Decomposition methods for two stage stochastic programming

Vincent Leclère

15/10/2018

Name: ______________________

By "type of problem", we mean "Linear Program" denoted (LP), "Mixed Integer Linear Program" denoted (MILP), "Quadratic Program" denoted (QP) (linear constraint, convex quadratic objective, continuous variables), "Mixed Integer Quadratic Program" denoted (MIQP)... By size of a problem we mean the number of integer and linear variables.

1 A manufacturing problem

Consider a manufacturing unit that produces items A and B, from raw material α, β and γ. The production unit manager want to satisty the demand while minimizing expected costs.

Raw material α can be bought either one month in advance advance, by case of 100 kilos, for 500€per case, or at the last minute, at the price of 20€per kilo (any quantity). Raw material β can be bought either one month in advance, by case of 200 kilos, for 600€per case, or at the last minute, at the price of 10€per kilo (any quantity). Raw material γ can only be bought by case of 50 kilos, one month in advance, at the price of 100€per case. Raw material (in kilo) ordered in advance are denoted by $(x_\alpha, x_\beta, x_\gamma)$, raw material ordered at the last minute are denoted by (y_A, y_B).

In order to produce one unit of item A, we need 1 kilos of α and 3 kilos of β. In order to produce one unit of item B, we need 2 kilos of α, 1 kilos of β and 1 kilos of γ.

Consequently, one month before starting the producing process, the production unit manager need to decide how many case of each product he should order (he cannot order half-cases). Two days before the production process starts, the actual demand in product a and b is revealed, he can then order raw material by "last minute" process. The demand has to be satisfied in any cases.

One month in advance, the demand in each item is assumed to be random with law given in 1.

s	1	2
p_s	0.6	0.4
(d_A, d_B)	(50,100)	(100,50)

Table 1: law of demand

1. (2 points) Justify that the problem is a two-stage stochastic programm by specifying: first stage control, second stage control (or recourse), noise and cost function. What are all the random variables of the problem ?

2. (2 points) Write the associated stochastic program (with probabilistic notation: $\mathbb{E}$, etc.)

3. (2 points) Write explicitely (with all constraints, and all numerical values) the extended formulation.

4. Consider the progressive hedging algorithm applied to this problem.

   (a) (1 point) In which space lives the price of information λ ? Give its dimension.

(b) (2 points) For a given price of information λ^k, explicit the slave program associated to scenario 1 at iteration $k > 1$.

(c) (1 point) For an iteration of the progressive hedging algorithm, specify how many problem of which type and size are solved. Is the progressive hedging algorithm adapted to this problem ?

5. Consider the (single-cut) L-shaped algorithm applied to this problem.

   (a) (2 points) Write the (primal) slave problem associated to scenario 1.

(b) (2 points) Write the master problem at iteration K.

(c) (1 point) For an iteration of the single-cut L-shaped algorithm, specify how many problem of which type and size are solved. Is the (single-cut) L-shaped algorithm adapted to this problem ?