Interface Course 2019
Stochastic Optimization for Large-Scale Systems
♢
Spatial Decomposition Methods II

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Ultimate goal of the lecture

How to to obtain "good" strategies for a large scale stochastic optimal control problem, for example a problem corresponding to the optimal management over a given time horizon of a system involving a large amount of dynamical production units.

- In order to obtain decision strategies (closed-loop controls), we have to use Dynamic Programming or related methods.
  - **Assumption**: Markovian case,
  - **Difficulty**: curse of dimensionality.
- In order to to take into account the size of the system, we have to use decomposition/coordination techniques.
  - **Assumption**: convexity,
  - **Difficulty**: information pattern of the problem.

Mixture of spatial and temporal decompositions

Lecture outline

1. Mixing spatial and temporal decompositions
   - Problem formulation and price decomposition
   - Dual approximate dynamic programming (DADP)
   - Upper and lower bounds for large scale SOC problems
2. Application to dams management problems
   - Hydro valley modeling
   - Numerical experiments
3. Application to microgrids management problems
   - Urban microgrid modeling
   - Numerical experiments

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Optimization problem

We recall the SOC problem under consideration:

$$\min_{\boldsymbol{U},\boldsymbol{X}} \mathbb{E}\bigg(\sum_{i=1}^{N}\Big(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i)\Big)\bigg) , \qquad (\mathcal{P})$$

subject to dynamics constraints:

$$\boldsymbol{X}_0^i = f_{-1}^i(\boldsymbol{W}_0) ,$$
$$\boldsymbol{X}_{t+1}^i = f_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) ,$$

to measurability constraints:

$$\boldsymbol{U}_t^i \preceq \sigma(\boldsymbol{W}_0, \ldots, \boldsymbol{W}_t) ,$$ **Decision-Hazard** setting

and to instantaneous coupling constraints

$$\sum_{i=1}^{N} \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) = 0 .$$ **Feasible** constraints

Assumptions

Assumption 1 (White noise)

Noises $\boldsymbol{W}_0, \ldots, \boldsymbol{W}_T$ are independent over time.

We have also assumed full noise observation:

$$\boldsymbol{U}_t^i \preceq \sigma(\boldsymbol{W}_0, \ldots, \boldsymbol{W}_t) .$$

As a consequence of these assumptions, there is no optimality loss to seek the control $\boldsymbol{U}_t^i$ as a function of the state at time t rather than a function of the past noises:

$$\boldsymbol{U}_t^i \preceq \sigma(\boldsymbol{X}_t^1, \ldots, \boldsymbol{X}_t^N) .$$

We are in the Markovian case, and Dynamic Programming applies.

But DP faces the curse of dimensionality when N is large...

Lagrangian formulation

We dualize the coupling constraints and obtain the Lagrangian:

$$\mathcal{L}(\boldsymbol{X}, \boldsymbol{U}, \boldsymbol{\Lambda}) = \mathbb{E}\left(\sum_{i=1}^{N} \left(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i) + \sum_{t=0}^{T-1} \boldsymbol{\Lambda}_t \cdot \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \right) \right) ,$$

where the $\boldsymbol{\Lambda}_t$'s are $\sigma(\boldsymbol{W}_0, \ldots, \boldsymbol{W}_t)$-measurable random variables.

We assume that a saddle point of $\mathcal{L}$ exists,[1] so that

$$\min_{\boldsymbol{U},\boldsymbol{X}} \max_{\boldsymbol{\Lambda}} \mathcal{L}(\boldsymbol{X}, \boldsymbol{U}, \boldsymbol{\Lambda}) = \max_{\boldsymbol{\Lambda}} \min_{\boldsymbol{U},\boldsymbol{X}} \mathcal{L}(\boldsymbol{X}, \boldsymbol{U}, \boldsymbol{\Lambda}) .$$

[1] Such an assumption is **highly non-trivial** for the considered problem...

Uzawa algorithm

At iteration k of the algorithm,

1. Solve subproblem i, $i = 1, \dots, N$, with $\boldsymbol{\Lambda}^{(k)}$ fixed:

$$\min_{\boldsymbol{U}^i, \boldsymbol{X}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} \Big(L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + \boldsymbol{\Lambda}_t^{(k)} \cdot \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \Big) + K^i(\boldsymbol{X}_T^i) \right) ,$$

subject to

$$\boldsymbol{X}_{t+1}^i = f_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) ,$$
$$\boldsymbol{U}_t^i \preceq \sigma(\boldsymbol{W}_0, \dots, \boldsymbol{W}_t) ,$$

whose solution is denoted $\big(\boldsymbol{U}^{i,(k+1)}, \boldsymbol{X}^{i,(k+1)}\big)$.

2. Update the multipliers $\boldsymbol{\Lambda}_t$:

$$\boldsymbol{\Lambda}_t^{(k+1)} = \boldsymbol{\Lambda}_t^{(k)} + \rho_t \left(\sum_{i=1}^{N} \Theta_t^i\big(\boldsymbol{X}_t^{i,(k+1)}, \boldsymbol{U}_t^{i,(k+1)}\big) \right) .$$

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Main idea of the approximation

As already pointed out, $\boldsymbol{\Lambda}_t^{(k)}$ depends on $(\boldsymbol{W}_0, \dots, \boldsymbol{W}_t)$, so that solving a subproblem is as complex as solving the initial problem.

In order to overcome the difficulty, we choose at each time t and for each i a random variable $\boldsymbol{Y}_t^i$ which is measurable w.r.t. the past noises $\big(\boldsymbol{W}_0, \dots, \boldsymbol{W}_t\big)$. We call $\boldsymbol{Y}^i = \big(\boldsymbol{Y}_0^i, \dots, \boldsymbol{Y}_{T-1}^i\big)$ the information process for subsystem i.

The core idea of DADP is to replace the multiplier $\boldsymbol{\Lambda}_t^{(k)}$ by its conditional expectation w.r.t. $\boldsymbol{Y}_t^i$, that is, $\mathbb{E}(\boldsymbol{\Lambda}_t^{(k)} \mid \boldsymbol{Y}_t^i)$. From an intuitive point of view, this leads to a good approximation if

$\boldsymbol{Y}_t^i$ is (highly) correlated to the random variable $\boldsymbol{\Lambda}_t$.

Note that we require that the information process is not influenced by controls.

Subproblem approximation

Following this idea, we replace subproblem i in Uzawa algorithm by:

$$\min_{\boldsymbol{U}^i,\boldsymbol{X}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} \Big(L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + \mathbb{E}(\boldsymbol{\Lambda}_t^{(k)} \mid \boldsymbol{Y}_t^i) \cdot \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \Big) + K^i(\boldsymbol{X}_T^i) \right) ,$$

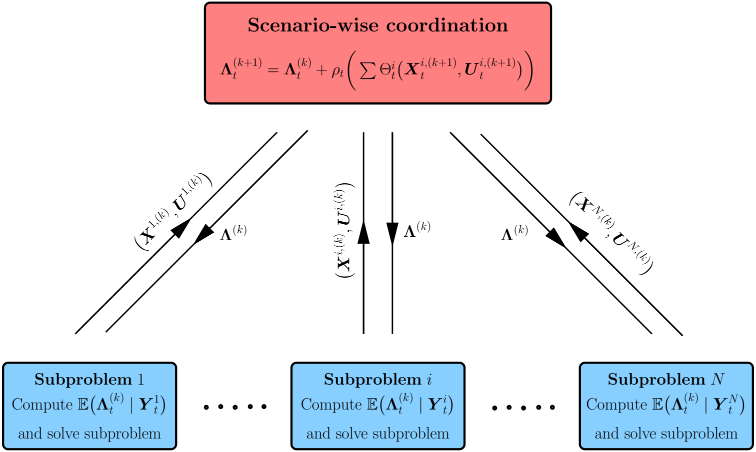
subject to

$$\boldsymbol{X}_{t+1}^i = f_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) ,$$
$$\boldsymbol{U}_t^i \preceq \sigma(\boldsymbol{W}_0, \dots, \boldsymbol{W}_t) .$$

The conditional expectation $\mathbb{E}(\boldsymbol{\Lambda}_t^{(k)} \mid \boldsymbol{Y}_t^i)$ corresponds to a given function of the variable $\boldsymbol{Y}_t^i$, so that subproblem i now involves 2 exogenous random processes, that is, $\boldsymbol{W}$ and $\boldsymbol{Y}^i$.

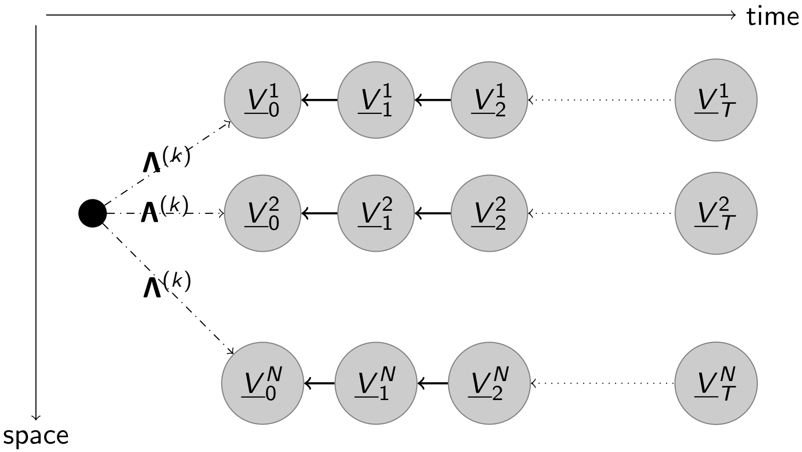
If $\boldsymbol{Y}^i$ is a short memory process, DP applies effectively.

DADP as a spatial decomposition (price) algorithm



Each subproblem is solved by DP: temporal decomposition.

Mix of spatial and temporal decompositions in DADP



Possible choices for the information process

1. **Perfect memory**: $\boldsymbol{Y}_t^i = \left(\boldsymbol{W}_0, \ldots, \boldsymbol{W}_t\right)$.
   - $\mathbb{E}(\boldsymbol{\Lambda}_t^{(k)} \mid \boldsymbol{Y}_t^i) = \boldsymbol{\Lambda}_t^{(k)}$: no approximation!
   - The state size of the subproblem increases with time...
2. **Minimal information**: $\boldsymbol{Y}_t^i \equiv \mathrm{cste}$.
   - $\boldsymbol{\Lambda}_t^{(k)}$ is approximated by its expectation $\mathbb{E}(\boldsymbol{\Lambda}_t^{(k)})$.
   - The information variable does not deliver any information...
3. **Static information**: $\boldsymbol{Y}_t^i = h_t^i\left(\boldsymbol{W}_t\right)$.
   - Such a choice is guided by the intuition that a part of $\boldsymbol{W}_t$ mostly "explains" the optimal multiplier.
4. **Dynamic information**: $\boldsymbol{Y}_{t+1}^i = h_t^i\left(\boldsymbol{Y}_t^i, \boldsymbol{W}_{t+1}\right)$.
   - In the Dynamic Programming equation, the state vector is augmented by embedding $\boldsymbol{Y}_t^i$, that is, the necessary memory to compute the information variable at the next time step.

Dynamic Programming equation

In the last case (dynamic information), the DP equation writes:

$$\underline{V}^i_T(x,y) = K^i(x) \; ,$$

$$\underline{V}^i_t(x,y) = \min_u \mathbb{E}\Bigg(\Big(L^i_t(x,u,\boldsymbol{W}_{t+1}) + \mathbb{E}(\boldsymbol{\Lambda}^{(k)}_t \mid \boldsymbol{Y}^i_t = y) \cdot \Theta^i_t(x,u) + \underline{V}^i_{t+1}\big(\boldsymbol{X}^i_{t+1}, \boldsymbol{Y}^i_{t+1}\big)\Big)\Bigg) \; ,$$

subject to the dynamics:

$$\boldsymbol{X}^i_{t+1} = f^i_t(x,u,\boldsymbol{W}_{t+1}) \; ,$$
$$\boldsymbol{Y}^i_{t+1} = h^i_t(y,\boldsymbol{W}_{t+1}) \; .$$

About the coordination

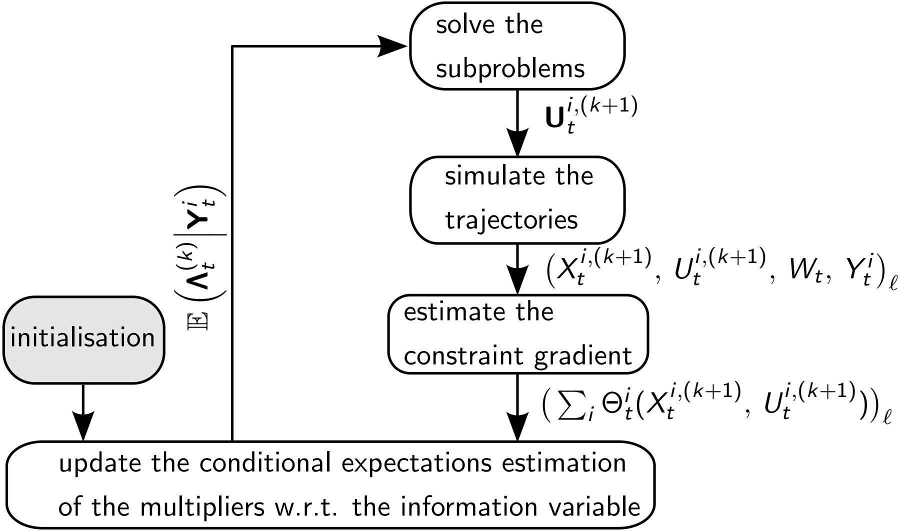
The task of coordination is performed in a scenario-wise manner.

- A set of noise scenarios is drawn once for all. Trajectories of the information process $\boldsymbol{Y}^i$ are simulated along the scenarios.
- At iteration k, the optimal trajectories of the state process $\boldsymbol{X}^{i,(k+1)}$ and of the control process $\boldsymbol{U}^{i,(k+1)}$ are simulated along the noise scenarios, for all subsystems.
- The dual multipliers are updated along the noise scenarios according to the formula:

$$\boldsymbol{\Lambda}_t^{(k+1)} = \boldsymbol{\Lambda}_t^{(k)} + \rho_t \left(\sum_{i=1}^{N} \Theta_t^i \big(\boldsymbol{X}_t^{i,(k+1)}, \boldsymbol{U}_t^{i,(k+1)} \big) \right) .$$

- The conditional expectations $\mathbb{E}(\boldsymbol{\Lambda}_t^{(k+1)} \mid \boldsymbol{Y}_t^i)$ are obtained by regression of the trajectories of $\boldsymbol{\Lambda}_t^{(k+1)}$ on those of $\boldsymbol{Y}_t^i$.

DADP flowchart (based on scenarios)



Interpretation of DADP

The approximation made on the dual process allows to obtain a tractable way for solving the subsystems. It also provides an interpretation of what has been made in terms of constraints.

From now on, assume that the information variable $\boldsymbol{Y}_t$ is the same for all subsystems. We consider a new problem derived from $(\mathcal{P})$:

$$\min_{\boldsymbol{U},\boldsymbol{X}} \mathbb{E}\Bigg(\sum_{i=1}^{N}\Big(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i,\boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i)\Big)\Bigg) , \qquad (\mathcal{P}_r)$$

subject to the modified coupling constraints:

$$\mathbb{E}\Big(\sum_{i=1}^{N}\Theta_t^i(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i) \;\Big|\; \boldsymbol{Y}_t\Big) = 0 .$$

Interpretation of DADP II

Proposition 1

Assume that the Lagrangian associated with Problem $(\mathcal{P}_r)$ has a saddle point. Then the DADP algorithm can be interpreted as the Uzawa algorithm applied to Problem $(\mathcal{P}_r)$.

Proof. Since the term $\mathbb{E}\big(\mathbb{E}(\boldsymbol{\Lambda}_t^{(k)} \mid \boldsymbol{Y}_t) \cdot \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i)\big)$ which appears in the cost function of subproblem i in DADP can be written:

$$\mathbb{E}\big(\mathbb{E}(\boldsymbol{\Lambda}_t^{(k)} \mid \boldsymbol{Y}_t) \cdot \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i)\big) = \mathbb{E}\big(\boldsymbol{\Lambda}_t^{(k)} \cdot \mathbb{E}(\Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \mid \boldsymbol{Y}_t)\big) \ ,$$

the global constraint really handled by DADP is:

$$\mathbb{E}\left(\sum_{i=1}^{N} \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \;\middle|\; \boldsymbol{Y}_t\right) = 0 \ . \qquad \square$$

DADP thus consists in replacing an almost-sure constraint by its conditional expectation w.r.t. the information variable $\boldsymbol{Y}_t$.

Summary

To summarize, DADP leads to solve the approximated problem:

$$\min_{\boldsymbol{U},\boldsymbol{X}} \mathbb{E}\bigg(\sum_{i=1}^{N}\sum_{t=0}^{T-1}\Big(L_t^i(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i,\boldsymbol{W}_t)+K^i(\boldsymbol{X}_T^i)\Big)\bigg) \text{ s.t. } \mathbb{E}\Big(\sum_{i=1}^{N}\Theta_t^i(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i)\;\Big|\;\boldsymbol{Y}_t\Big)=0\,,$$

whereas the true problem is:

$$\min_{\boldsymbol{U},\boldsymbol{X}} \mathbb{E}\bigg(\sum_{i=1}^{N}\sum_{t=0}^{T-1}\Big(L_t^i(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i,\boldsymbol{W}_t)+K^i(\boldsymbol{X}_T^i)\Big)\bigg) \text{ s.t. } \sum_{i=1}^{N}\Theta_t^i(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i)=0\,.$$

The conditional expectation constraint handled by DADP is a relaxed version of the almost sure constraint of the true problem.

An immediate consequence is that the DADP optimal value is an exact lower bound of the true problem optimal value.

Some questions

⋆ What is the suitable theoretical framework of the algorithm?

The convergence of Uzawa's algorithm is granted provided that:

- the problem is posed in Hilbert spaces,
- and it exists a saddle point.

It thus seems natural to place ourselves in a Hilbert space. But it is known (works by Rockafellar and Wets) that a saddle point doesn't exist in Hilbert spaces for such problems. . .

⋆ Does the approximate solution converge to the true solution?

Epiconvergence results are available w.r.t. the information delivered by $\boldsymbol{Y}_t$. But epiconvergence raises difficult technical problems when addressed to stochastic optimization problems.

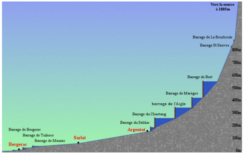
⋆ How to obtain a feasible solution from the approximate solution?

Use an appropriate heuristic (to be explained later on)!

Progress status

- First, we have obtained a lower bound for a global optimization problem with coupling constraints thanks to a price decomposition and coordination scheme (spatial decomposition).
- Second, we have computed the lower bound by dynamic programming (temporal decomposition)
- Using the price Bellman value functions, we have an heuristic procedure to devise an online policy for the global problem

We will apply this decomposition scheme to dams management problems



We now investigate **two** decomposition schemes (price and resource) to obtain lower **and** upper bounds for a global optimization problem.

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An abstract optimization problem

We consider the following optimization problem

$$V_0^\sharp = \min_{u^1 \in \mathcal{U}^1_{\mathrm{ad}}, \cdots, u^N \in \mathcal{U}^N_{\mathrm{ad}}} \sum_{i=1}^{N} J^i(u^i)$$

$$\text{s.t.} \quad \underbrace{\left(\Theta^1(u^1), \cdots, \Theta^N(u^N)\right) \in -S}_{\text{coupling constraint}}$$

with

- $u^i \in \mathcal{U}^i$ be a local decision variable
- $J^i : \mathcal{U}^i \to \mathbb{R}, \ i \in [\![1, N]\!]$ be a local objective
- $\mathcal{U}^i_{\mathrm{ad}}$ be a subset of $\mathcal{U}^i$
- $\Theta^i : \mathcal{U}^i \to \mathcal{C}^i$ be a local constraint mapping
- S be a subset of $\mathcal{C} = \mathcal{C}^1 \times \cdots \times \mathcal{C}^N$

We denote by $S^\star$ the dual cone of S

$$S^\star = \left\{\lambda \in \mathcal{C}^\star \mid \langle \lambda , r \rangle \geq 0 \quad \forall r \in S \right\}$$

Price and resource value functions

For each $i \in [\![1, N]\!]$,

- for any price $\lambda^i \in (\mathcal{C}^i)^\star$, we define the local price value
$$\underline{V}_0^i[\lambda^i] = \min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) + \langle \lambda^i , \Theta^i(u^i) \rangle$$
- for any resource $r^i \in \mathcal{C}^i$, we define the local resource value
$$\overline{V}_0^i[r^i] = \min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

Theorem 1 (Upper and lower bounds for optimal value)

- *For any admissible price* $\lambda = (\lambda^1, \cdots, \lambda^N) \in S^\star$
- *For any admissible resource* $r = (r^1, \cdots, r^N) \in -S$

$$\sum_{i=1}^N \underline{V}_0^i[\lambda^i] \;\leq\; V_0^\sharp \;\leq\; \sum_{i=1}^N \overline{V}_0^i[r^i]$$

The case of multistage stochastic optimization I

Assume that the local price value

$$\underline{V}_0^i[\lambda^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) + \langle \lambda^i , \Theta^i(u^i) \rangle ,$$

corresponds to a stochastic optimal control problem

$$\underline{V}_0^i[\boldsymbol{\Lambda}^i](x_0^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\bigg(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + \langle \boldsymbol{\Lambda}_t^i , \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \rangle + K^i(\boldsymbol{X}_T^i) \bigg)$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = f_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) , \quad \boldsymbol{X}_0^i = f_{-1}^i(\boldsymbol{W}_0)$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots , \boldsymbol{W}_t)$$

This local control problem can be solved by Dynamic Programming (DP) under restrictive assumptions:

- the noise process $\boldsymbol{W}$ is a white noise process
- the price process $\boldsymbol{\Lambda}^i$ follows a dynamics in small dimension

DP leads to a collection $\big\{ \underline{V}_t^i[\boldsymbol{\Lambda}^i] \big\}_{t \in [\![0,T]\!]}$ of local price value functions

The case of multistage stochastic optimization II

Similar considerations hold true for the local resource value

$$\overline{V}_0^i[r^i] = \min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

considered as a stochastic optimal control problem

$$\begin{aligned} \overline{V}_0^i[\boldsymbol{R}^i](x_0^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\bigg(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i) \bigg) \\ \text{s.t. } \boldsymbol{X}_{t+1}^i = f_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) \ , \ \ \boldsymbol{X}_0^i = f_{-1}^i(\boldsymbol{W}_0) \\ \sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t) \\ \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) = \boldsymbol{R}_t^i \end{aligned}$$

Provided that the dynamics of the resource process $\boldsymbol{R}^i$ is small,
DP leads to a collection $\big\{\overline{V}_t^i[\boldsymbol{R}^i]\big\}_{t \in [\![0,T]\!]}$ of local resource value functions

Mix of spatial and temporal decompositions I

For any admissible price process $\boldsymbol{\Lambda} \in S^{\star}$ and any admissible resource process $\boldsymbol{R} \in -S$, we have bounds of the optimal value $V_0^{\sharp}$

$$\sum_{i=1}^{N} \underline{V}_0^i[\boldsymbol{\Lambda}^i](x_0^i) \leq V_0^{\sharp} \leq \sum_{i=1}^{N} \overline{V}_0^i[\boldsymbol{R}^i](x_0^i)$$

1. To obtain the bounds, we perform spatial decompositions giving
   - a collection $\left\{\underline{V}_0^i[\boldsymbol{\Lambda}^i](x_0^i)\right\}_{i \in [\![1,N]\!]}$ of price local values
   - a collection $\left\{\overline{V}_0^i[\boldsymbol{R}^i](x_0^i)\right\}_{i \in [\![1,N]\!]}$ of resource local values

   The computation of these local values can be performed in parallel

2. To compute each local value, we perform temporal decomposition based on Dynamic Programming. For each i, we obtain
   - a sequence $\left\{\underline{V}_t^i[\boldsymbol{\Lambda}^i]\right\}_{t \in [\![0,T]\!]}$ of price local value functions
   - a sequence $\left\{\overline{V}_t^i[\boldsymbol{R}^i]\right\}_{t \in [\![0,T]\!]}$ of resource local value functions

   The computation of these local values functions is done sequentially

Mix of spatial and temporal decompositions II

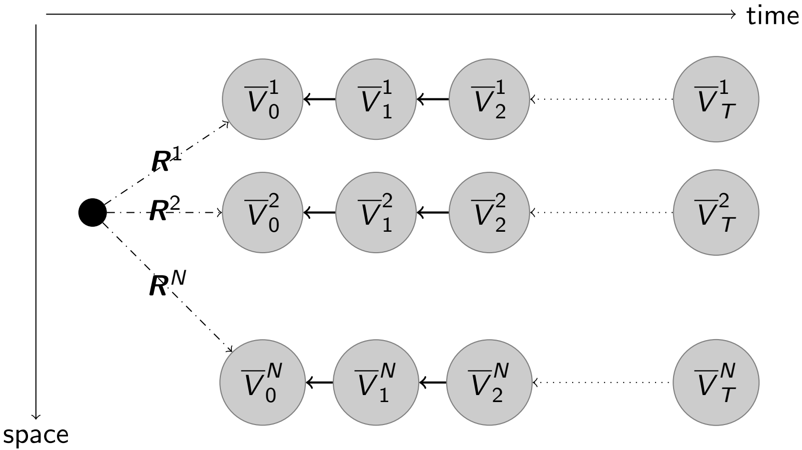


Figure: The case of resource decomposition

The case of deterministic price and resource processes

We assume that $\boldsymbol{W}$ is a white noise process, and we restrict ourselves to **deterministic** admissible processes $r \in -S$, $\lambda \in S^\star$

- The local value functions $\underline{V}_t^i[\lambda^i]$ and $\overline{V}_t^i[r^i]$ are easy to compute because they only depend on the local state variable x^i

- It is easy to obtain tighter bounds by maximizing the lower bound w.r.t. prices and minimizing the upper bound w.r.t. resources

$$\sup_{\lambda \in S^\star} \sum_{i=1}^{N} \underline{V}_0^i[\lambda^i](x_0^i) \leq V_0^\sharp \leq \inf_{r \in -S} \sum_{i=1}^{N} \overline{V}_0^i[r^i](x_0^i)$$

Heuristic procedure to produce online admissible policies

The local value functions $\underline{V}_t^i[\lambda^i]$ and $\overline{V}_t^i[r^i]$ allow the computation of global policies by solving static optimization problems

- In the case of local price value functions, the policy is obtained as

$$\underline{\gamma}_t(x_t^1, \cdots, x_t^N) \in \underset{u_t^1, \cdots, u_t^N}{\arg\min} \, \mathbb{E}\left(\sum_{i=1}^{N} L_t^i(x_t^i, u_t^i, \boldsymbol{W}_{t+1}) + \sum_{i=1}^{N} \underline{V}_{t+1}^i[\lambda^i](\boldsymbol{X}_{t+1}^i) \right)$$

$$\text{s.t.} \quad \boldsymbol{X}_{t+1}^i = f_t^i(x_t^i, u_t^i, \boldsymbol{W}_{t+1}) \;, \quad \forall i \in [\![1, N]\!]$$

$$\big(\Theta_t(x_t^1, u_t^1), \cdots, \Theta_t(x_t^N, u_t^N)\big) \in -S_t$$

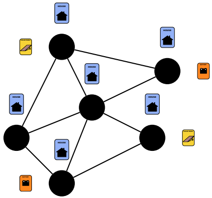
- A global policy based on resource value functions is also available

Estimating the expected cost of such policies by Monte Carlo simulation leads to a **statistical** upper bound of the optimal cost of the problem

Progress status

- First, we have obtained lower and upper bounds for a global optimization problem with coupling constraints thanks to two spatial decomposition schemes
  - price decomposition
  - resource decomposition
- Second, we have computed the lower and upper bounds by dynamic programming (temporal decomposition)
- Using the price and resource Bellman value functions, we have devised two online policies for the global problem

We will apply these decomposition schemes to large-scale network problems



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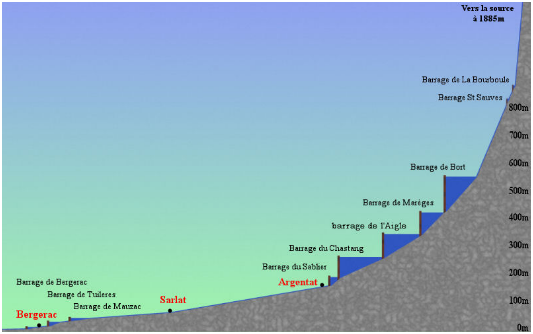
The Durance cascade

Motivation

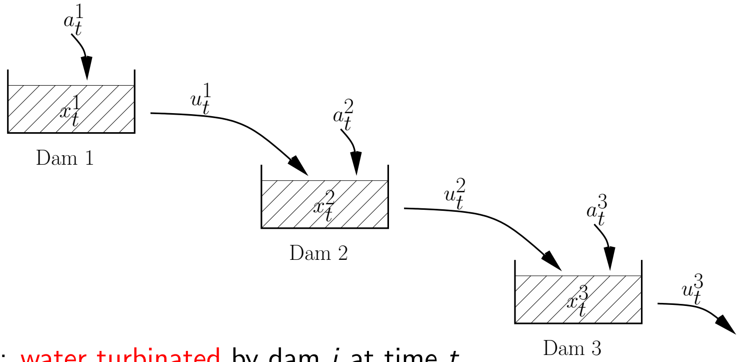
Electricity production management for hydro valleys



- *1 year time horizon*: compute each month the "values of water" (Bellman functions)
- *stochastic framework*: rain, market prices
- *large-scale valley*: 5 dams and more

We wish to remain within the scope of Dynamic Programming.

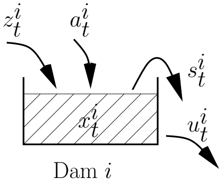
Operating scheme



u_t^i : water turbinated by dam i at time t,
x_t^i : water volume of dam i at time t,
a_t^i : water inflow at dam i at time t,
p_t^i : market price at dam i at time t,

Randomness: $w_t^i = (a_t^i, p_t^i)$ and $w_t = (w_t^1, \ldots, w_t^N)$.

Dynamics and costs functions



Dam dynamics:

$$x_{t+1}^i = f_t^i(x_t^i, u_t^i, w_t^i, z_t^i) \; ,$$
$$= x_t^i - u_t^i + a_t^i + z_t^i - s_t^i \; ,$$

z_t^{i+1} being the outflow of dam i:

$$z_t^{i+1} = g_t^i(x_t^i, u_t^i, w_t^i, z_t^i) \; ,$$
$$= u_t^i + \underbrace{\max\left\{0, x_t^i - u_t^i + a_t^i + z_t^i - \overline{x}^i\right\}}_{s_t^i} \; .$$

We assume the Hazard-Decision information structure (u_t^i is chosen once w_t^i is observed), so that $\underline{u}^i \leq u_t^i \leq \min\left\{\overline{u}^i, x_t^i + a_t^i + z_t^i - \underline{x}^i\right\}$.

Gain at time $t < T$: $\quad L_t^i(x_t^i, u_t^i, w_t^i, z_t^i) = p_t^i u_t^i - \epsilon (u_t^i)^2$.

Final gain at time T: $\quad K^i(x_T^i) = -a^i \min\{0, x_T^i - \widehat{x}^i\}^2$.

Stochastic optimization problem

The global optimization problem reads:

$$\max_{(\boldsymbol{X},\boldsymbol{U},\boldsymbol{Z})} \mathbb{E}\left(\sum_{i=1}^{N}\left(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i,\boldsymbol{W}_t^i,\boldsymbol{Z}_t^i) + K^i(\boldsymbol{X}_T^i)\right)\right),$$

subject to:

$$\boldsymbol{X}_{t+1}^i = f_t^i(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i,\boldsymbol{W}_t^i,\boldsymbol{Z}_t^i) \ , \ \ \forall i \ , \ \ \forall t \ ,$$

$$\boldsymbol{U}_t^i \preceq \sigma\big(\boldsymbol{W}_0,\dots,\boldsymbol{W}_t\big) \ , \qquad \forall i \ , \ \ \forall t \ ,$$

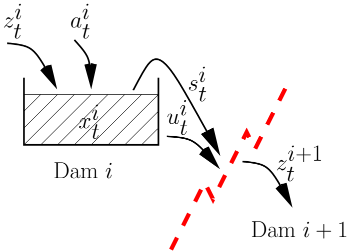
$$\boldsymbol{Z}_t^{i+1} = g_t^i(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i,\boldsymbol{W}_t^i,\boldsymbol{Z}_t^i) \ , \ \ \forall i \ , \ \ \forall t \ .$$

Assumption. Noises $\boldsymbol{W}_0,\dots,\boldsymbol{W}_{T-1}$ are *independent over time*.

Standard price decomposition

- Dualize the coupling constraints $\boldsymbol{Z}_t^{i+1} = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i)$.
  Note that the associated multiplier $\boldsymbol{\Lambda}_t^{i+1}$ is a random variable.

- Minimize the dual problem (using a gradient-like algorithm).



- At iteration k, the duality term:

  $$\boldsymbol{\Lambda}_t^{i+1,(k)} \cdot \big(\boldsymbol{Z}_t^{i+1} - g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i)\big) ,$$

  is the difference of two terms:

  - $\boldsymbol{\Lambda}_t^{i+1,(k)} \cdot \boldsymbol{Z}_t^{i+1}$ $\rightsquigarrow$ dam $i+1$,
  - $\boldsymbol{\Lambda}_t^{i+1,(k)} \cdot g_t^i(\cdots)$ $\rightsquigarrow$ dam i.

- Dam by dam decomposition for the maximization in $(\boldsymbol{X}, \boldsymbol{U}, \boldsymbol{Z})$ at $\boldsymbol{\Lambda}_t^{i+1,(k)}$ fixed.

Application of DADP

The i-th subproblem writes:

$$\max_{\boldsymbol{U}^i,\boldsymbol{Z}^i,\boldsymbol{X}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} \Big(L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) + \boldsymbol{\Lambda}_t^{i,(k)} \cdot \boldsymbol{Z}_t^i - \boldsymbol{\Lambda}_t^{i+1,(k)} \cdot g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) \Big) + K^i(\boldsymbol{X}_T^i) \right) ,$$

but $\boldsymbol{\Lambda}_t^{i,(k)}$ depends on the whole past of noises $(\boldsymbol{W}_0, \ldots, \boldsymbol{W}_t)$...

We recall that the core idea of DADP is

- to replace the constraint $\boldsymbol{Z}_t^{i+1} - g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) = 0$ by its conditional expectation with respect to $\boldsymbol{Y}_t^i$:

$$\mathbb{E}\big(\boldsymbol{Z}_t^{i+1} - g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) \bigm| \boldsymbol{Y}_t^i\big) = 0 ,$$

- where $(\boldsymbol{Y}_0^i, \ldots, \boldsymbol{Y}_{T-1}^i)$ is a “well-chosen” information process.

Subproblems in DADP

DADP thus consists of a constraint relaxation, which is equivalent to replace the multiplier $\boldsymbol{\Lambda}_t^{i,(k)}$ by its conditional expectation $\mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k)} \mid \boldsymbol{Y}_t^{i-1}\big)$.

The expression of the i-th subproblem becomes:

$$\max_{\boldsymbol{U}^i,\boldsymbol{Z}^i,\boldsymbol{X}^i} \mathbb{E}\Bigg(\sum_{t=0}^{T-1}\Big(L_t^i\big(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i,\boldsymbol{W}_t^i,\boldsymbol{Z}_t^i\big) + \mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k)} \mid \boldsymbol{Y}_t^{i-1}\big)\cdot\boldsymbol{Z}_t^i \\ - \mathbb{E}\big(\boldsymbol{\Lambda}_t^{i+1,(k)} \mid \boldsymbol{Y}_t^{i}\big)\cdot g_t^i\big(\boldsymbol{X}_t^i,\boldsymbol{U}_t^i,\boldsymbol{W}_t^i,\boldsymbol{Z}_t^i\big)\Big) \\ + K^i\big(\boldsymbol{X}_T^i\big)\Bigg) .$$

If each process $\boldsymbol{Y}^i$ follows a dynamical equation, DP applies.

A crude relaxation: $\boldsymbol{Y}_t^i \equiv \text{cste}$

1. The multipliers $\boldsymbol{\Lambda}_t^{i,(k)}$ appear only in the subproblems by means of their expectations $\mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k)}\big)$, so that each subproblem involves a 1-dimensional state variable.

2. For the gradient algorithm, the coordination task reduces to:

$$\mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k+1)}\big) = \mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k)}\big) - \rho_t \mathbb{E}\Big(\boldsymbol{Z}_t^{i+1,(k)} - g_t^i\big(\boldsymbol{X}_t^{i,(k)}, \boldsymbol{U}_t^{i,(k)}, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^{i,(k)}\big)\Big) .$$

3. The constraints taken into account by DADP are

$$\mathbb{E}\Big(\boldsymbol{Z}_t^{i+1} - g_t^i\big(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i\big)\Big) = 0 .$$

The DADP solutions do not satisfy the initial constraints: we need to use an heuristic method to regain admissibility.

Admissible online policies for the global problem

We have computed N local Bellman functions $\underline{V}_t^i$ at each time t, each depending on a single state variable x^i, whereas we need one global Bellman function V_t depending on the global state $(x^1, \ldots, x^N)$ in order to design the decisions at time t.

Heuristic procedure: form the following global Bellman function:

$$\widehat{V}_t(x^1, \ldots, x^N) = \sum_{i=1}^{N} \underline{V}_t^i(x^i) \ ,$$

and solve at each time t the one-step DP problem:

$$\max_{u,z} \ \sum_{i=1}^{N} L_t^i(x^i, u^i, w_t^i, z^i) + \widehat{V}_{t+1}(x_{t+1}^1, \ldots, x_{t+1}^N) \ ,$$

$$\text{s.t.} \quad x_{t+1}^i = f_t^i(x^i, u^i, w_t^i, z^i) \ , \quad z^{i+1} = g_t^i(x^i, u^i, w_t^i, z^i) \quad \forall i \ .$$

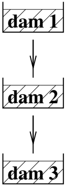
Bounds for the problem optimal cost

Let $V_0^\sharp$ be the optimal value of the global optimization problem.

1. As already noticed, the optimal value computed by DADP, that is, the sum of the optimal values of the subproblems once the optimal multiplier $\mathbb{E}(\boldsymbol{\Lambda}_t)$ have been obtained, is an exact upper bound[2] of $V_0^\sharp$.
2. The expected value associated to the admissible policy induced by the sum of the local Bellman functions is a lower bound of global problem optimal value. This expected value being evaluated by Monte Carlo, we in fact have at disposal a statistical lower bound of $V_0^\sharp$.

[2] and not a lower bound because we are dealing with a maximization problem

Four case studies



Discretization

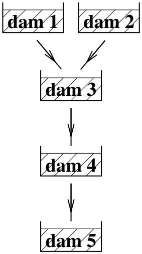
$T \rightsquigarrow 12$

$\boldsymbol{X} \rightsquigarrow 41$

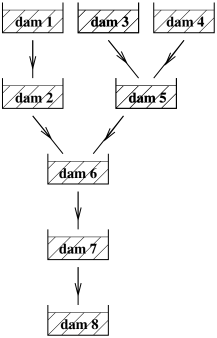
$\boldsymbol{U} \rightsquigarrow 6$

$\boldsymbol{W} \rightsquigarrow 10$

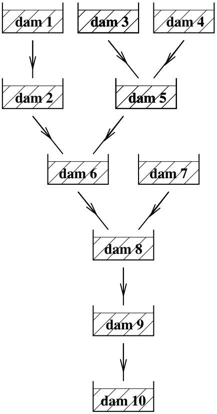
3-Dams



5-Dams



8-Dams



10-Dams

Results

Valley	3-Dams	5-Dams	8-Dams	10-Dams
DP CPU time	*5*'	*461200*'	N.A.	N.A.
DP value	2482.3	4681.6	N.A.	N.A.
SDDP exact UB	2491.3	4694.1	11958.3	17256.0
SDDP value	2481.6	4680.9	11834.4	17069.3
SDDP CPU time	*3*'	*7*'	*13*'	*50*'

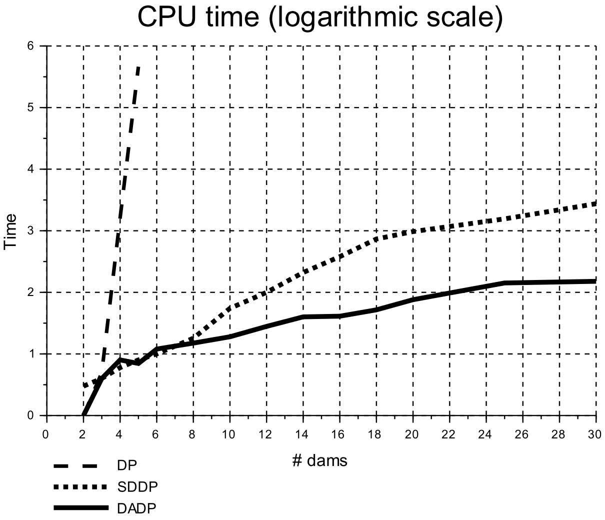
Table: Results obtained by DP and SDDP

Valley	3-Dams	5-Dams	8-Dams	10-Dams
DADP CPU time	*3*'	*5*'	*12*'	*24*'
DADP exact UB	2687.5	4885.9	12451.0	17933.5
DADP value	2401.6	4633.7	11573.0	16759.8
Gap with SDDP	**−3.2%**	**−1.0%**	**−2.2%**	**−1.8%**

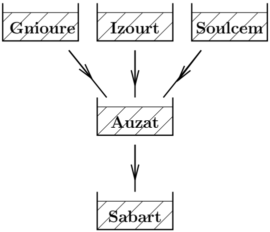
Table: Results obtained by DADP

Results obtained using a 4 cores – 8 threads Intel®Core i7 based computer.

Challenging the curse of dimensionality



Two realistic valleys

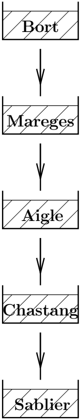


Discretization

$T \rightsquigarrow 12$, $W \rightsquigarrow 10$

realistic grids for U and X

Vicdessos



Dordogne

Results

Valley	**Vicdessos**	**Dordogne**
SDDP CPU time	*9*'	*17*'
SDDP exact UB	2258.0	22310.0
SDDP value	2244.3	22136.1

Table: Results obtained by SDDP

Valley	**Vicdessos**	**Dordogne**
DADP CPU time	*10*'	*210*'
DADP exact UB	2285.6	22991.1
DADP value	2237.4	21650.8
Gap with SDDP	$-\mathbf{0.3}\%$	$-\mathbf{2.2}\%$

Table: Results obtained by DADP

Conclusions and perspectives

Conclusions for this study

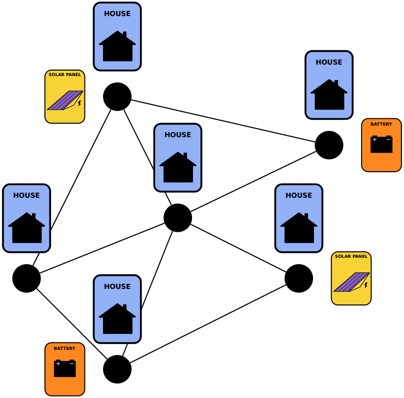
- Fast numerical convergence of the DADP method.
- Near-optimal results even when using a “crude” relaxation.
- Method that can be used for very large valleys

General perspectives

- Apply to more complex topologies (microgrids).
- Use other decomposition methods (resource, prediction).
- Study the theoretical questions (convergence. . .).

Motivation

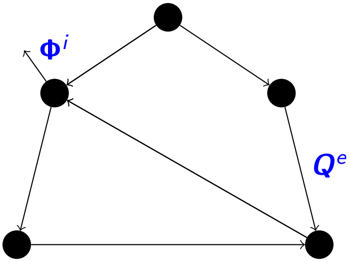
We consider a *peer-to-peer* microgrid where houses exchange energy, and we formulate it as a large-scale stochastic optimization problem



How to manage it in an (sub)optimal manner?

Network and flows

Directed graph $G = (\mathcal{V}, \mathcal{E})$



- $\boldsymbol{Q}_t^e$ flow through edge e,
- $\boldsymbol{\Phi}_t^i$ flow imported at node i

Let A be the *node-edge* incidence matrix

Each node corresponds to a building with its own devices (battery, hot water tank, solar panel. . .)

At each time $t \in [\![0, T-1]\!]$, the Kirchhoff current law couples node and edge flows

$$A\boldsymbol{Q}_t + \boldsymbol{\Phi}_t = 0$$

Optimization problem at a given node

At each **node** $i \in \mathcal{V}$, given a node flow process $\boldsymbol{\Phi}^i$, we minimize the house cost

$$J^i_{\mathcal{V}}(\boldsymbol{\Phi}^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\bigg(\sum_{t=0}^{T-1} L^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1}) + K^i(\boldsymbol{X}^i_T) \bigg)$$

subject to, for all $t \in [\![0, T-1]\!]$

i) nodal dynamics constraints (battery, hot water tank)

$$\boldsymbol{X}^i_{t+1} = f^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1})$$

ii) non-anticipativity constraints (future remains unknown)

$$\sigma(\boldsymbol{U}^i_t) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_{t+1})$$

iii) nodal load balance equations (demand - production = import)

$$\Delta^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1}) = \boldsymbol{\Phi}^i_t$$

Transportation cost and global optimization problem

We define the network cost as the sum over time and **edges** of the costs of flows $\boldsymbol{Q}_t^e$ through the edges of the network

$$J_{\mathcal{E}}(\boldsymbol{Q}) = \mathbb{E}\left(\sum_{t=0}^{T-1} \sum_{e \in \mathcal{E}} l_t^e(\boldsymbol{Q}_t^e)\right)$$

This transportation cost is additive in space, in time and in uncertainty!

The global optimization problem is obtained by gathering all elements

$$V_0^\sharp = \min_{\boldsymbol{\Phi}, \boldsymbol{Q}} \sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(\boldsymbol{\Phi}^i) + J_{\mathcal{E}}(\boldsymbol{Q})$$

$$\text{s.t. } A\boldsymbol{Q} + \boldsymbol{\Phi} = 0$$

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$$\text{s.t. } A\boldsymbol{Q} + \boldsymbol{\Phi} = 0$$

Price and resource decompositions

- Price problem:

$$\underline{V}_0[\boldsymbol{\Lambda}] = \min_{\boldsymbol{\Phi},\boldsymbol{Q}} \sum_{i \in \mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{\Phi}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) + \langle \boldsymbol{\Lambda} \, , A\boldsymbol{Q} + \boldsymbol{\Phi} \rangle$$

$$= \sum_{i \in \mathcal{V}} \underbrace{\Big(\min_{\boldsymbol{\Phi}_i} \; J^i_{\mathcal{V}}(\boldsymbol{\Phi}^i) + \langle \boldsymbol{\Lambda}^i \, , \boldsymbol{\Phi}^i \rangle \Big)}_{\text{Node } i\text{'s subproblem}} + \underbrace{\Big(\min_{\boldsymbol{Q}} \; J_{\mathcal{E}}(\boldsymbol{Q}) + \langle A^\top \boldsymbol{\Lambda} \, , \boldsymbol{Q} \rangle \Big)}_{\text{Network subproblem}}$$

- Resource problem:

$$\overline{V}_0[\boldsymbol{R}] = \min_{\boldsymbol{\Phi},\boldsymbol{Q}} \sum_{i \in \mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{\Phi}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) \quad \text{s.t.} \quad A\boldsymbol{R} + \boldsymbol{\Phi} = 0 \, , \quad \boldsymbol{Q} = \boldsymbol{R}$$

$$= \sum_{i \in \mathcal{V}} \Big(\min_{\boldsymbol{\Phi}_i} \; J^i_{\mathcal{V}}(\boldsymbol{\Phi}^i) \;\text{s.t.}\; \boldsymbol{\Phi}^i = -(A\boldsymbol{R})^i \Big) + \Big(\min_{\boldsymbol{Q}} \; J_{\mathcal{E}}(\boldsymbol{Q}) \;\text{s.t.}\; \boldsymbol{Q} = \boldsymbol{R} \Big)$$

Find **deterministic** processes $\widehat{\lambda}$ and $\widehat{r}$ with a gap as small as possible

$$\sup_{\lambda} \underline{V}_0[\lambda] \leq V_0^\sharp \leq \inf_{r} \overline{V}_0[r]$$

Price and resource decompositions

- Price problem:

$$\underline{V}_0[\boldsymbol{\Lambda}] = \min_{\boldsymbol{\Phi},\boldsymbol{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{\Phi}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) + \langle \boldsymbol{\Lambda} \,, A\boldsymbol{Q} + \boldsymbol{\Phi} \rangle$$

$$= \sum_{i\in\mathcal{V}} \underbrace{\Big(\min_{\boldsymbol{\Phi}_i} J^i_{\mathcal{V}}(\boldsymbol{\Phi}^i) + \langle \boldsymbol{\Lambda}^i \,, \boldsymbol{\Phi}^i \rangle \Big)}_{\text{Node } i\text{'s subproblem}} + \underbrace{\Big(\min_{\boldsymbol{Q}} J_{\mathcal{E}}(\boldsymbol{Q}) + \langle A^\top \boldsymbol{\Lambda} \,, \boldsymbol{Q} \rangle \Big)}_{\text{Network subproblem}}$$

- Resource problem:

$$\overline{V}_0[\boldsymbol{R}] = \min_{\boldsymbol{\Phi},\boldsymbol{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{\Phi}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) \quad \text{s.t.} \quad A\boldsymbol{R} + \boldsymbol{\Phi} = 0 \,, \quad \boldsymbol{Q} = \boldsymbol{R}$$

$$= \sum_{i\in\mathcal{V}} \Big(\min_{\boldsymbol{\Phi}_i} J^i_{\mathcal{V}}(\boldsymbol{\Phi}^i) \ \text{s.t.} \ \boldsymbol{\Phi}^i = -(A\boldsymbol{R})^i \Big) + \Big(\min_{\boldsymbol{Q}} J_{\mathcal{E}}(\boldsymbol{Q}) \ \text{s.t.} \ \boldsymbol{Q} = \boldsymbol{R} \Big)$$

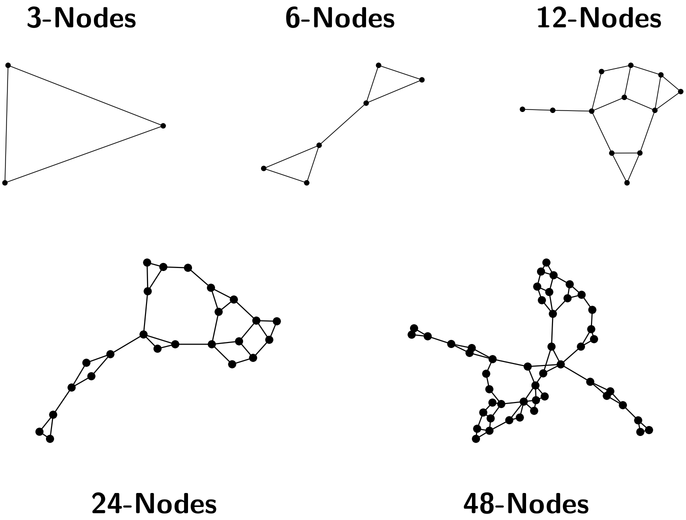
Find **deterministic** processes $\widehat{\lambda}$ and $\widehat{r}$ with a gap as small as possible

$$\sup_{\lambda} \underline{V}_0[\lambda] \leq V_0^\sharp \leq \inf_{r} \overline{V}_0[r]$$

Progress status

- We have formulated a multistage stochastic optimization problem on a graph
- We are able to handle the coupling Kirchhoff constraints by the two methods presented earlier
  - Price decomposition
  - Resource decomposition
- Now, we show the scalability of decomposition algorithms (we solve problems with up to 48 buildings)

Different urban configurations



Problem settings

Thanks to the periodicity of demands and electricity tariffs, the microgrid management problem can be solved day by day

- One day horizon with a 15mn time step: $T = 96$
- Weather corresponds to a sunny day in Paris *(June 28, 2015)*
- We mix three kinds of buildings
  1. battery + electrical hot water tank
  2. solar panel + electrical hot water tank
  3. electrical hot water tank

  and we suppose that all consumers are sharing their devices

Algorithms implemented on the problem

SDDP

We use the SDDP algorithm to solve the problem globally...

- but noises $\boldsymbol{W}_t^1, \cdots, \boldsymbol{W}_t^N$ are independent node by node, so that the support size of the noise may be huge ($|\text{supp}(\boldsymbol{W}_t^i)|^N$). We must resample the noise to be able to compute the cuts

Price decomposition

Spatial decomposition and maximization w.r.t. a deterministic price λ

- Each nodal subproblem solved by a DP-like method
- Maximisation w.r.t. λ by Quasi-Newton (BFGS) method

$$\lambda^{(k+1)} = \lambda^{(k)} + \rho^{(k)} H^{(k)} \nabla \underline{V}_0[\lambda^{(k)}]$$

- Oracle $\nabla \underline{V}_0[\lambda]$ estimated by Monte Carlo ($N^{scen} = 1,000$)

Resource decomposition

Spatial decomposition and minimization w.r.t. a deterministic resource process r

Exact upper and lower bounds on the global problem

	Network	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
State dim.	$\lvert\mathbb{X}\rvert$	4	8	16	32	64
SDDP	time	1'	3'	10'	79'	453'
SDDP	LB	225.2	455.9	889.7	1752.8	3310.3
Price	time	6'	14'	29'	41'	128'
Price	LB	213.7	447.3	896.7	1787.0	3396.4
Resource	time	3'	7'	22'	49'	91'
Resource	UB	253.9	527.3	1053.7	2105.4	4016.6

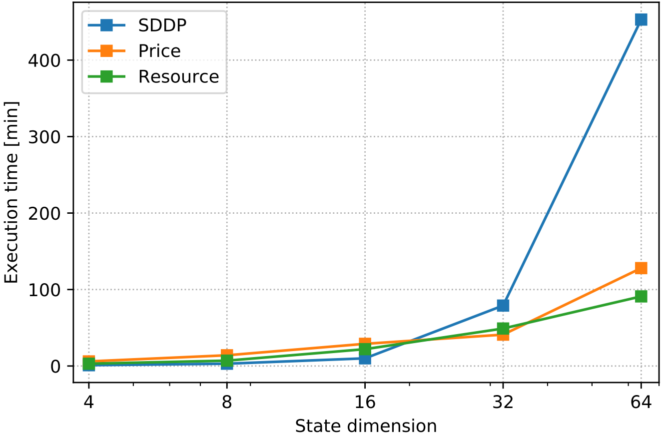
For the **48-Nodes** microgrid,

- price decomposition gives a (slightly) better exact lower bound than SDDP

$$\underbrace{3310.3}_{\underline{V}_0[sddp]} \leq \underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^\sharp \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

- **price decomposition is more than 3 times faster than SDDP**

Time evolution



Policy evaluation by Monte Carlo (1,000 scenarios)

	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
SDDP policy	226 ± 0.6	471 ± 0.8	936 ± 1.1	1859 ± 1.6	3550 ± 2.3
Price policy	228 ± 0.6	464 ± 0.8	923 ± 1.2	1839 ± 1.6	3490 ± 2.3
Gap	+0.9 %	-1.5%	-1.4%	-1.1%	-1.7%
Resource policy	229 ± 0.6	471 ± 0.8	931 ± 1.1	1856 ± 1.6	3503 ± 2.2
Gap	+1.3 %	0.0%	-0.5%	-0.2%	-1.2%

All the cost values above are statistical upper bounds of $V_0^\sharp$

For the **48-Nodes** microgrid,

- price policy beats SDDP policy and resource policy

$$V_0^\sharp \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{3550}_{C[sddp]}$$

- the exact upper bound given by resource decomposition is not so tight

$$\underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^\sharp \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

gap <3% ≈ 3% >18%

Conclusions

- We have two algorithms that decompose spatially and temporally a large-scale optimization problem under coupling constraints.
- In our case study, price decomposition beats SDDP for large instances (≥ 24 nodes)
  - in computing time (more than twice faster)
  - in precision (more than 1% better)
- Price decomposition gives (in a surprising way) a tight lower bound, whereas the upper bound given by resource decomposition is weak (which is understandable on the case study)
- Can we obtain tighter bounds? *especially for resource decomposition...*
  If we select properly price $\boldsymbol{\Lambda}$ and resource $\boldsymbol{R}$ processes among the class of Markovian processes (instead of deterministic ones) we can obtain "better" nodal value functions (with an extended local state)

Further details in

F. Pacaud
Decentralized Optimization Methods for Efficient Energy Management under Stochasticity
PhD Thesis, Université Paris Est, 2018

P. Carpentier, J.-P. Chancelier, M. De Lara and F. Pacaud
Computation by Decomposition of Upper and Lower Bounds for Large Scale Multistage Stochastic Optimization Problems
Working paper, 2019

THANK YOU FOR YOUR ATTENTION

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