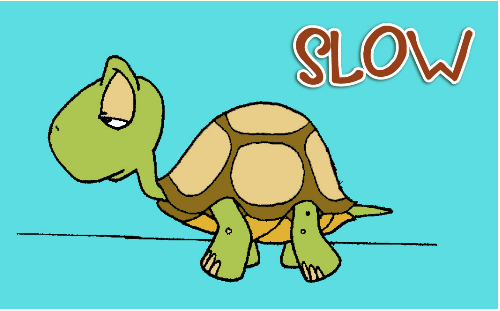
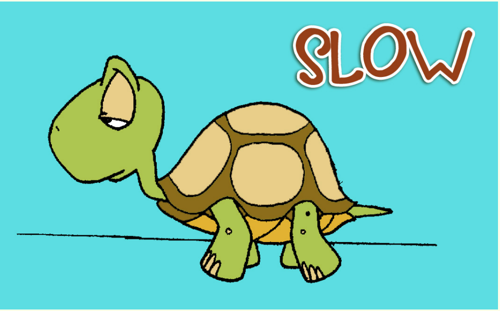


Souce: quotemaster.org

Speed of Generalized Benders Decomposition in our setting

Great idea . . . but



Souce: quotemaster.org

Speed of Generalized Benders Decomposition in our setting

(desperately slow!)

Speeding up the Master: *à la* level-bundle

Replace the Generalized Benders master

$$\begin{cases} \min & \langle c, x\rangle + \mathbf{W}_k(x) \\ \text{s.t.} & x \in \{0,1\} \\ & Ax = a \end{cases}$$

by the following stabilized variant:

$$\begin{cases} \min & \frac{1}{2}\|x - x^{best}\|^2 \\ \text{s.t.} & x \in \{0,1\} \\ & \langle c, x\rangle + \mathbf{W}_k(x) \leq \ell\textbf{evel} \\ & Ax = a \end{cases}$$

$$= \frac{1}{2}(x + x^{best}) - \langle x, x^{best}\rangle$$

remains an LP!

Speeding up the Master: *à la* level-bundle with a twist

Replace the Generalized Benders master

$$\left\{ \begin{array}{ll} \min & \langle c, x \rangle + \mathbf{W}_k(x) \\ \text{s.t.} & x \in \{0,1\} \\ & Ax = a \end{array} \right.$$

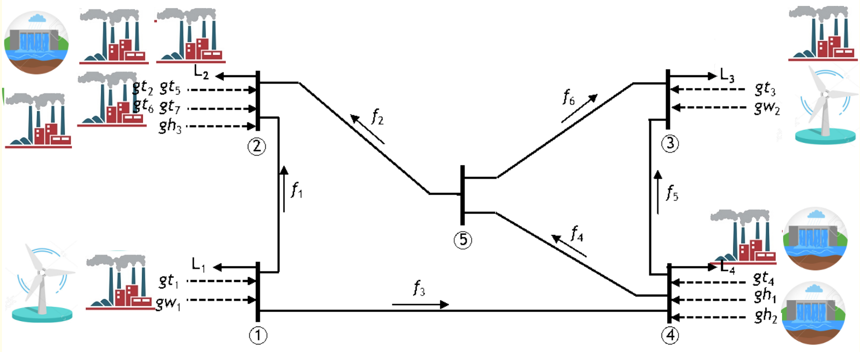
by the following stabilized variant:

$$\left\{ \begin{array}{ll} \min & \frac{1}{2}\|x - x^{best}\|^2 \qquad \equiv \frac{1}{2}(x + x^{best}) - \langle x, x^{best} \rangle \\ \text{s.t.} & x \in \{0,1\} \\ & \langle c, x \rangle + \mathbf{W}_k(x) \leq \ell_{\text{evel}} \\ & Ax = a \end{array} \right.$$

remains an LP!

A toy -yet realistic- power system

- System with 21 units, 6 transmission lines and 5 buses, 4 of load (distributed energy)



L_k : energy demand of bus k

f_j : energy flow of line j

gx : energy generation of source x (x can be t: thermal, w: wind or h: hydro)

A toy -yet realistic- power system

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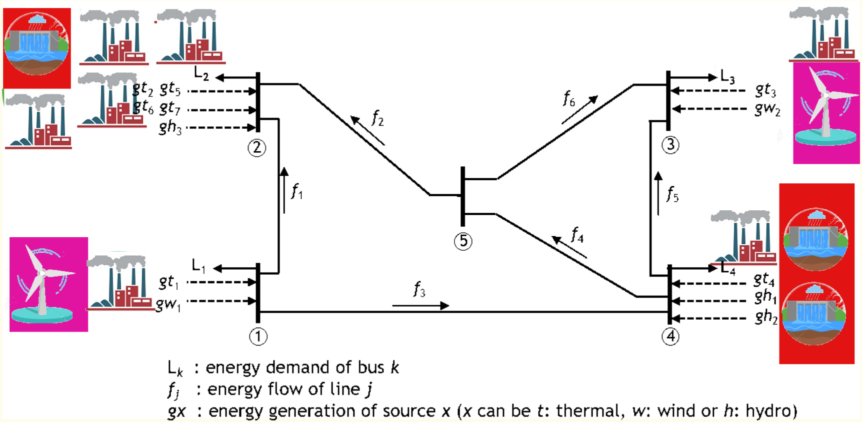
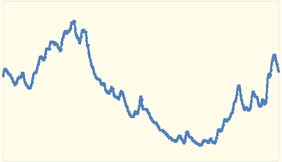
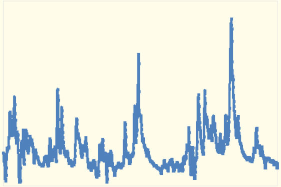


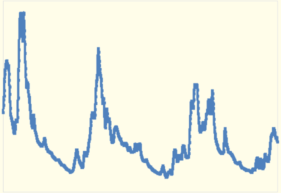
- Random variables
  - Inflows in gh_1, gh_2 and gh_3
  - Wind generation gw_1 and gw_2

Uncertainty in inflows and wind

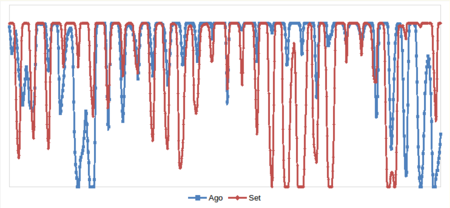
Incremental Inflow profiles

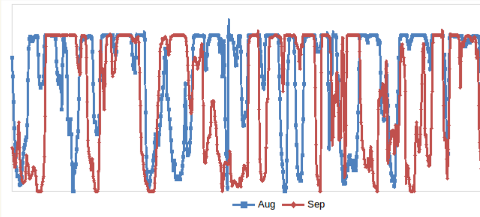




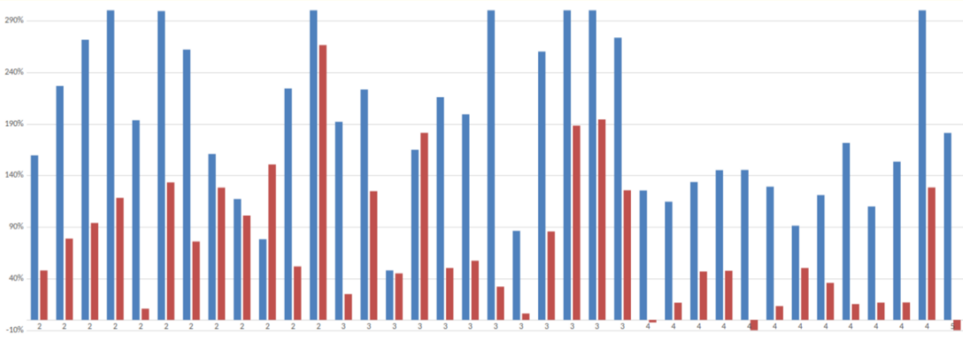


Wind profiles

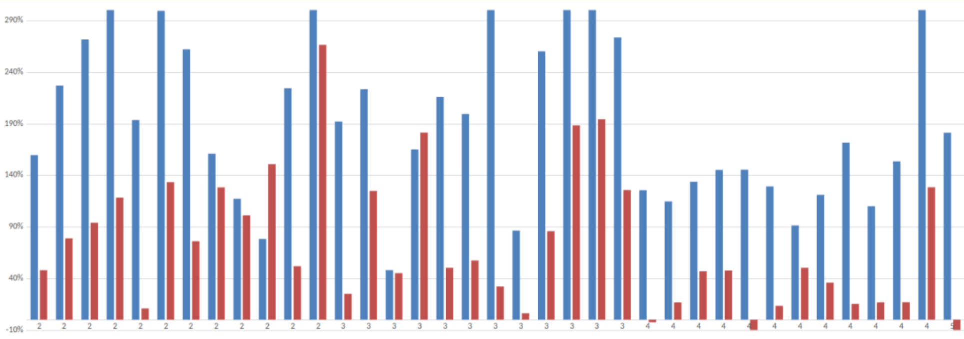




Is the decomposition beneficial? And the stabilization?

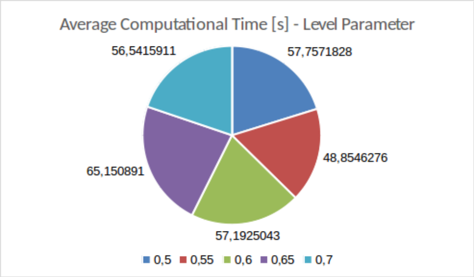


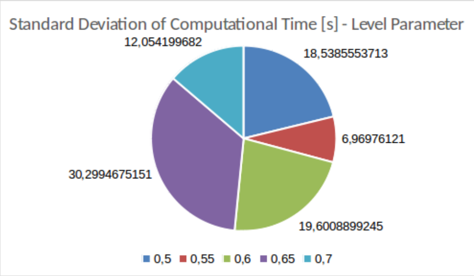
Is the **decomposition** beneficial? And the **stabilization?**



Already for small instances there is a gain

Tuning parameters: which ℓevel?





For some level choices, the variability in solving times is higher

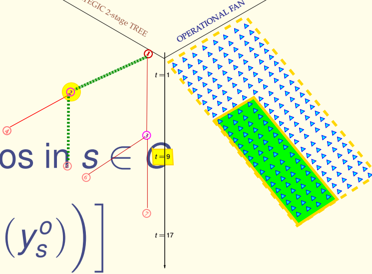
Mathematical Formulation if uncertainty

Calculations done over 27 scenarios in $s \in O$

$$
\left\{
\begin{array}{ll}
\min & \mathbb{E}_s\Big[\langle c_s, x_s\rangle + \mathbb{E}_{o\in O(s)}\Big(f_s^o(y_s^o)\Big)\Big] \\
\text{s.t.} & x_s \in \{0,1\}\,, y_s^o \geq 0 \\
 & A_s x_s = a_s \\
 & B_s^o y_s^o = b_s^o \\
 & x_s\, y_{low} \leq y_s^o \leq x_s\, y^{up} \\
 & \mathrm{hp}(y_s^o) = 0
\end{array}
\right.
$$

- $A_s x_s = a_s$: { start-up, shut-down
- $B_s^o y_s^o = b_s^o$: { water balance, ramp , flow limits, demand
- $x_s\, y_{low} \leq y_s^o \leq x_s\, y^{up}$: { generation only if switched on
- $\mathrm{hp}(y_s^o) = 0$: { hydro-production function

Mathematical Formulation if uncertainty



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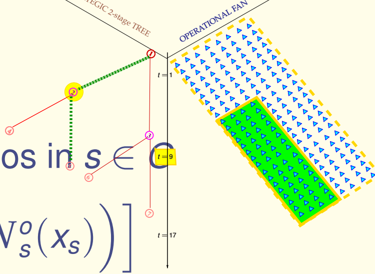

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Tuning parameters: which starting point?

General scheme:

1. Given a commitment x_k, solve operational problems $\mathbb{W}^o_s(x_k(s))$ (with *conv* $\mathrm{hp}(y) \leq 0$)
2. Use the optimal dispatch to find a feasible $y^\circ(s)$ for the nonconvex problem
3. Starting with $y^\circ(s)$, solve $\mathbb{V}^o_s(x_k(s))$ (with $\mathrm{hp}(y) = 0$) (IPOPT)
4. Stop if $\mathbb{E}[\mathbb{V}^o_s(x_k(s)) - \mathbb{W}^o_s(x_k(s))]$ is sufficiently small
5. Otherwise, add Benders cut from $\mathbb{E}\mathbb{W}^o_s(x_k(s))$ to master problem to compute x_{k+1} and loop

Tuning parameters: which starting point?

General scheme:

1. Given a commitment x_k, solve operational problems $\mathbb{W}_s^o(x_k(s))$ (with *conv* $\mathrm{hp}(y) \leq 0$)
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Total CPU time: 3h

Assessing the quality of the commitment

Simulation over 1000 scenarios in O^{out}

- Take $s \in O^{out}$, find the closest scenario $(s^*, o^*) \in O$
- Solve operational problem with dispatch $x^*(s^*)$: compute $\mathbb{V}_s^{o^*}(x^*)$
- Compute $\Delta(s) :=$% deficit w.r.t demand(s)
- Compute $Cost(s)$

Take averages, standard deviations and compare for 5 different sets O^{out}

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What is the closest scenario?

One scenario has very heterogeneous components

- wind at different locations
- inflows to different reservoirs
- demand at different buses

Three options

- Brute force: compute $\mathbb{V}_s^o(x)$ for all $s \in O$, take x^* giving the smallest cost
- Pseudo-distance: as in scenario selection
- Sustainable measure: prioritize demand satisfaction

[1]

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[1] W. de Oliveira, C. S., et al. Optimal scenario tree reduction for stochastic streamflows in power generation planning problems. OMS 2010, V 25 pp. 917-936

What is the closest scenario?

One scenario has very heterogeneous components

- wind at different locations
- inflows to different reservoirs
- demand at different buses **+ wind = - deficit**

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[2]

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Numerical results

For a convex $\mathrm{hp}(y)$, over 24000h

Scen sel	Avg. cost ($\times 10^6$)	St.Dev. cost ($\times 10^6$)	# Rel. deficit $\geq 1\%$	Avg. rel. deficit $\geq 1\%$	St.Dev. rel. deficit $\geq 1\%$
Brute	5.6	0.3	17	2.2%	0.6%
Pseu	6.5	2.8	140	9.2%	6.4%
Sust	7.1	3.4	210	9.8%	5.7%

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For a nonconvex $\mathrm{hp}(y)$

Scen sel	Avg. cost ($\times 10^6$)	St.Dev. cost ($\times 10^6$)	# Rel. deficit $\geq$ 1%	Avg. rel. deficit $\geq$ 1%	St.Dev. rel. deficit $\geq$ 1%
Pseu	13.1	100.7	1414	17.2%	38.0%
Sust	9.9	11.25	1473	8.8%	6.2%
Pseu	13.2	12.9	1958	10.1%	6.7%
Sust	12.9	82.7	1472	14.4%	30.1%

Concluding Comments

- Changing the UC along the day reduces costs
- Slave parallelization should increase the gain in computational time
- Stabilizing Benders with level bundle improves convergence speed
- MIP tuning is crucial (off-the shelf not good)

Our aim: solve a real-life instance

- toy system with 3 hydro and 7 thermal units, 6 transmission lines and 5 buses with 4 scenarios took more than 14h.
- Brazilian Interconnected System: 1000 hydro units, 150 thermal units, 600 lines, 4000 buses
- Would want to consider 50-100 scenarios . . .