Optimal Electricity Demand-Response Contracting

SESO 2018 International Thematic Week
Smart Energy and Stochastic Optimization
ENPC, Marne La Vallée — May, 2018

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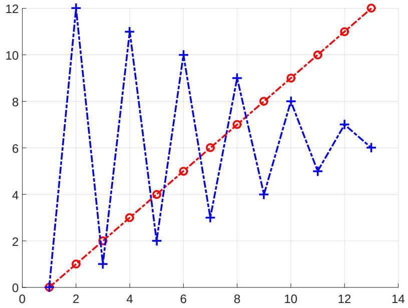
Agenda

Problem

- How to cope with intermittent sources of energy in power systems?
- The need for more flexibility in electric systems can be satisfied either
- ... by batteries or ...
- ... a better use of demand flexibility potential.
- Possible to use distributed control of appliances (Meyn et al. (2015), Tindemans et al. (2015))
- Also possible to use demand-response.
- Important demand-response (DR) and smart grid world wide. EU investment in smart metering: 45 billions € to reach 200 millions smart meters.
- DR programs reduce consumption level. What about volatility?



Total consumption X = Total consumption X

$\langle X \rangle = 1^2 + 1^2 + ... + 1^2 = 12$ $\langle X \rangle = 12^2 + 11^2 + 10^1 .. + 1^2 = 650$

Questions

- Is it possible to encite consumers to reduce the volatility of their consumption?

Results

- We designed a volatility risk trade model between one producer and one consumer in the framework of continuous-time optimal contract theory.
- We obtain closed-form expression for the optimal contract in the case of linear energy value discrepancy between producer and consumer.
- Optimal contract allows the system to bear more risk as it may lead to an increase of consumption volatility.
- We obtained closed-form expression of the first-best optimal contract problem. The first-best is equal to the second-best only in the case where the consumer values more energy than the producer. Same result regarding the potential of an increase of volatility.

The model

The consumer (The Agent)

Dynamics of the deviation from baseline consumption

$$X_t^{a,b} = X_0 + \int_0^t (-\sum_{i=1}^N a_i(s))ds + \int_0^t \sum_{i=1}^N \sigma_i \sqrt{b_i(s)} dW_s^i$$

Cost function for efforts $\nu := (a, b)$:

$$c(a,b) := \underbrace{\frac{1}{2}\sum_{i=1}^N \frac{{a_i}^2}{\mu_i}}_{c_1(a)} + \underbrace{\frac{1}{2}\sum_{i=1}^N \frac{\sigma_i({b_i}^{-\eta_i} - 1)}{\lambda_i \eta_i}}_{c_2(b)}, \; 0 \le a_i, \;\; 0 < b_i \le 1.$$

Consumer's criterion:

$$J_A(\xi,\nu) := \mathbb{E}^\nu \left[U_A\Big(\xi + \int_0^T \left(f(X_s^\nu) - c(\nu_s)\right) ds\Big)\right],$$

with $U_A(x) = -e^{-r\,x}$.

The producer (The Principal)

$$J_{\mathrm{P}}(\xi, \mathbb{P}^{\nu}) := \mathbb{E}^{\mathbb{P}^{\nu}}\left[U\left(-\xi - \int_0^T g(X_s)ds - \frac{h}{2}\langle X\rangle_T\right)\right]$$

- g generation cost function, convexe centered at zero
- h direct unitary cost of volatility
- $U(x) = -e^{-p\,x}$.

The producer's problem is:

$$V^{\mathrm{P}} := \sup_{\xi \in \Xi} \sup_{\mathbb{P}^{\nu} \in \mathcal{P}^{\star}(\xi)} J_{\mathrm{P}}(\xi, \mathbb{P}^{\nu}).$$

with the participation constraint: the consumer enters in the contract only if his expected utility is above $R := R_0 e^{-r\pi}$ where

$$R_0 := \sup_{\mathbb{P}^{\nu} \in \mathcal{P}} J_{\mathrm{A}}(0, \mathbb{P}^{\nu}) = \mathbb{E}^{\mathbb{P}^{\nu}}\left[U_A\left(\int_0^T (f(X_s) - c(\nu_s))\,ds\right)\right],$$

is the utility he gets without contract and π is a premium.

Remarks

- The consumer has never an interest in making an effort to reduce consumption without contract.
- Because of risk-aversion, the consumer has an interest in making an effort to reduce volatility even without contract

Consumer's reservation utility

The consumer's reservation utility is given by $R_0 = -e^{-ru(0,X_0)}$, where u is the unique viscosity solution of the HJB equation

$$-\partial_t u = f + H(u_x, u_{xx} - ru_x^2), \text{ with } u(T,.) = 0,$$

where H is the consumer's Hamiltonian

$$H(z,\gamma) := \sup_{(a,b)} \left\{ -a \cdot \mathbf{1} z + \frac{1}{2} |\sigma(b)|^2 \gamma - c(a,b) \right\}, \ \mathbf{1} := (1, \cdots, 1),$$

which optimizers are

$$\widehat{a}(z) := \mu z^-, \ \widehat{b}_j(\gamma) := 1 \wedge (\lambda_j \gamma^-)^{-\frac{1}{1+\eta_j}}.$$

If in addition u is smooth, then the optimal efforts of the consumer are

$$a^0 := 0, \ b_j^0 := 1 \wedge \left(\lambda_j \left(u_{xx} - ru_x^2 \right)^- \right)^{-\frac{1}{1+\eta_j}}.$$

Optimal contract

The optimal contract

- Cvitanic, Possamaï & Touzi (2015) proves the optimal contract is of the form

$$Y^{Y_0,Z,\Gamma} := Y_0 + \int_0^t Z_s dX_s + \frac{1}{2}\int_0^t (\Gamma_s + r{Z_s}^2) d\langle X\rangle_s - \int_0^t \big(H(Z_s,\Gamma_s) + f(X_s)\big) ds.$$

- Y_0 is going to be the certainty equivalent of reservation utility of the consumer.
- Payment ($Z_t \leq 0$) if consumption decreases ($dX \leq 0$)
- Payment ($\Gamma_t \leq 0$) if volatility decreases
- Compensation for induced volatility cost rZ_s^2
- Minus the natural benefits the consumer earns when making efforts induced by (Z_t, Γ_t), i.e. $H(Z_s, \Gamma_s) + f(X_s)$

Solution of the producer's problem

$V^{\mathrm{P}} = -e^{-p(v(0,X_0)-L_0)}$ with $L_0 = -r^{-1}\log(-R)$ and where v is the unique viscosity solution of the PDE

$$-\partial_t v = (f-g) + \frac{1}{2}\bar{\mu}\, v_x^2 - \frac{1}{2}\inf_{z\in\mathbb{R}}\big\{F_0\big(q(v_x, v_{xx}, z)\big) + \bar{\mu}(z^- + v_x)^2\big\},$$

$$v(T,x) = 0,$$

with $\bar{\mu} := \sum_i \mu_i$, $\bar{\lambda} = \max_i \lambda_i$ and

$$F_0(q) = q|\widehat{\sigma}(-q)|^2 + \widehat{c}_2(-q),\ \ q(v_x, v_{xx}, z) := h - v_{xx} + rz^2 + p(z - v_x)^2,$$

and

$$\gamma^\star := -\Big(q(v_x, v_{xx}, z^\star) \vee \frac{1}{\bar{\lambda}}\Big),$$

and $z^\star$ satisfies $z^\star \in \big(v_x, \frac{p}{r+p}v_x\big)$, when $v_x \le 0$, and $z^\star = \frac{p}{r+p}v_x$ when $v_x \ge 0$.

Remarks

- Assume that $(f-g)(x) = \delta x$.
- Then, we guess that $v(t,x) = A(t)x + B(t)$ with

$$
\begin{aligned}
-A'(t) &= \delta, \\
-B'(t) &= \frac{1}{2}\bar{\mu}A^2(t) - \frac{1}{2}\inf_{z\in\mathbb{R}}\left\{F_0\big(h + rz^2 + p(z-A(t))^2\big) + \bar{\mu}(z^- + A(t))^2\right\}, \\
A(T) &= B(T) = 0.
\end{aligned}
$$

- Thus, we have $A(t) = \delta(T-t)$ and the sign of v_x is given by the sign δ.

Linear Case

Consumer's reservation utility in the linear case $f(x) = \kappa\, x$

- Then, the reservation utility of the consumer is

$$R_0 = -\exp\big(-r\big(\kappa X_0 T + E(T)\big)\big),$$

where $E(T) := -\frac{1}{2}\int_0^T F_0(-\gamma_s^0)\,ds,\ \gamma_s^0 := -r\kappa^2(T-s)^2.$

- The consumer's optimal effort is

$$a^0 = 0, \text{ and } b_j^0(t) := 1 \wedge \Big(\lambda_j r\kappa^2(T-t)^2\Big)^{-\frac{1}{1+\eta_j}},$$

thus inducing the dynamics

$$dX_t^0 = \sigma^0 \cdot dW_t,$$

with $\sigma^0 := \widehat{\sigma}(\gamma_t^0)$.

Optimal contract when energy has more value for the consumer $\delta \geq 0$

If $\delta \geq 0$, the optimal payments rate are

$$z_t^\star = \frac{p}{r+p}\delta(T-t), \quad \gamma_t^\star = -\left[\left(h + \rho\,\delta^2(T-t)^2\right) \vee \frac{1}{\bar{\lambda}}\right], \;\; \frac{1}{\rho} := \frac{1}{r} + \frac{1}{p}.$$

The dynamics of the consumption deviation is

$$dX_t^\star = \sigma_t^\star \cdot dW_t,$$

with $\sigma_t^\star := \widehat{\sigma}(\gamma_t^\star)$. And the optimal contract is

$$\xi^\star = L_0 + \int_0^T \left(\frac{1}{2}\widehat{c}_2(\gamma_t^\star) - \kappa X_t + \frac{1}{2}\frac{rp^2\delta^2}{(p+r)^2}(T-t)^2|\sigma_t^\star|^2\right)dt + \int_0^T z_t^\star \sigma_t^\star \cdot dW_t.$$

Remark

- If $\delta = 0$ and $h = 0$, the producer induces no effort from the consumer and thus, the volatility under optimal contract is $|\sigma|^2 \geq |\sigma_t^0|^2$.

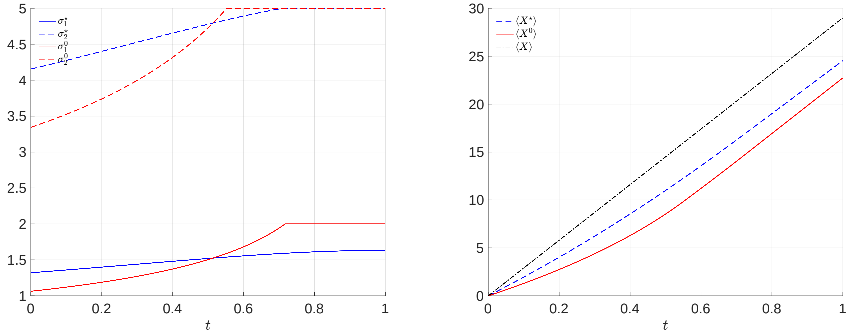


Figure: (Left) Volatilities of two usages without contract (red) and with optimal contract (blue). (Right) Quadratic variation when no efforts are done (black) without contract (red) and with optimal contract (blue).

$\mu = (1, 5)$, $\sigma = (2.0, 5.0)$, $\lambda = (1/2, 1/5)$, $\eta = (1, 1)$, $r = 1$, $\pi = 0$, $p = 2$, $h = 4.5$, $\kappa = 5$ $\delta = 3$

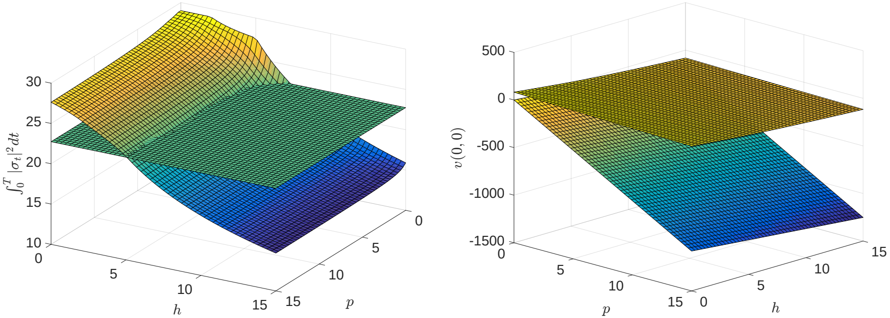


Figure: (Left) Total volatility of consumption deviation under optimal contract as a function of the direct volatility cost h and the risk-aversion parameter p of the consumer compared to the total volatility without contract (flat surface). (Right) Certainty equivalent of the producer with contract and without contract as a function of the direct volatility cost h and the risk-aversion parameter p.

Certainty equivalent gain

When $\delta \geq 0$, the certainty equivalent gain from the contract for the producer is:

$$G^P = -\pi + \frac{1}{2}\int_0^T F_0(-\gamma_s^0)ds - \frac{1}{2}\int_0^T \widehat{c}_2(\gamma_s^\star)ds + \frac{h}{2}\left(\int_0^T \left(|\sigma_s^0|^2 - |\sigma_s^\star|^2\right)ds\right)$$

$$+ \underbrace{\frac{p}{2}\int_0^T \left((\kappa-\delta)^2|\sigma_s^0|^2 - \frac{r}{r+p}\delta^2|\sigma_s^\star|^2\right)(T-s)^2 ds}_{\text{Indirect volatility cost compromise}}.$$

Optimal contract with $\delta \leq 0$

If $\delta \leq 0$ and $h + r\delta^2 T^2 \leq \frac{1}{\bar{\lambda}}$, the optimal payments rate are

$$\gamma_t^\star = -\frac{1}{\bar{\lambda}}, \quad z_t^\star = \Lambda\delta(T-t), \text{with } \Lambda := \frac{1 + p\frac{|\sigma|^2}{\bar{\mu}}}{1 + (r+p)\frac{|\sigma|^2}{\bar{\mu}}}$$

The dynamics of the consumption deviation is

$$dX_t^\star = \bar{\mu} z_t^\star dt + \sigma \cdot dW_t.$$

And the optimal contract is

$$\xi^\star = L_0 + \frac{1}{2}\int_0^t \left(\bar{\mu} + r|\sigma|^2\right)(z_s^\star)^2 ds - \int_0^t \kappa X_s ds + \int_0^T z_s^\star \sigma \cdot dW_s$$

Certainty equivalent gain

When $\delta \leq 0$ and $h + r\delta^2 T^2 \leq \frac{1}{\lambda}$, the certainty equivalent gain from the contract for the producer is:

$$G^P = -\pi + \kappa T X_0 + \frac{1}{2}\int_0^T F_0(-\gamma_t^0)dt + \frac{h}{2}\int_0^T (|\sigma_t^0|^2 - |\sigma|^2)dt + \int_0^T \delta\bar{\mu}(T-t)z_t^\star dt - \frac{1}{2}\int_0^T (\bar{\mu} + r|\sigma|^2)(z_t^\star)^2 dt + \frac{p}{2}\int_0^T (\kappa^2|\sigma_t^0|^2 - (1-\Lambda)^2\delta^2|\sigma|^2(z_t^\star)^2)(T-t)^2 dt.$$

Remark

- The positive term $\delta\bar{\mu}(T-t)z_t^\star$ is the rate of revenue from the energy reduction while $\frac{1}{2}(\bar{\mu} + r|\sigma|^2)(z_t^\star)^2$ is the rate of cost.
- This cost is made of two terms: the direct cost of effort made by the consumer to reduce consumption $(\bar{\mu}(z_t^\star)^2)$ and the indirect cost of volatility induced by this reduction on the mean consumption $(r|\sigma|^2(z_t^\star)^2)$.

Conclusion & Perspectives

Conclusion

- Trading volatility of consumption can benefit to both generation and consumption, allowing the system to bear more risk.

Future work

- Calibration to publicly available demand-response programs (London, Austin)
- Extension to a group of consumers
- Identification of consumers types (adverse selection)

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