

Interface Course 2019
on
Stochastic Optimization for Large-Scale Systems

November 4 to 8, 2019

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Abstract

Sensors and data abound. This spatial and temporal information is supposed to allow a better management — in new energy systems, transport or eco-industrial parks, to quote a few examples. This leads to problems of large-scale optimization, the formulation of which is delicate. Indeed, one needs to take into account at least three dimensions: *temporal* (stages of decision, dynamics, inertia), *spatial* (different decision units connected by flows), *stochastic* (various scenarios are possible) and thus risk. This leads to multi-stage stochastic optimization problems which enter the class of *large-scale systems*.

The 4 days session Interface 2019 *Stochastic Optimization for Large-Scale Systems* will present tools and methods to formulate and solve such problems. For this purpose, courses, computing sessions and interactive sessions will alternate.

The school addresses a mixed public, in the academy and in the industry. These four days of immersion in the CIRM (International Center of Mathematical Meetings, Marseille) will make discussions and exchanges possible, be it during or outside the training sessions. The institutional organizers are the ENSTA ParisTech and the École des Ponts ParisTech.

Keywords: stochastic programming, scenario trees, progressive hedging, stochastic optimal control, stochastic dynamic programming, stochastic dual dynamic programming, spatial decomposition.

Link: <http://cermics.enpc.fr/~delara/SES0/courseSES02019/courseSES02019/>

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1 Organizing committee, speakers and participants

1.1 Organizing committee

The organizing committee is made of the following French researchers

- Pierre Carpentier (ENSTA ParisTech)
<http://uma.ensta-paristech.fr/~pcarpent>
- Jean-Philippe Chancelier (ENPC ParisTech)
<http://cermics.enpc.fr/~jpc/>
- Michel De Lara (ENPC ParisTech)
<http://cermics.enpc.fr/~delara/>
- Vincent Leclère (ENPC ParisTech)
<http://cermics.enpc.fr/~leclerev/>

1.2 Scientific committee

- R. Tyrrell Rockafellar (University of Washington, Seattle, USA)
<http://www.math.washington.edu/~rtr/mypage.html>
- Alexander Shapiro (Georgia Tech, USA)
<http://www2.isye.gatech.edu/~ashapiro/>
- Roger Wets (UC Davis, USA)
<https://www.math.ucdavis.edu/~rjbw/mypage/Home.html>

1.3 Target audience and prerequisites

Target audience

The course mainly deals with optimization in a dynamical setting and subject to uncertain perturbations. The iconic example consists of managing a stock when facing an uncertain demand over a given time span. The decisions of refilling the stock are taken at given discrete instants, knowing the past demand uncertainties.

Such optimization problems occur in most industries and services dealing with stochastic dynamical systems, such as dam management, battery aging, smart-grid control in the energy field, or managing dynamic equilibrium between demand and offer in a context where production and transport are coupled via a network (petroleum industry).

Prerequisites

Prerequisites are:

- basic notions in Probability (random variables, probability distribution, expectation and conditional expectation, Monte Carlo approximation),
- basic notions in Optimization (linear programming, optimality conditions, Lagrangian duality, gradient-like algorithms).

Computer and softwares:

- bring your personal laptop for computer practical sessions

- we will be using `julia`, `jupyter` and a linear solver like `GLPK` that you should have installed before arriving: see instructions available at <https://github.com/leclere/TP-CIRM>

2 Program

During the first part of the course (sessions 1 to 3), we will present the two standard mathematical approaches dealing with stochastic multistage optimization problems, that is, stochastic programming and stochastic optimal control. The sessions include several practical works in Julia.

Multistage stochastic optimization problems are often too large to be solved in a straightforward manner. During the second part of the course (sessions 5 to 8), we will present advanced methods allowing to solve such problems by decomposition, that is, by iteratively (in parallel or sequentially) solving subproblems of a reasonable size rather than the whole one. We will recall the notions from convex optimization and duality theory necessary for devising such methods.

2.1 Session 0: Monday November 4, 2019 (morning).

Participants Reception

Speaker: Vincent Leclère (10:00–12:00)

Vincent Leclère will be available to help participants install the software Julia. Indeed, practical sessions will be done with the Julia language, relying on the JuMP library. It would be largely preferable to have `julia` installed when you arrive for the Interface session. Please contact vincent.leclere@enpc.fr if you have difficulties with the installation.

The code for the practical sessions and instructions for installation are available on <https://github.com/leclere/TP-CIRM>

2.2 Session 1: Monday November 4, 2019 (afternoon).

Introduction to Stochastic Optimization

- 14:00–14:30: Roundtable

Speaker: Michel De Lara (14:30–16:00)

The course starts with basics in stochastic optimization.

- 14:30–16:00: One-stage Stochastic programming
- break

Speaker: Vincent Leclère (16:30–18:00)

- 16:30–18:00: Practical work: the newsvendor problem

2.3 Session 2: Tuesday November 5, 2019 (morning).

Stochastic Multistage Optimization

Speaker: Vincent Leclère (09:00–12:30)

When the problem incorporates several time steps, the stochastic programming approach proposes to build a scenario tree of uncertainties, and to attach one control per node, hence leading to an equivalent deterministic formulation. We present alternative equivalent deterministic formulations.

We discuss the importance of information structures in multistage optimization, as well as how to use such structures to obtain computable bounds.

We end the session with an hands-on project, that tackles a multistage version of the problem studied in Session 1.

- 09:00–10:30: Stochastic programming: from one-stage to two-stage (and multistage) problems
- break
- 11:00–12:30: Practical work: the flower girl problem

2.4 Session 3: Tuesday November 5, 2019 (afternoon).

Stochastic Optimal Control and Dynamic Programming

Speaker: Jean-Philippe Chancelier and Michel De Lara (14:00–15:30)

The stochastic optimal control approach looks at the problem from a Markovian point of view, where uncertainties are stagewise independent. We present a generic way of solving this problem by Dynamic Programming.

- 14:00–15:00: Stochastic optimal control. Dynamic Programming approach
- 15:00–15:30: Management of reservoirs
- break

Speaker: Vincent Leclère (16:00–17:30)

We end the session by applying Dynamic Programming to the problem studied in Session 2.

- 16:00–17:30: Practical work: the flower girl problem (continued)

2.5 Session 4: Wednesday November 6, 2019 (morning).

Decomposition Methods. Progressive Hedging Algorithm.

Speaker: Michel De Lara (09:00–10:00)

Multistage stochastic optimization problems are, by essence, complex because their solutions are indexed both by stages (time) and by uncertainties (scenarios). Quite often, solutions are also indexed by decision units, like nodes in a graph (space), or agents in a team. Hence, their large scale nature makes decomposition methods appealing. We present, in an unified framework, three main approaches and methods to decompose multistage stochastic optimization problems for numerical resolution: time decomposition (and state-based resolution methods, like Stochastic Dynamic Programming, in Stochastic Optimal Control); scenario decomposition (like Progressive Hedging in Stochastic Programming); spatial decomposition (price or resource decompositions).

- 09:00–10:00: General overview of decomposition methods

- break

Speaker: Jean-Philippe Chancelier and Vincent Leclère (10:30–12:30)

When a stochastic multistage optimization optimization is formulated on a large number of scenarios, decomposition techniques can be used to iteratively solve a collection of subproblems, each formulated along a single scenario.

We present two approaches: (augmented) Lagrangian decomposition along scenarios, leading to the Progressive Hedging algorithm (morning), and Bender’s decomposition leading to the L-Shaped method (afternoon).

- 10:30–12:30: Progressive Hedging algorithm

2.6 Session 5: Wednesday November 6, 2019 (afternoon).

L-Shaped method. Stochastic Dual Dynamic Programming (SDDP)

Speaker: Vincent Leclère (15:30–19:00)

- 15:30–17:00: L-Shaped method
- break

In the framework of dynamic programming, the SDDP (Stochastic Dual Dynamic Programming) method allows to push forward the limits of the curse of dimensionality. Taking advantage of linearity and convexity, SDDP builds (lower) approximations of the Bellman value functions in an iterative way.

- 17:30–19:00: Stochastic Dual Dynamic Programming algorithm

2.7 Session 6: Thursday November 7, 2019 (morning).

Practical Works on SDDP

Speaker: Vincent Leclère (09:00–12:30)

The morning is devoted to a computer session aiming at discovering SDDP algorithm through the Julia software (SDDP.jl package).

- 09:00–12:30: Practical work: SDDP in action

2.8 Session 7: Thursday November 6, 2019 (afternoon).

Interactive Session

Speaker: Participants and Michel De Lara (14:00–17:30)

During this afternoon, industrials and PhD students can present their current problems. It will be the occasion to address some specific difficulties encountered when dealing with real-life problems.

- 14:00–15:30: Discussion with industrials and PhD students presenting their current problems.
- 16:00–17:00 Renewal and operation of a battery: mixing time blocks and price/resource decompositions methods

- 17:00–18:30: Discussion with industrials and PhD students presenting their current problems.

Among possible questions, let us mention

- how to take into account correlated noises,
- how to incorporate risk in the modeling,
- how to devise a good final cost.

Based on industrial participant’s proposals, we will organize small groups to tackle specific issues during the numerous “course free” periods enabled by the immersive nature of the Interface program.

2.9 Session 8: Friday November 8, 2019 (morning). *Spatial Decomposition Methods*

Speaker: Pierre Carpentier (08:30–12:00)

During this last session, we will explore different decomposition schemes allowing to split a stochastic optimal control problem — involving a large number of units — so as to obtain several small-scale subproblems. These methods allow to solve the subproblems by Dynamic Programming or SDDP.

- 08:30–10:00: Mixing decomposition techniques and Dynamic Programming
- break
- 10:30–12:00: Dual Approximate Dynamic Programming and related methods

Speaker: Stéphanie Vareilles (12:00–12:30)

Wrap-up session.

References

- [Ber00] D. P. Bertsekas. *Dynamic Programming and Optimal Control*. Athena Scientific, Belmont, Massachusetts, second edition, 2000. Volumes 1 and 2.
- [CCCD15] P. Carpentier, J.-P. Chancelier, G. Cohen, and M. De Lara. *Stochastic Multi-Stage Optimization. At the Crossroads between Discrete Time Stochastic Control and Stochastic Programming*. Springer-Verlag, Berlin, 2015.
- [RW91] R.T. Rockafellar and R. J-B. Wets. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of operations research*, 16(1):119–147, 1991.
- [SDR09] A. Shapiro, D. Dentcheva, and A. Ruszczyński. *Lectures on stochastic programming: modeling and theory*. The society for industrial and applied mathematics and the mathematical programming society, Philadelphia, USA, 2009.