Unit commitment

Progressive Hedging:
dealing with binary variables & Chance Constraints

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Transmission Network

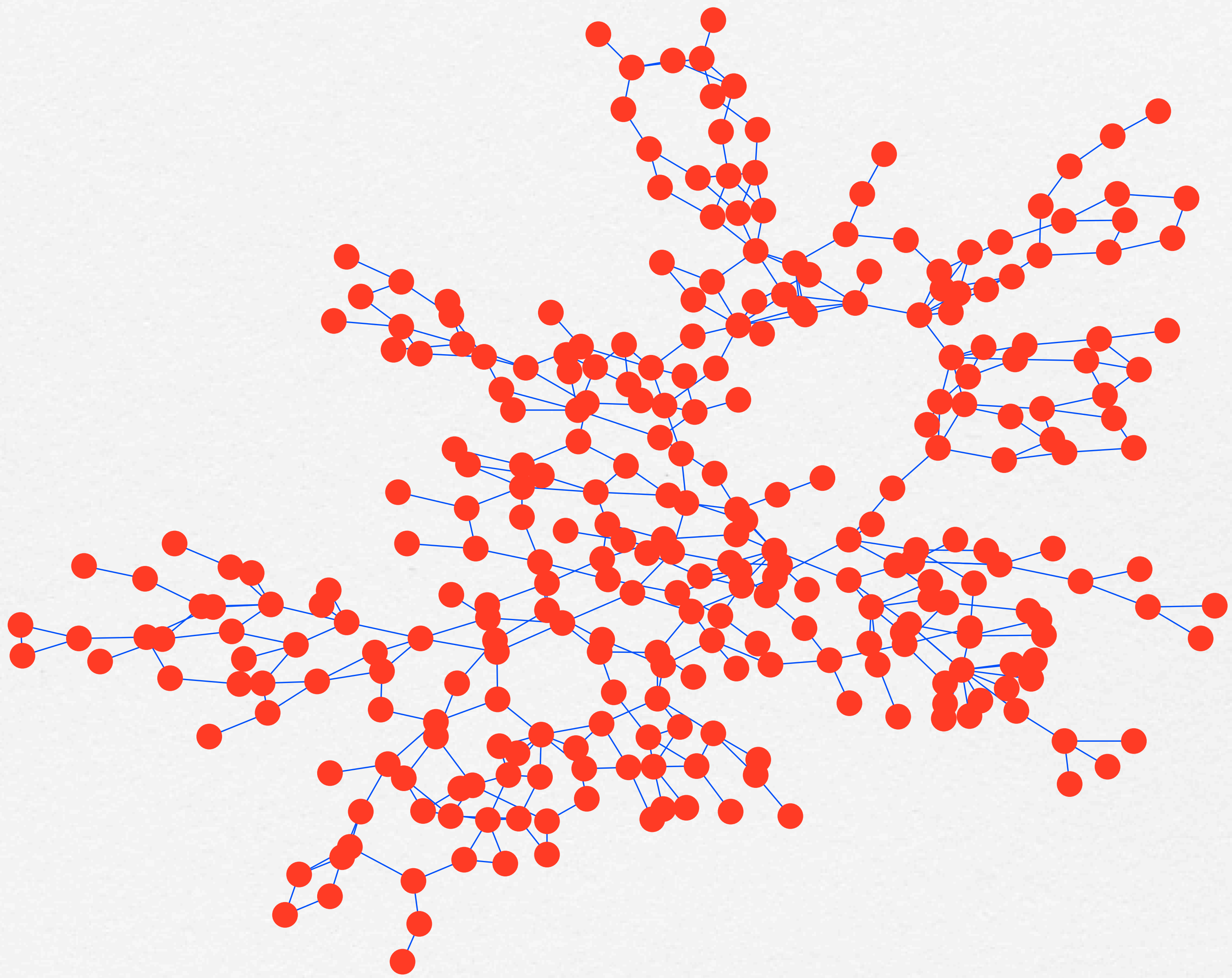
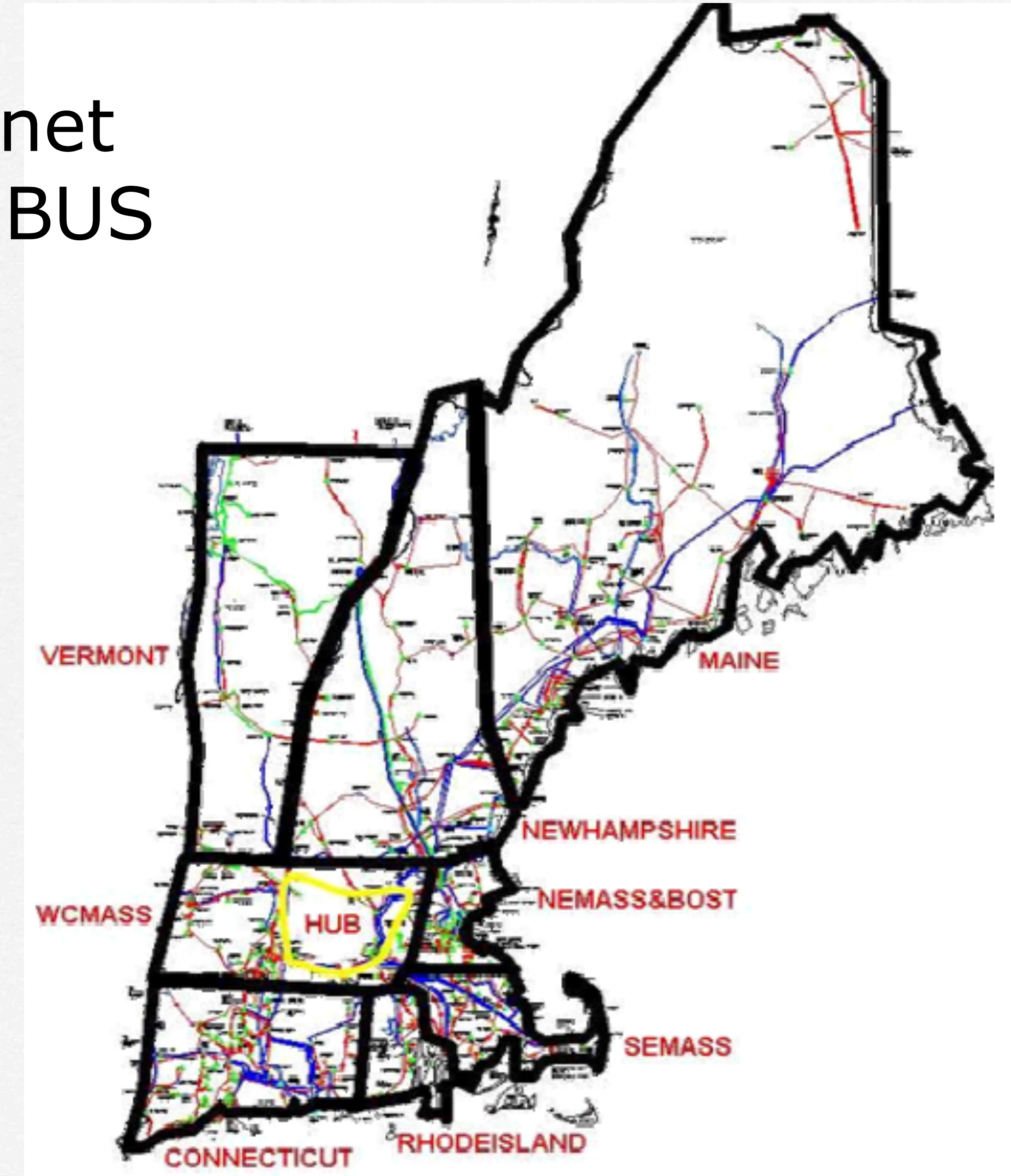


Figure 1. Topology of the IEEE 300 node system

Transmission Network

NE-ISO net
~30,000 BUS



FERC

Federal Energy Regulatory Commission

RTO

In the US is an organization that is responsible for moving electricity over large interstate areas; coordinates, controls and monitors an electricity transmission grid that is larger with much higher voltages than the typical power company's distribution grid.

ISO

Is an organization formed at the direction or recommendation of the ***FERC,*** *in the areas where an ISO is established, it coordinates, controls and monitors the operation of the electrical power system, usually within a single US State, but sometimes encompassing multiple states.*

ISO New England Inc. (*ISO-NE*) *is an independent, non-profit RTO, serving Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island and Vermont. Its Board of Directors and its over 400 employees* have no financial interest or ties to any company doing business in the region's wholesale electricity marketplace.

Energy Sources

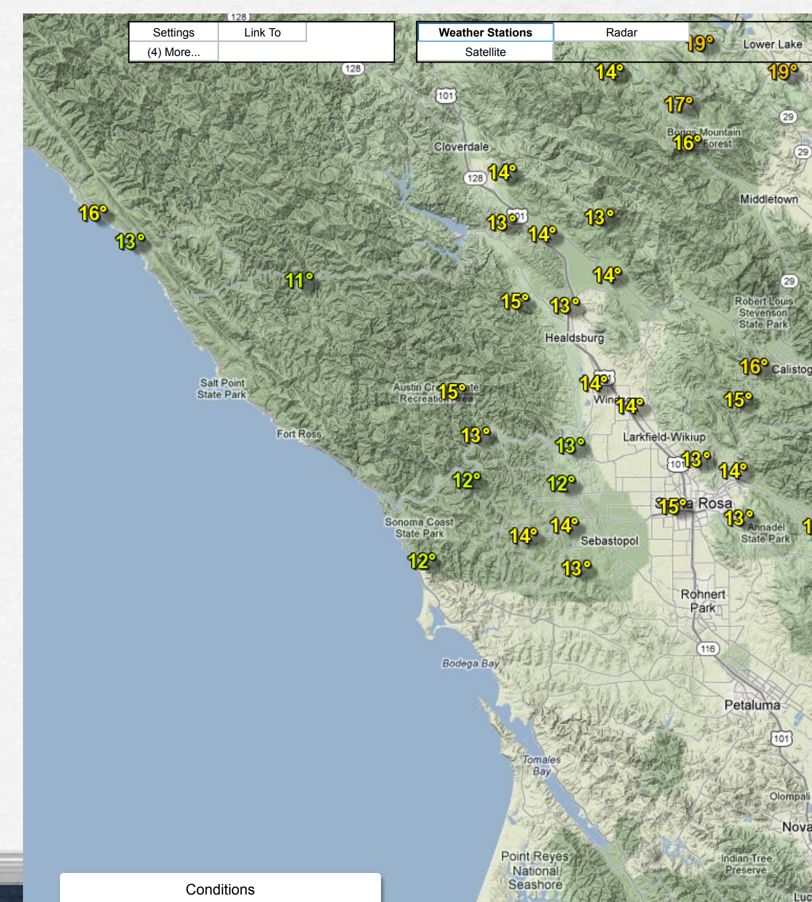
- nuclear energy
- hydro-power
- thermal plants (coal, oil, shale oil, bio, rubish, . . .)
- gas turbines (natural gas, from ”cracking’)
- renewables (wind, solar, ..., ocean waves)

different characteristics

Uncertainties

- WEATHER: demand & supply (especially renewables)
- industrial-commercial environment (demand)
- seasonal, day of the week, time of the day
- contingencies: transmission lines, generators

El Cerrito, California (94530) Conditions & Forecast http://www.wunderground.com/cgi-bin/findweather/getForecast...



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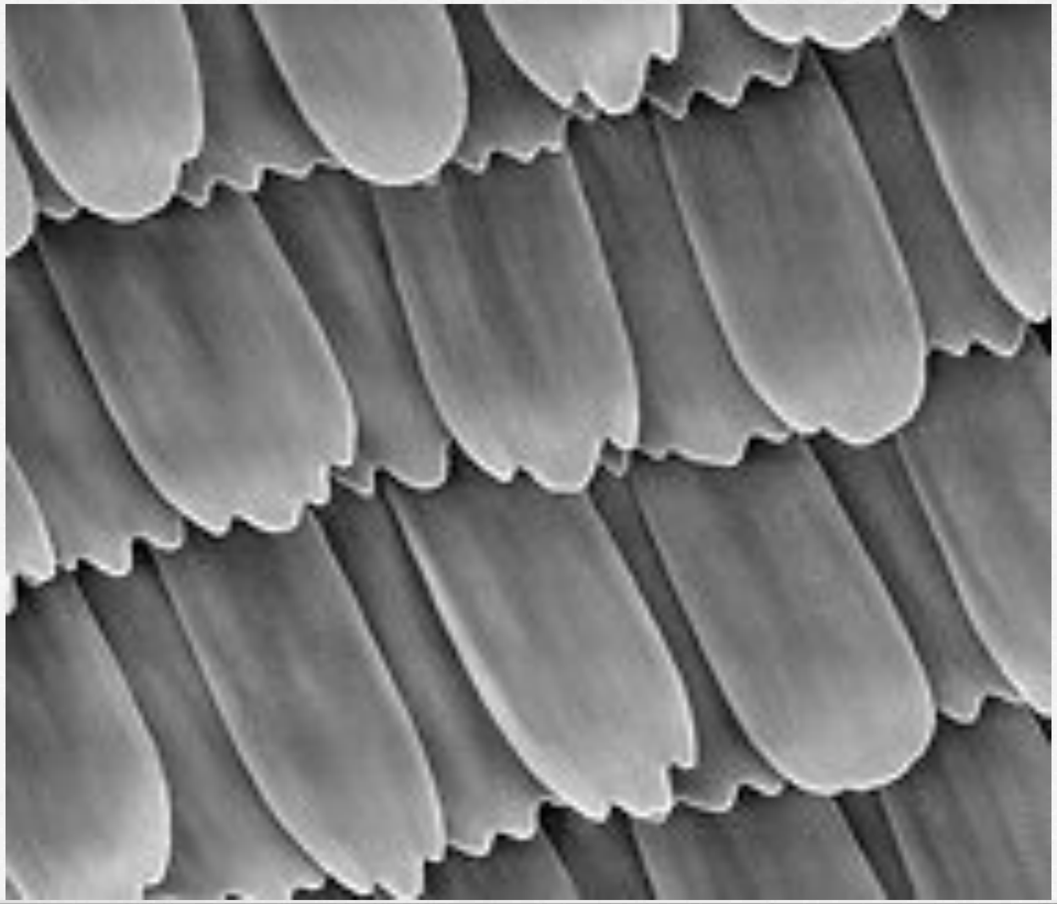

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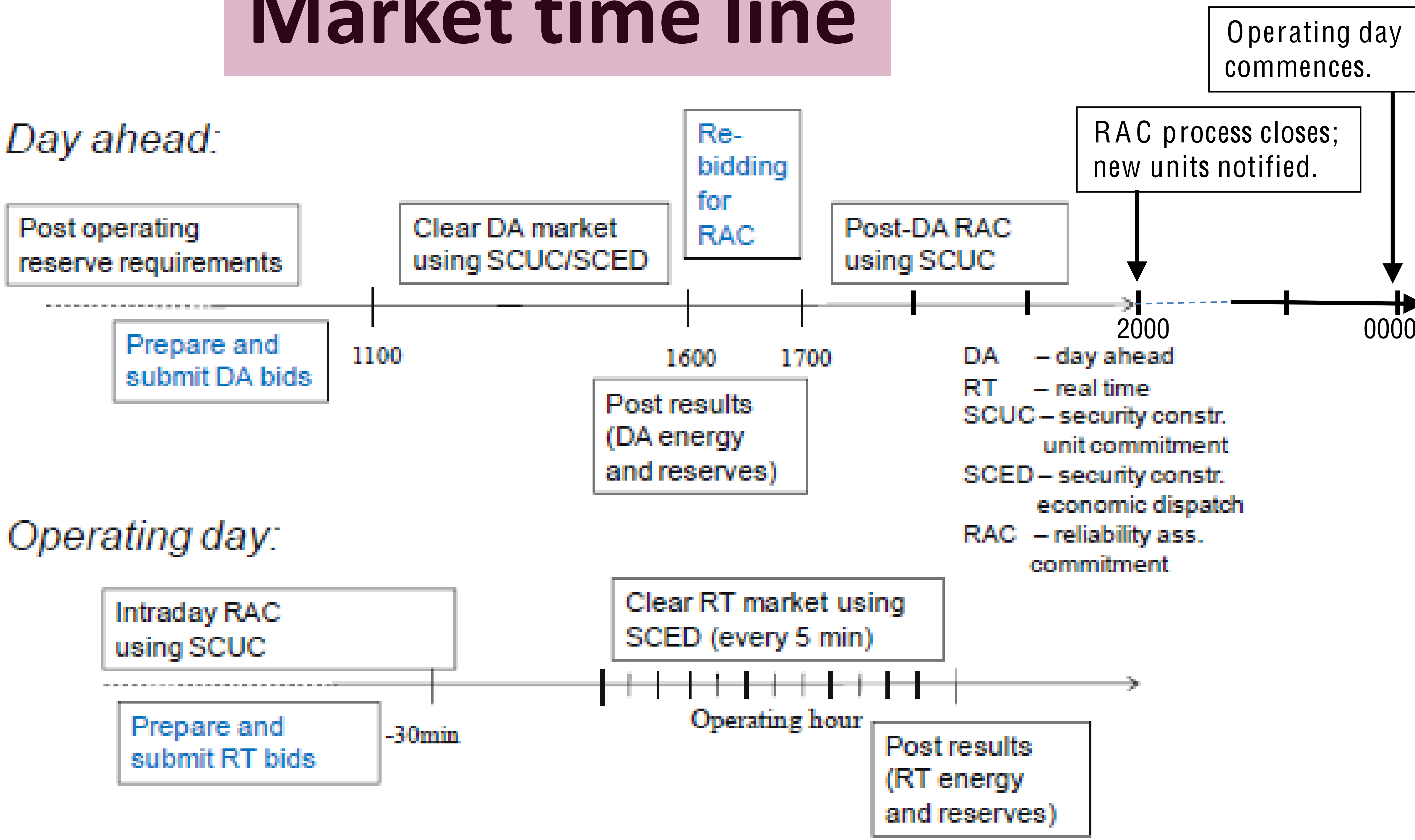
Market time line



	MISO	NYISO	PJM	ERCOT	CAISO
Market timeline	DA offers due: 11am DA results: 4pm Re-bidding due: 5pm RT offers due: OH -30 min	DA offers due: 5 am DA results: 11 am RT offers due: OH -75 min	DA offers due: noon DA results: 4pm RT offers due: 6pm DA	DA bids due (reserves): 1pm/4pm DA results (reserves): 1.30pm/6pm RT offers due: OH -60 min	DA offers: 10am DA results: 1pm RT offers: OH - 75 min

Ref: A. Botterud, J. Wang, C. Monteiro, and V. Miranda "Wind Power Forecasting and Electricity Market Operations," available at www.usaee.org/usaee2009/submissions/OnlineProceedings/Botterud_etal_paper.pdf

Short history of ISO-management techniques

- *RT: deterministic optimization with LMP (dual variables associated with demand(s) constraints).*
- *SCUC/SCED: Lagrangian relaxation with conservative reliability constraints*
- *SCUC/SCED: deterministic MIP with conservative RUT*
- *ARPA-"E (project): "take into account uncertainty"*

A collection of sto-programs

- DA-SCUC/SCED unit commitment binaries
- DA-RAC rebidding assessment bidding (binaries)
- DA-RUT - reliability commitments (spinning, N-1)
- RT - 3 min (real time adjustments) LMP's
- SCED2 - 3 or 4 hours schedule to foresee ramp ups/down, etc.

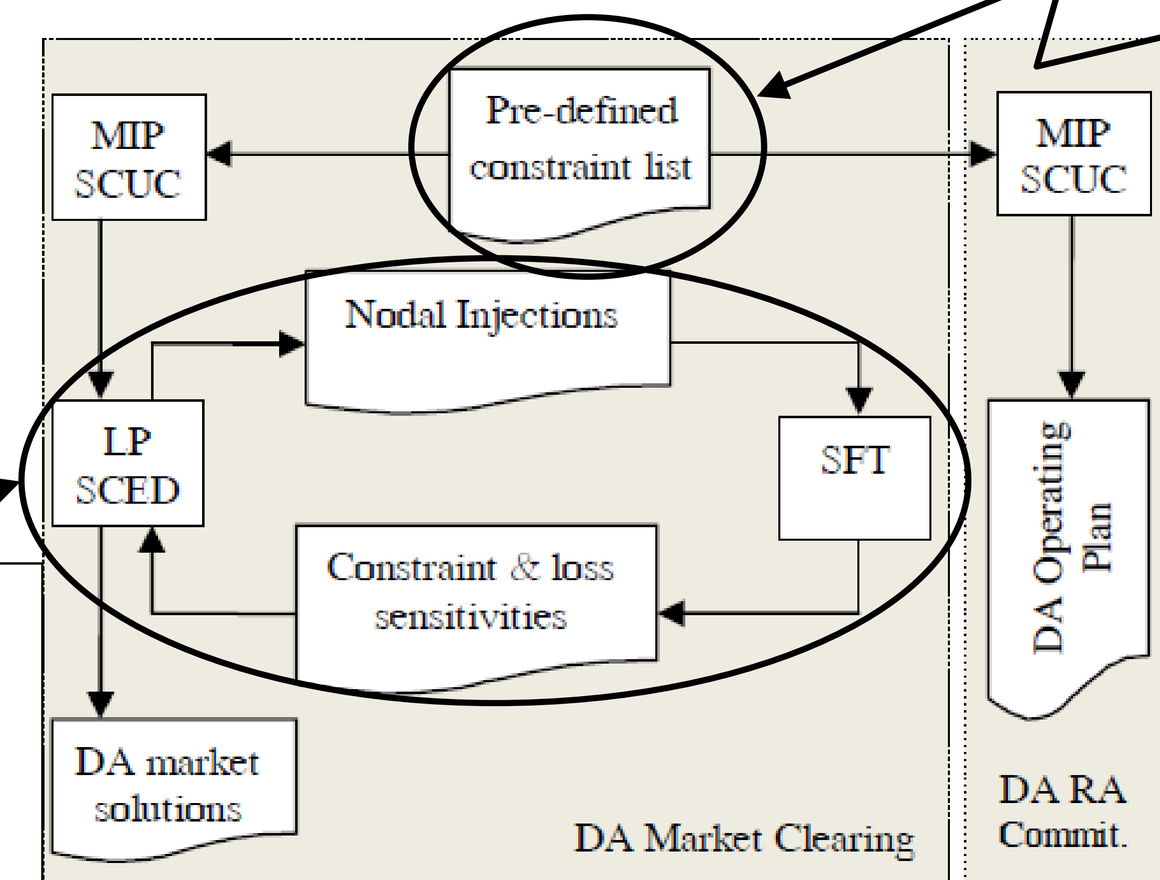
DA = day ahead

Team composition:

- SCUC/SCED model designers+ optim. implementation: UCD (Woodruff + ,Wets), Sandia National Labs (Watson, Silvia, Siirola, Ross + Sandia Livermore).
- Uncertainty description: UCD (Wets), Iowa State (Ryan, Tesfatsion, Alipantis +)
- Software prototype: Alstom (Kwok Cheung +)
- ISO-mentor: NE-ISO (Eugene Litvinov + ...)
- Market modifications: Iowa State (Tesfatsion, Alipantis + all)

Day-Ahead Market

SCUC enforces limited number of transmission constraints on the commitment solution.



Each hourly SCED performs SFT, which tests all contingencies in a list and for violations, imposes appropriate constraints in SCED and resolves it.

Ref: Xingwang Ma, Haili Song, Mingguo Hong, Jie Wan, Yonghong Chen, Eugene Zak, “The Security-constrained Commitment and Dispatch For Midwest ISO Day-ahead Co-optimized Energy and Ancillary Service Market,” Proc. of the 2009 IEEE PES General Meeting.

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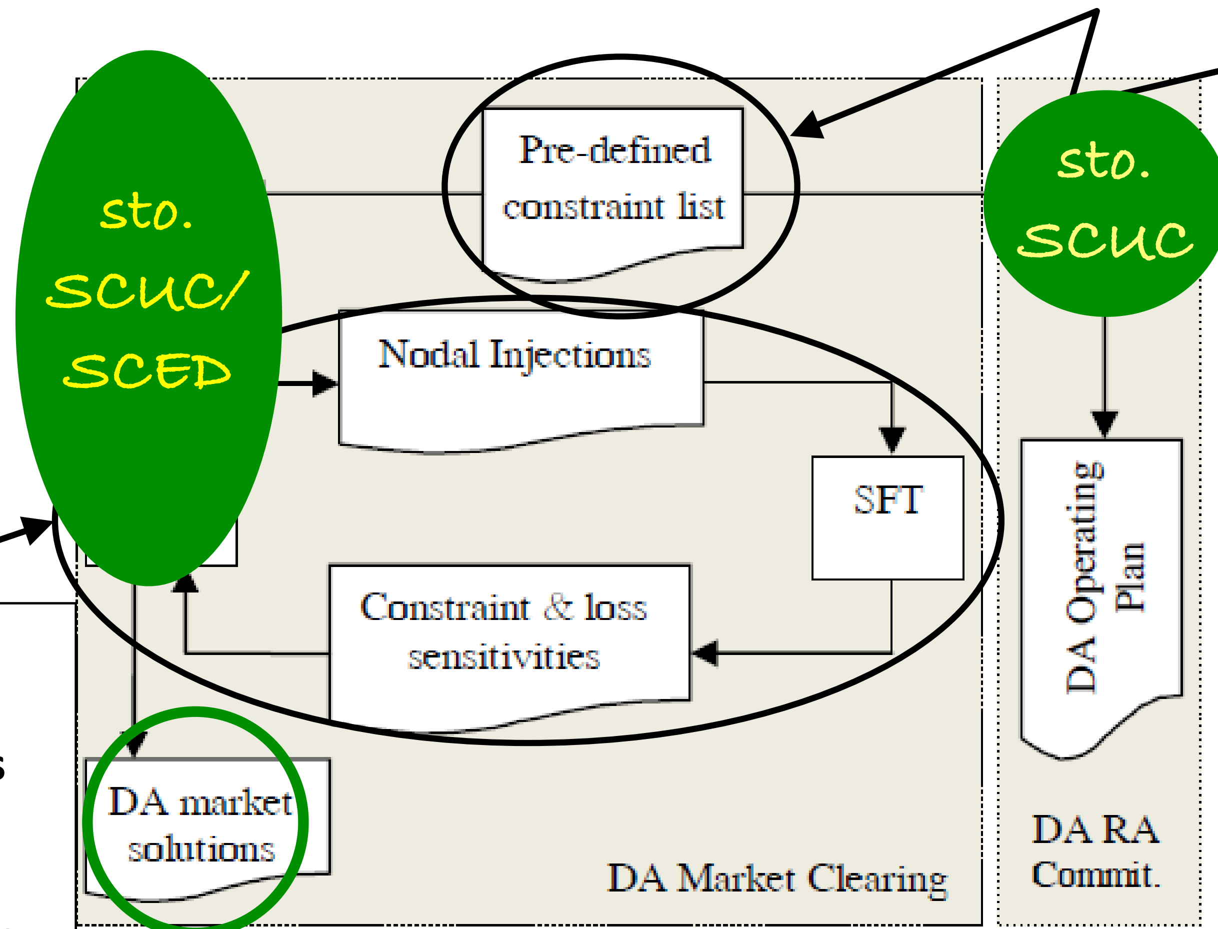

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Abstract Unit Commitment

production cost *startup cost* *shutdown cost*

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \qquad \text{with}$$

K time periods *J generating units*

demand

power output
$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

spinning reserve

max power output
$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \overline{p}_j(k) \in \Pi, \quad \forall j \in J, \; \forall k \in K$$

Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

min. expectation
(actually: risk measure)
with penalties

production cost startup cost shutdown cost

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \quad \text{with}$$

K time periods J generating units

demand

power output $$\sum_{j \in J} \underset{\sim}{p_j}(k) = \underset{\sim}{D}(k), \quad \forall k \in K$$

adjust node
balance eq'ns

spinning reserve

max power output $$\sum_{j \in J} \underset{\sim}{\bar{p}_j}(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \underset{\sim}{\Pi}, \quad \forall j \in J, \ \forall k \in K$$

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"Stochastic Version"

between a rock and a hard place

CPLEX-MIP: can handle a few scenarios

PH : not designed for binary vairables

Progressive Hedging Algorithm

0. w_ξ^0 such that $\mathbb{E}\{w_\xi^0\} = 0,\ \nu = 0.$ Pick $\rho > 0$ Review

1. for all ξ :

$$(x_\xi^{1,\nu}, x_\xi^{2,\nu}) \in \arg\min f(\xi; x^1, x^2) + \langle w_\xi^\nu, x^1 \rangle + \frac{\rho}{2}\left|x^1 - \bar{x}^{1,\nu-1}\right|^2$$

$$x^1 \in C^1 \subset \mathbb{R}^{n_1},\ x^2 \in C^2(\xi, x^1) \subset \mathbb{R}^{n_2}$$

2. $\bar{x}^{1,\nu} = \mathbb{E}\{x_\xi^{1,\nu}\}$. Stop if $\left|x_\xi^{1,\nu} - \bar{x}^{1,\nu}\right| = 0$ (approx.)

otherwise $w_\xi^{\nu+1} = w_\xi^\nu + \rho\left[x_\xi^{1,\nu} - \bar{x}^{1,\nu}\right]$, return to 1. with $\nu = \nu + 1$

Implementation: bundling, $\rho \to \rho_s$, ...
Watson & Woodruff (Hart, Siirola, ...)
Chile: Sistemas Complejos de Ingeneria (L.F. Solari, ...)
& Centro de Modelamiento Matematico
Carl Laird (Texas A& M), Ryan Sarah (Iowa), ...

PH: binary variables

$\min\langle c, x\rangle + \sum_{\xi\in\Xi} p_\xi \langle q_\xi, y_\xi\rangle$ such that

$x \in C_1,\ \ y_\xi \in C_2(\xi, x)\ \forall\, \xi \in \Xi$

binary (integer) variables: some x's, some y_ξ's.

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binary (integer) variables: some x's, some y_ξ's.

Choice of $\rho \to \rho_j$ depending on $c_j, |x_j|, \ldots$

Variable Fixing, in particular binaries, $x_j(s) =$ constant (k iterations)
Variable Slamming: aggressive variable fixing $x_j(s) \approx$ constant (& $c_j x_j(s)$)
"Sufficient" variable convergence $\sim$ for small values of $c_j x_j(s)$

Termination criterion: variable slamming when $x_j^\nu(\xi) - x_j^{\nu+1}(\xi)$ small

Detecting cycling behavior: (simple) hashing scheme

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Detecting cycling behavior: (simple) hashing scheme

Enough variables fixed ⇒ clean up with CPLEX-MIP

Large Scale Chance-Constraints

Generalized Chance Constraints

$$\min\langle c, x\rangle + \textstyle\sum_{\xi\in\Xi} d_\xi p_\xi \langle q_\xi, y_\xi\rangle$$
$$\text{such that } (x, y_\xi) \in C_\xi, \quad \forall\, \xi \in [\,\mathcal{S} : d_\xi = 1\,]$$
$$\textstyle\sum_{\xi\in\mathcal{S}} p_\xi d_\xi \geq 1-\alpha, \;\; d_\xi \in \{0,1\}, \xi \in \mathcal{S}$$

Aircraft sustainability problem: $\min\langle c, x\rangle$ such that

$$A_\xi s \geq b_\xi d_\xi, \;\; \forall \xi \in \mathcal{S}, \;\; x \in \mathbb{R}_+$$
$$d_\xi \in \{0,1\}, \; \forall\, \xi \in \mathcal{S}, \quad \textstyle\sum_{\xi\in\mathcal{S}} d_\xi \geq (1-\alpha)|\mathcal{S}|$$

x: inventory policy, resource levels, time-index variables ($> 10^6$)
A_ξ, b_ξ discrete event simulation of operation sustainability
$|\Xi| \approx 5\cdot 10^6, \;\; \alpha \sim 0.04.$

PH & G.Chance Constraints

Relaxation:

$$\min\langle c, x\rangle + \textstyle\sum_{\xi\in\Xi} d_\xi p_\xi \langle q_\xi, y_\xi\rangle - \lambda\Big(\sum_{\xi\in\mathcal{S}} p_\xi d_\xi - (1-\alpha)\Big)$$

$$\text{such that } (x, y_\xi) \in C_\xi, \quad \forall\, \xi \in [\, \mathcal{S} : d_\xi = 1\,]$$

$$d_\xi \in \{0,1\}, \xi \in \mathcal{S}$$

for all $\lambda \geq 0$ yields a lower bound for the generalized C.C. problem

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Ignoring coupling C.C., for $\xi \in \mathcal{S}$: let

$$(\bar{x}_\xi, y_\xi) \in \operatorname{argmin}\langle c, x\rangle + \langle q_\xi, y\rangle$$

$$\text{such that } (x, y) \in C_\xi$$

PH & G.Chance Constraints

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"Decomposed" calculation of d_ξ:

$$\min\langle c, \bar{x}_\xi\rangle - \lambda d_\xi$$

$$\text{such that } (\bar{x}_\xi, y_\xi) \in C_\xi, \;\; d_\xi \in \{0,1\}$$

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Solution: let

$d_\xi = 1$ when $\langle c, x_\xi\rangle + \langle q_\xi, y_\xi\rangle \leq \lambda$

otherwise $d_\xi = 0$

PH & G.Chance Constraints

assuming λ given

0. w_ξ^0 such that $\mathbb{E}\left\{w_\xi^0\right\} = 0,\ \nu = 0.$ Pick $\rho > 0$

1. for all $\xi \in S$:

$$(x_\xi^\nu, y_\xi^\nu) \in \arg\min \langle c, x\rangle + \langle q, y\rangle + \langle w_\xi^\nu, x\rangle + \frac{\rho}{2}\left|x^1 - \overline{x}^{1,\nu-1}\right|^2, \quad (x,y) \in C_\xi$$

2. if $\left\langle c, x_\xi^\nu\right\rangle + \left\langle q, y_\xi^\nu\right\rangle \le \lambda,$ set $d_\xi = 1$ otherwise $d_\xi = 0$

3. $\overline{x}^\nu = \left(\sum_{\xi\in S} p_\xi d_\xi x_\xi^\nu\right) / \left(\sum_{\xi\in S} p_\xi d_\xi\right)$ and

$$w_\xi^{\nu+1} = w_\xi^\nu + \rho\left[x_\xi^{1,\nu} - \overline{x}^{1,\nu}\right]$$

4. if $\left(1 / \sum_{\xi\in S} p_\xi d_\xi\right)\sum_{\xi\in S} p_\xi d_\xi \mid x_\xi^\nu - \overline{x}^\nu \mid > \varepsilon$; return to 1. with $\nu = \nu + 1$

   otherwise Stop

Biasing the d_ξ variables

Strategy: PH $\rightarrow x^*_\xi$, $\lambda_{\max} \searrow \lambda^*$:

augmentation function (minimization of discontinuous functions):

limit function: $\tau = \langle c, x_\xi \rangle + \langle q_\xi, y_\xi \rangle$

$$\psi(\tau, \lambda, \lambda_{\max}) = \begin{cases} 1 & \text{when } 0 \le \tau \le \lambda, \\ 0 & \text{for } \lambda < \tau \le \lambda_{\max} \end{cases}$$

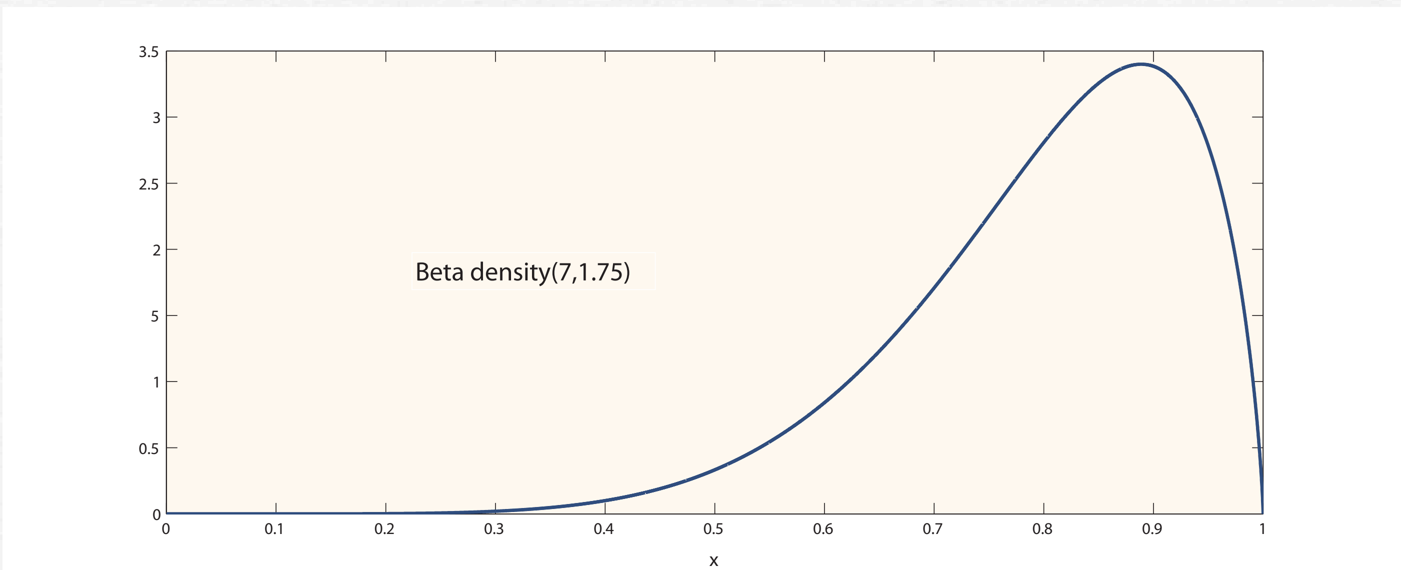
mollifiers: Beta densities on $[0, \Delta]$, $\Delta \searrow 0$ as PH converges (gap)

$$\varphi(\Delta; z) = \begin{cases} \frac{1/\Delta}{B(7,1.75)} (z/\Delta)^6 (1 - z/\Delta)^{0.75} & \text{when } z \in [0, \Delta] \\ 0 & \text{elsewhere} \end{cases}$$

Beta function $B(a, b) = \int_0^1 z^{a-1} (1 - z)^{b-1} \, dz$

Augmentation function

$$m(\Delta, \lambda_{\max}; z, \lambda) = \int_0^{\Delta} \psi(z - s, \lambda, \lambda_{\max})\varphi(\Delta; s)\, ds, \quad z \in [0, \lambda_{\max}]$$



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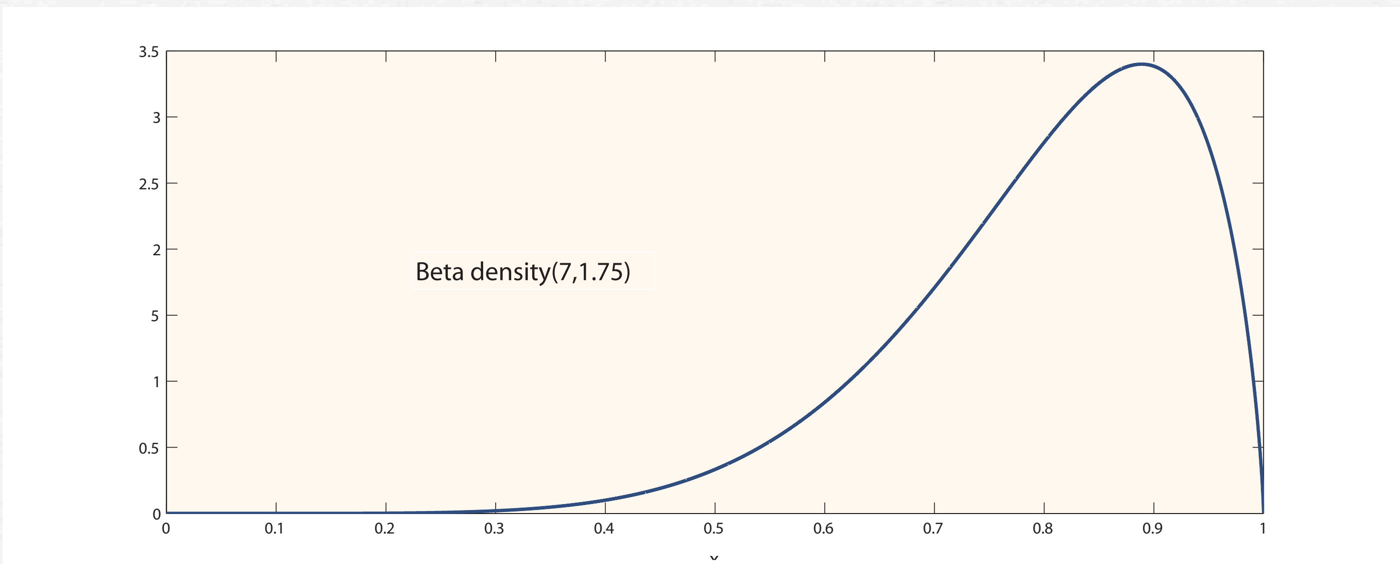
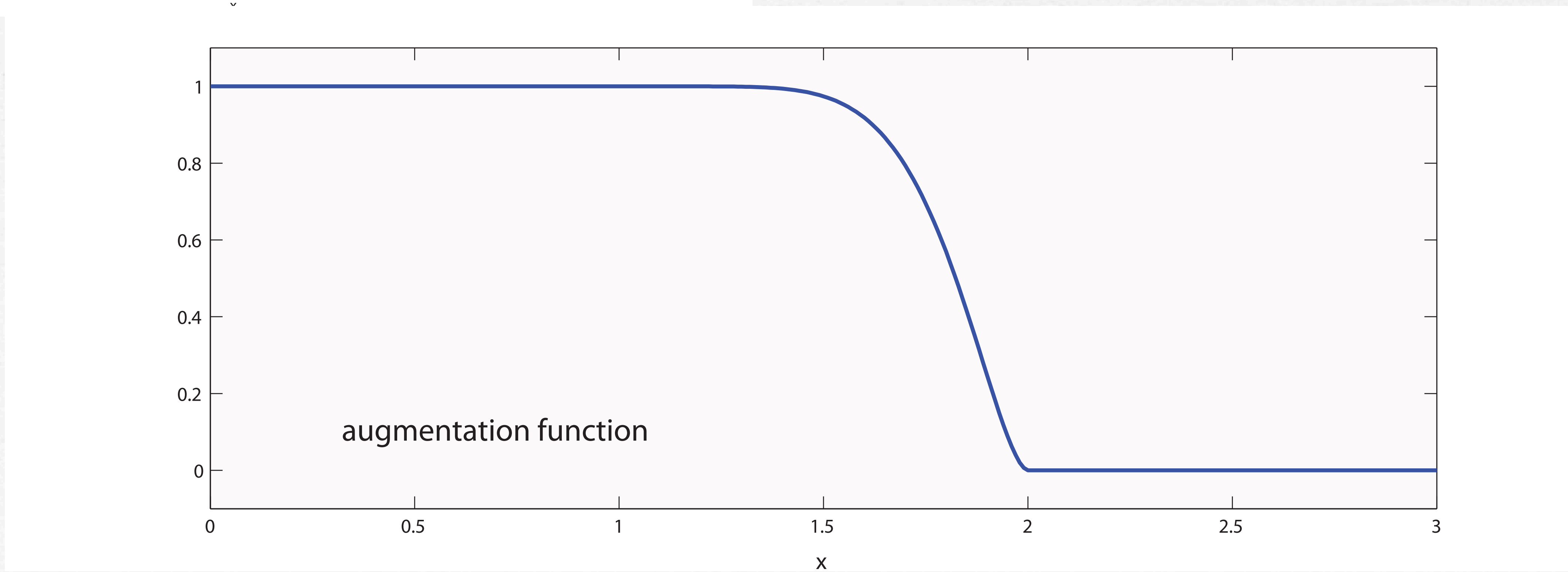




PH & G.Chance Constraints

0. w_ξ^0 such that $\mathbb{E}\left\{w_\xi^0\right\} = 0,\ \nu = 0.$ Pick $\rho > 0, \varepsilon > 0,\ \Delta = 1$

1. for all $\xi \in S$: ($\nu = 0$ ignore the proximal term, fix $\lambda_{\max}$ an upper bound on cost fcn)

$$(x_\xi^\nu, y_\xi^\nu) \in \arg\min \langle c, x\rangle + \langle q, y\rangle + \langle w_\xi^\nu, x\rangle + \frac{\rho}{2}\left|x^1 - \bar{x}^{1,\nu-1}\right|^2, \quad (x,y) \in C_\xi$$

2. $(\lambda, d) = \arg\min_{\lambda \le \lambda_{\max}} \lambda$ such that for all $\xi \in S$,

$$d_\xi = m\left(\Delta, \lambda_{\max}; \left\langle c, x_\xi^{\nu-1}\right\rangle + \left\langle q_\xi, y_\xi\right\rangle, \lambda\right) \ \&\ \sum_{\xi \in S} p_\xi d_\xi \ge 1 - \alpha$$

3. $\bar{x}^\nu = \left(\sum_{\xi \in S} p_\xi d_\xi x_\xi^\nu\right) / \left(\sum_{\xi \in S} p_\xi d_\xi\right)$ and

$$w_\xi^{\nu+1} = w_\xi^\nu + \rho\left[x_\xi^{1,\nu} - \bar{x}^{1,\nu}\right]$$

4. if $\left(1 / \sum_{\xi \in S} p_\xi d_\xi\right) \sum_{\xi \in S} p_\xi d_\xi \mid x_\xi^\nu - \bar{x}^\nu \mid > \varepsilon$; return to 1. with $\nu = \nu + 1$

otherwise Stop

Unit Commitment Modeling Load

HOW to model stochastic processes?

Example: load in Connecticut zone of ISO-NE

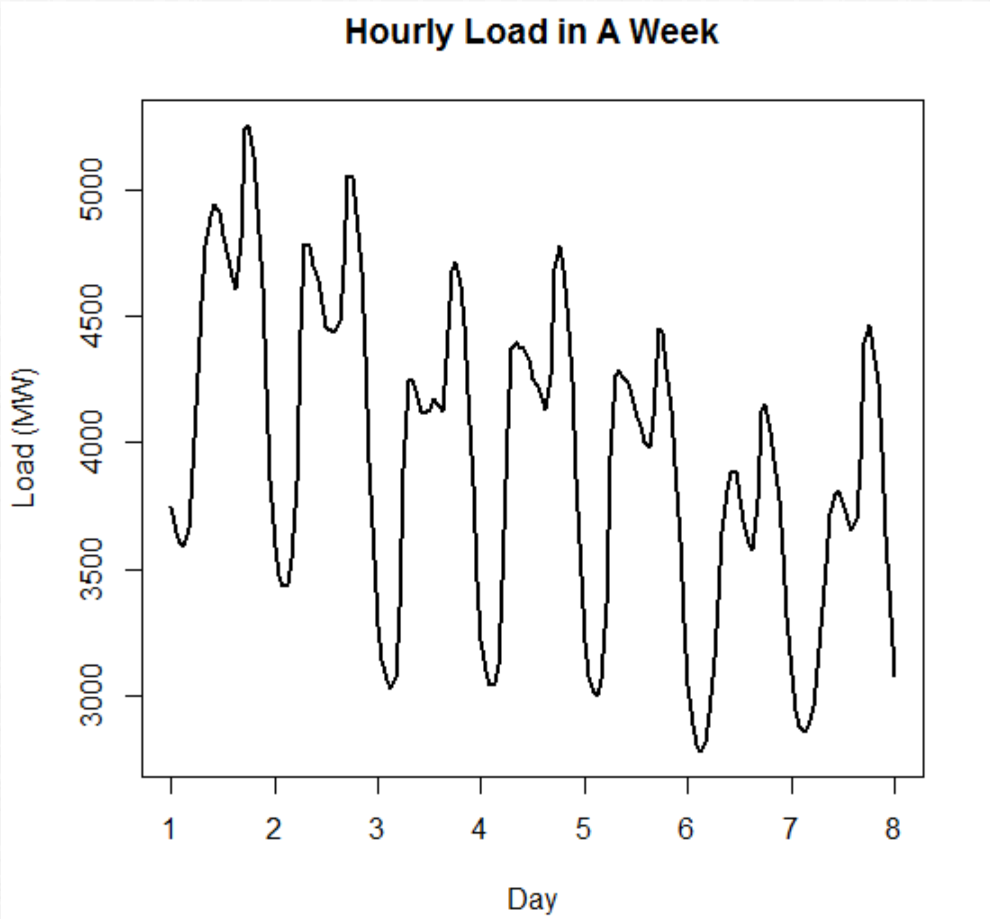
- Data from ISO-NE: For each of 8 load zones,
  - Date
  - Hour: 1-24
  - Temperature: Dry bulb in deg. F
  - Dew Point: Dew point temperature in deg. F
  - Demand
- CT accounts for 27.7% of electricity sales
- Consider data since 2006 after major market changes implemented by ISO-NE in 2005

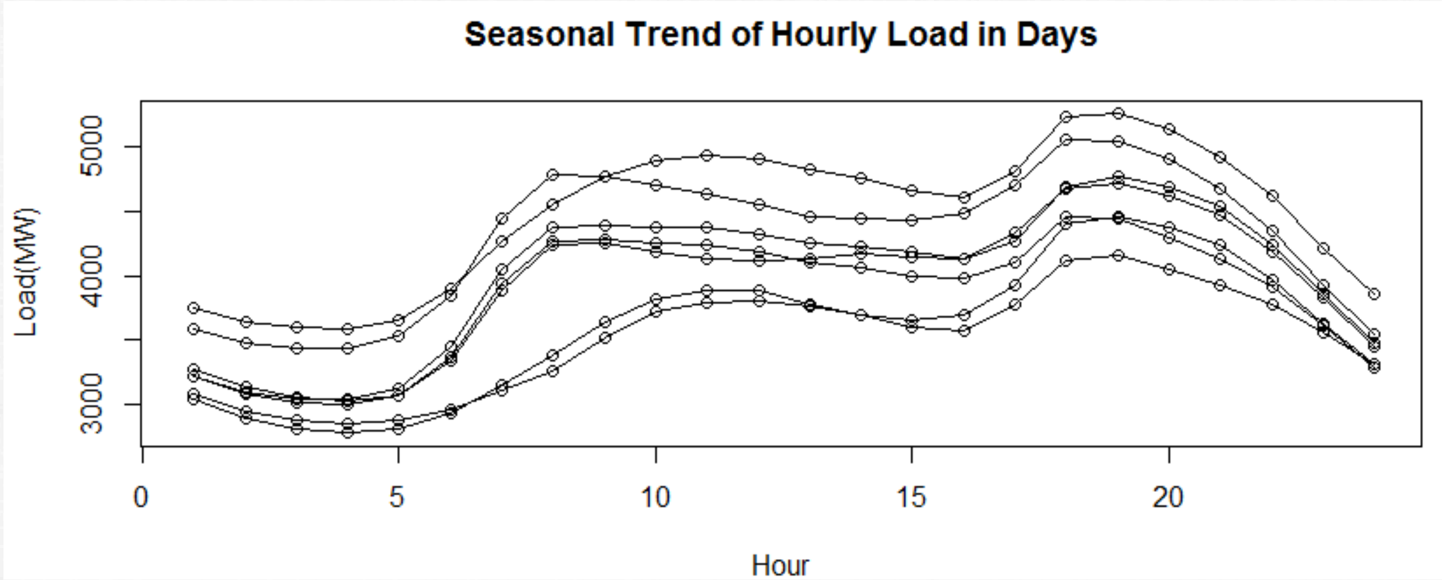
Load Modeling Process

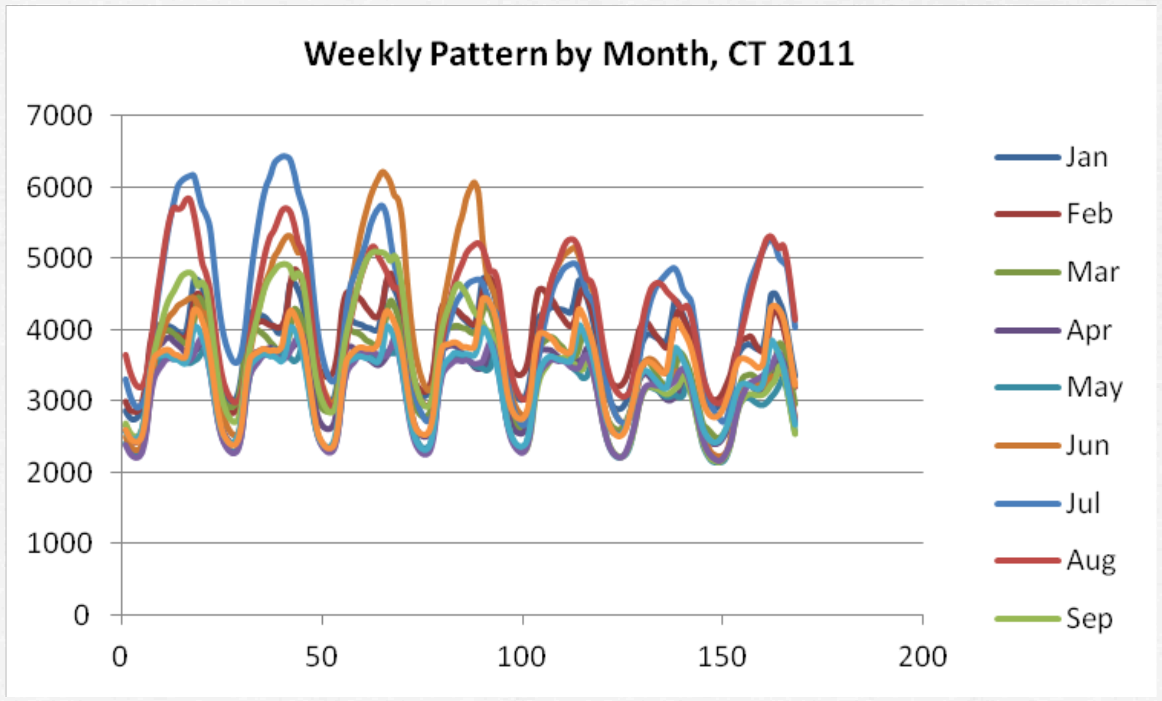
Example: load in Connecticut zone of ISO-NE

- Exploratory data analysis to determine major influences
- Data segmentation (excruciating data analysis)
- Multiple linear regression (MLR) within data segments to determine relationships
- Also experimented with:
  - Time series transfer functions within data segments
  - Semi-parametric time series approach
  - Multiple linear regression on whole data set with dummy variables for hour, day-type and month

CT load exploratory process

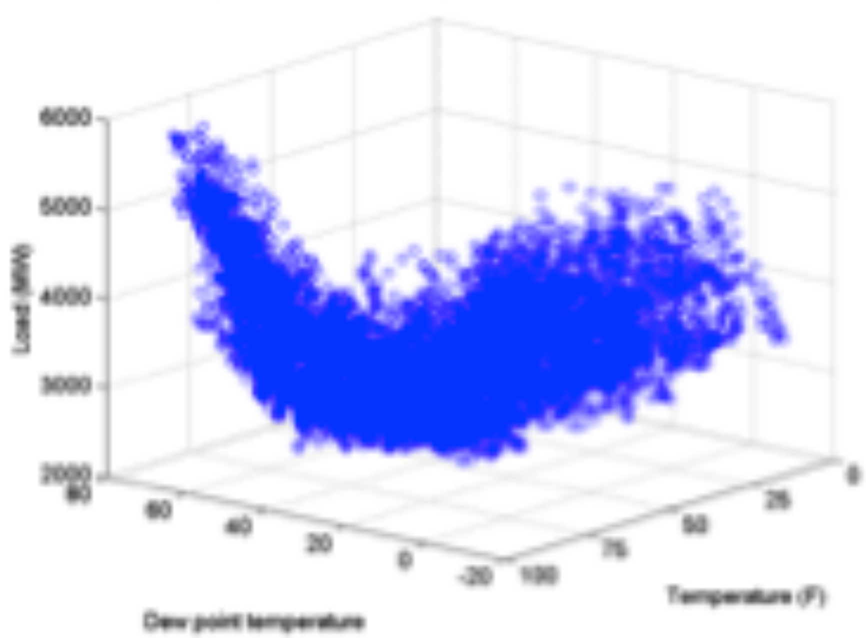




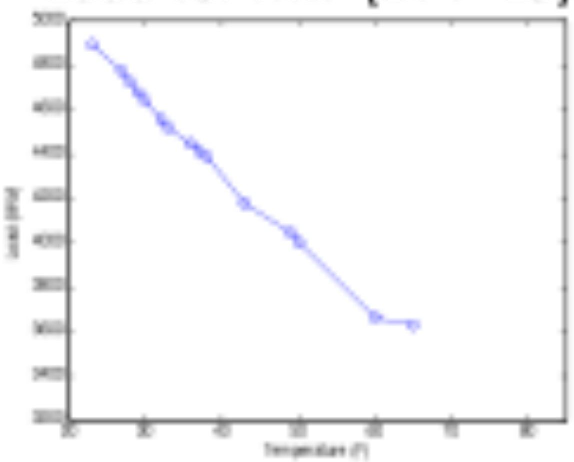


CT load vs. weather variables

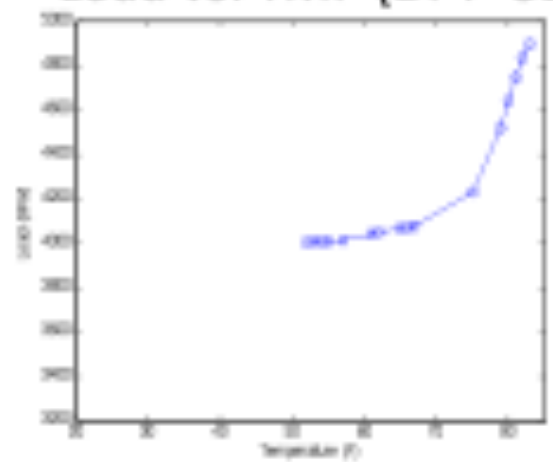
Load vs. temperature (TMP) and dew point temperature (DPT)



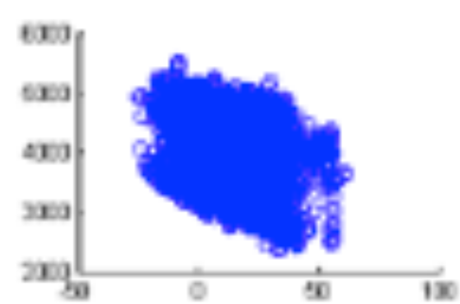
Load vs. TMP (DPT=19)



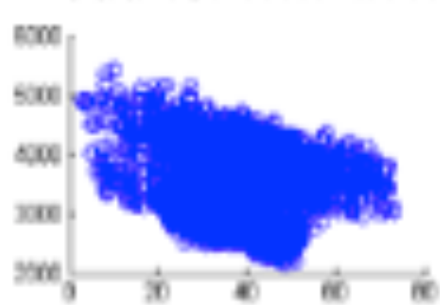
Load vs. TMP (DPT=52)



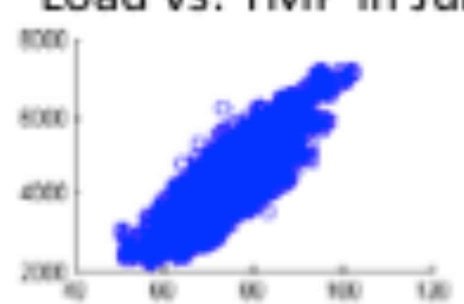
Load vs. DPT in Jan



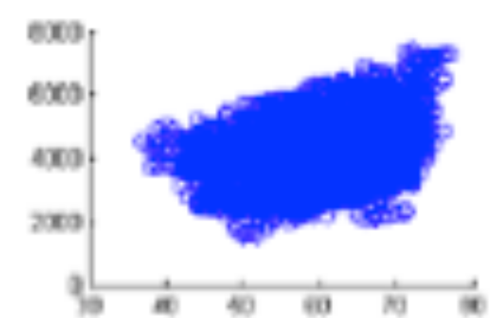
Load vs. TMP in Mar



Load vs. TMP in Jul



Load vs. DPT in Aug



Tentative CT Load Model

- Data segmented by season, day-type and hour
- For each segment, fit MLR model on 2006-2010 training data set

$$
\begin{aligned}
L(k) = \; & \beta_0 + \beta_1 TMP(k) + \beta_2 TMP(k)^2 + \beta_3 TMP(k)^3 + \beta_4 DPT(k) + \beta_5 DPT(k)^2 + \beta_6 DPT(k)^3 \\
& + \beta_7 TMP(k-1) + \beta_8 TMP(k-2) + \cdots + \beta_{14} TMP(k-8) + \beta_{15} TMP(k-24) + \beta_{16} TMP(k-168) \\
& + \beta_{17} DPT(k-1) + \beta_{18} DPT(k-2) + \cdots + \beta_{24} DPT(k-8) + \beta_{25} DPT(k-24) + \beta_{26} DPT(k-168) \\
& + \varepsilon(k)
\end{aligned}
$$

- Average relative error on training set ~3%
- Mean absolute percent error (MAPE) on 2011 test data ~6%
- To do: replace *TMP*(*k-m*), *DPT*(*k-m*) terms with *L*(*k-m*); consider interactions *TMP*(*k*)**DPT*(*k*)

⇒*Generating scenarios*