Summer Schools 2012, June 25 to July 6, 2012

STOCHASTIC OPTIMIZATION

Revenue Management Optimization in the Airline Industry

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Agenda

Revenue Management In an Airline

- What is Revenue Management? An introduction to Revenue Management Systems

Optimization in Revenue Management

- The OR techniques behind the screens

Revenue Management In an Airline

- What is revenue management? An introduction to Revenue Management Systems

A first example of Revenue Management

The value of a seat to go from Paris to Rome depends on the departure date

Your departure flight: Paris to Rome

Friday 18 November	Saturday 19 November	**Sunday 20 November**	Monday 21 November	Tuesday 22 November	Wednesday 23 November	Thursday 24 November
From **283 €**	From **76 €**	From **76 €**	From **192 €**	From **76 €**	From **102 €**	From **102 €**

Your return flight: Rome to Paris

Sunday 27 November	Monday 28 November	**Tuesday 29 November**	Wednesday 30 November	Thursday 1 December	Friday 2 December	Saturday 3 December
From **64 €**	From **90 €**	From **64 €**	From **64 €**	From **64 €**	From **64 €**	From **64 €**

Total amount including taxes for 1 Adult passenger(s) : 140 €

Lowest fare

5 A second example of Revenue Management

Even on the same flight prices may differ

Op. Flights Depart Arrive Aircraft	Tango Plus	Latitude	Executive First Lowest	Executive First Flexible
Direct Flights				
AC871 13:25 14:50 77W	€ 215	€ 1125	€ 1556	€ 2677
Cabin				
Any time change fee	**will apply**	**may apply**	**may apply**	**free**
Same day airport change fee	**$100 CAD**	**free**	**free**	**free**
Miles accumulated	**100%**	**100%**	**125%**	**150%**
Priority check-in		**applies**	**applies**	**applies**
Access to airport lounge	**$65 CAD**	**$55 CAD**	**applies**	**applies**

Products offered by Air Canada to go from Paris to Montreal on November 21st

Source: Air Canada website, request made on November 11th 2011

Revenue Management objective and means

Selling the right seat, to the right person at the right moment

Revenue Management determines product availability

- How many seats to protect on each flight, for each product, against what price, at each point in time?

To maximize network-wide revenue

- Protect seats for business passengers who bring high value and book late
- A connecting passenger brings a lower value than two non-stop passengers
- One connecting passenger brings another value than another connecting

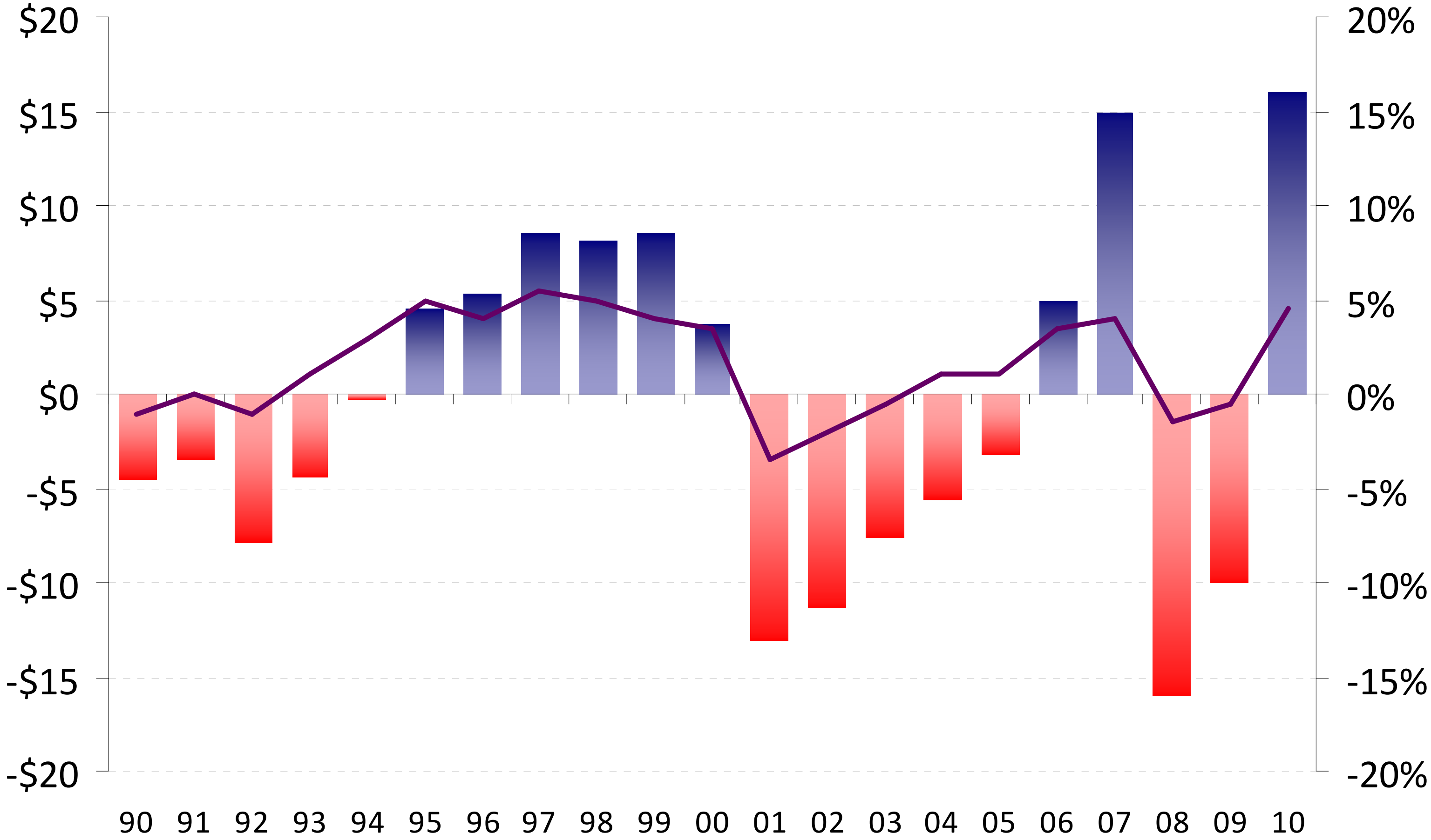
Dealing with uncertainty

- Seats correspond to a fixed amount of perishable resources
- Demand is uncertain and each empty seat will be a loss of revenue (spoilage)

Operations Research models are needed to solve this complex problem

Airline industry has a volatile profitability

In this context optimizing airline revenue is very important

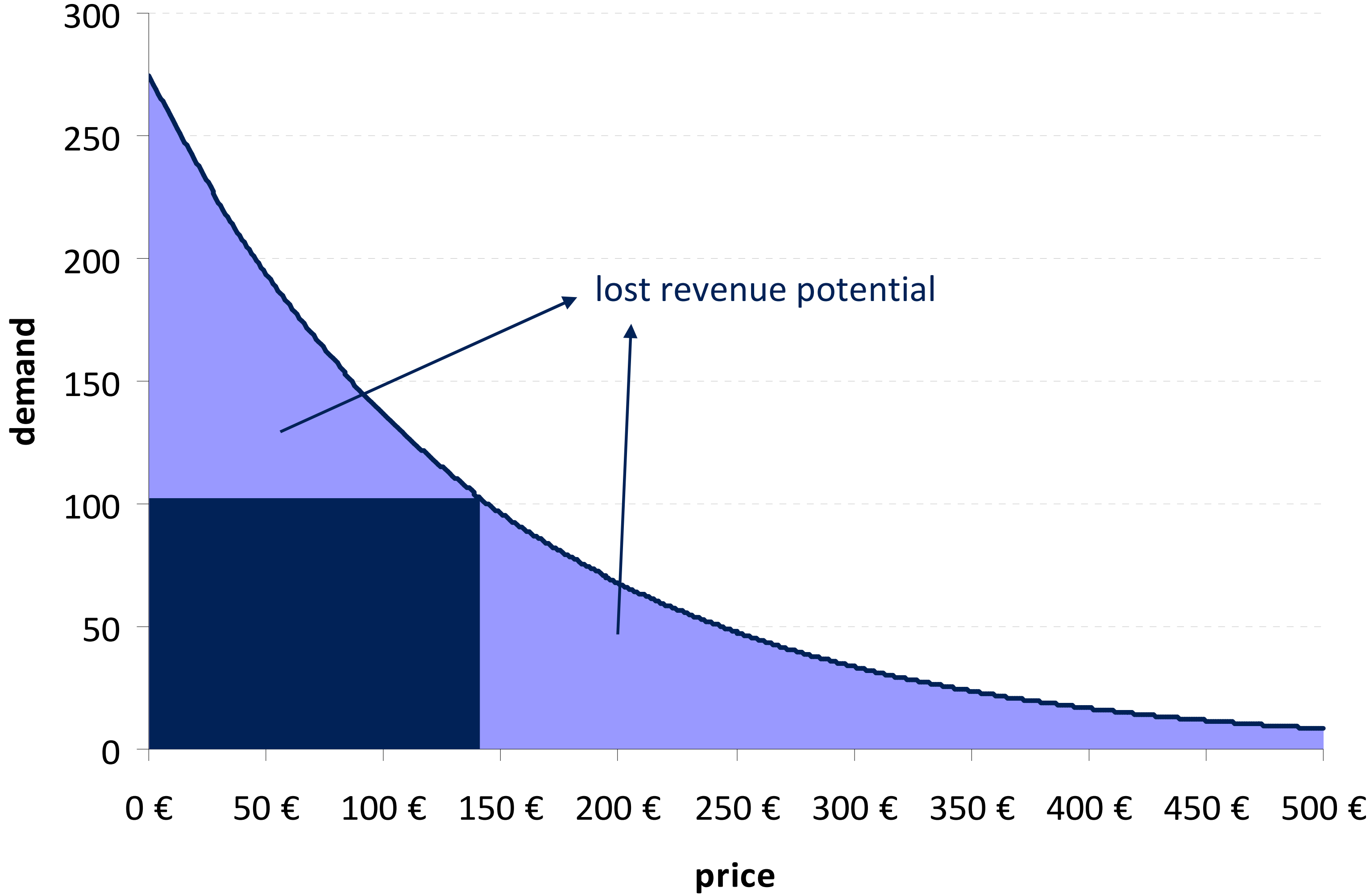


Global net profit of commercial airlines, in billions USD, and EBIT margin

Source: ICAO data

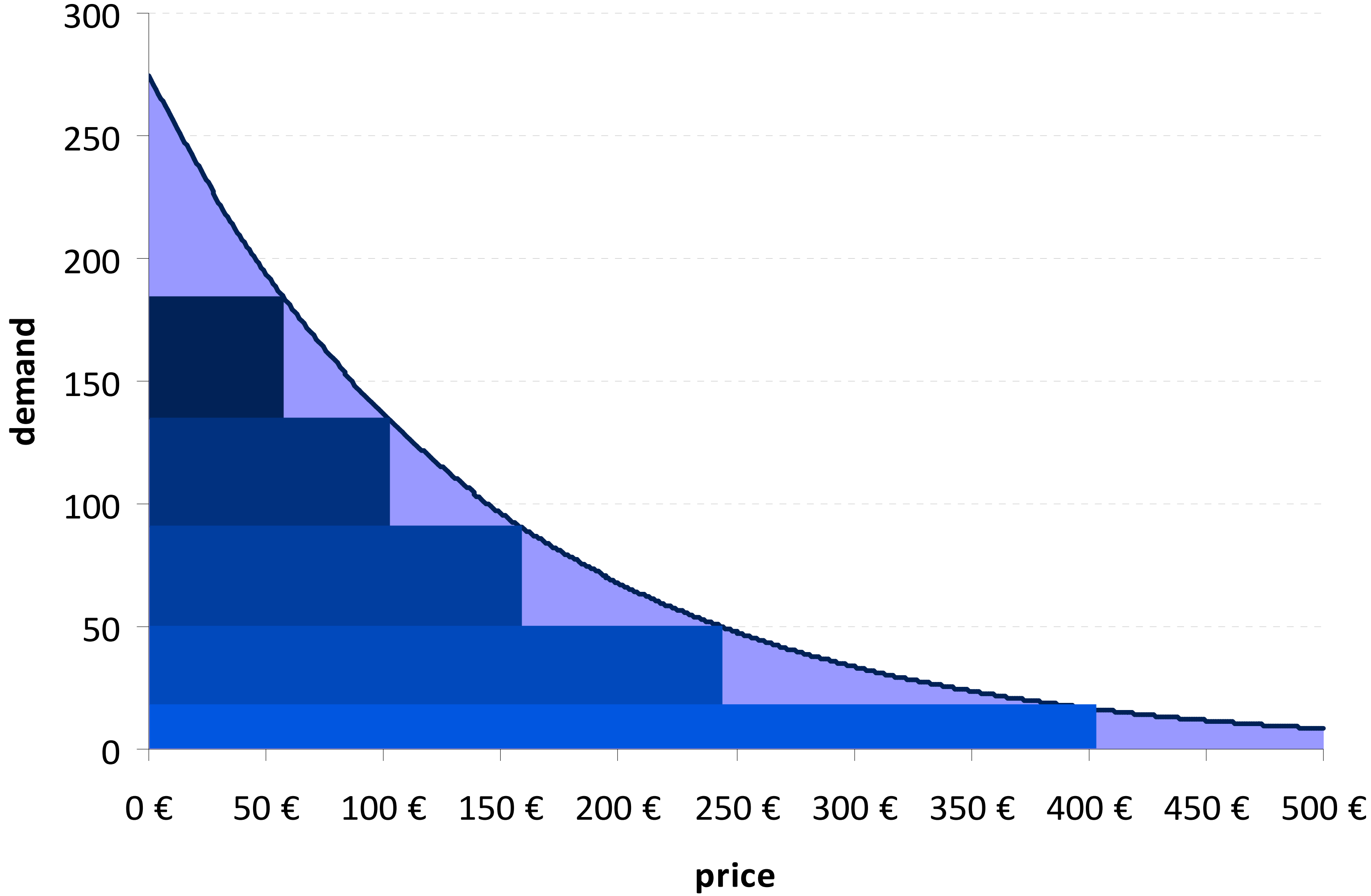
Price elasticity in demand

By offering only one fare, some revenue potential is lost



Segmentation of demand

By offering several fares, revenue can be increased



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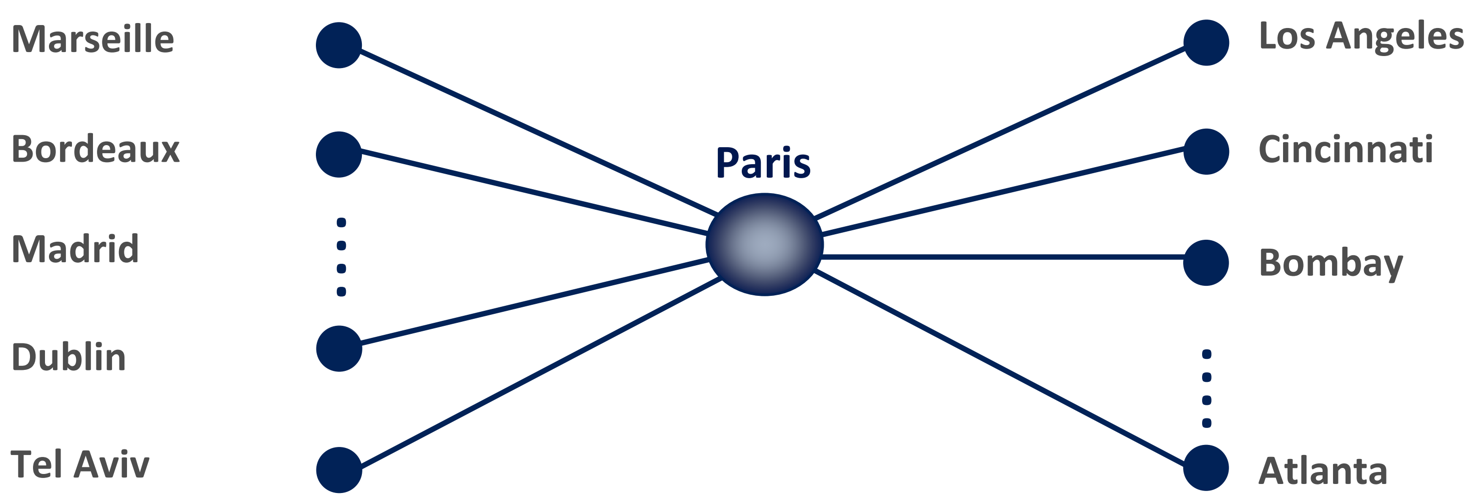
Example of a fare grid on a flight to Japan

Fare restrictions defined by pricing are designed to segment the market

Class	Cabin	One way	Return	Conditions
F	First	9 058 €	12 939 €	-
J	Business	5 591 €	7 986 €	-
C	Business	2 940 €	5 880 €	-
C	Business		4 200 €	min stay 3D
D	Business		3 490 €	min stay 4D, max stay 6M
Y	Eco	4 413 €	6 303 €	-
W	Eco	1 360 €	2 720 €	NRF
H	Eco	721 €	1 442 €	NRF, max stay 3M
B	Eco		1 280 €	NRF, min stay 3D, max stay 6M
H	Eco		1 030 €	NRF, min stay 5D, max stay 3M
K	Eco		910 €	NRF, min stay 5D, max stay 3M
V	Eco		700 €	NRF, min stay 7D, max stay 1M

The hub-and-spoke organization maximizes connecting trafic…

… which implies more complexity in the network optimization of the revenue



59 arrivals (medium haul) & 21 departures (long haul)

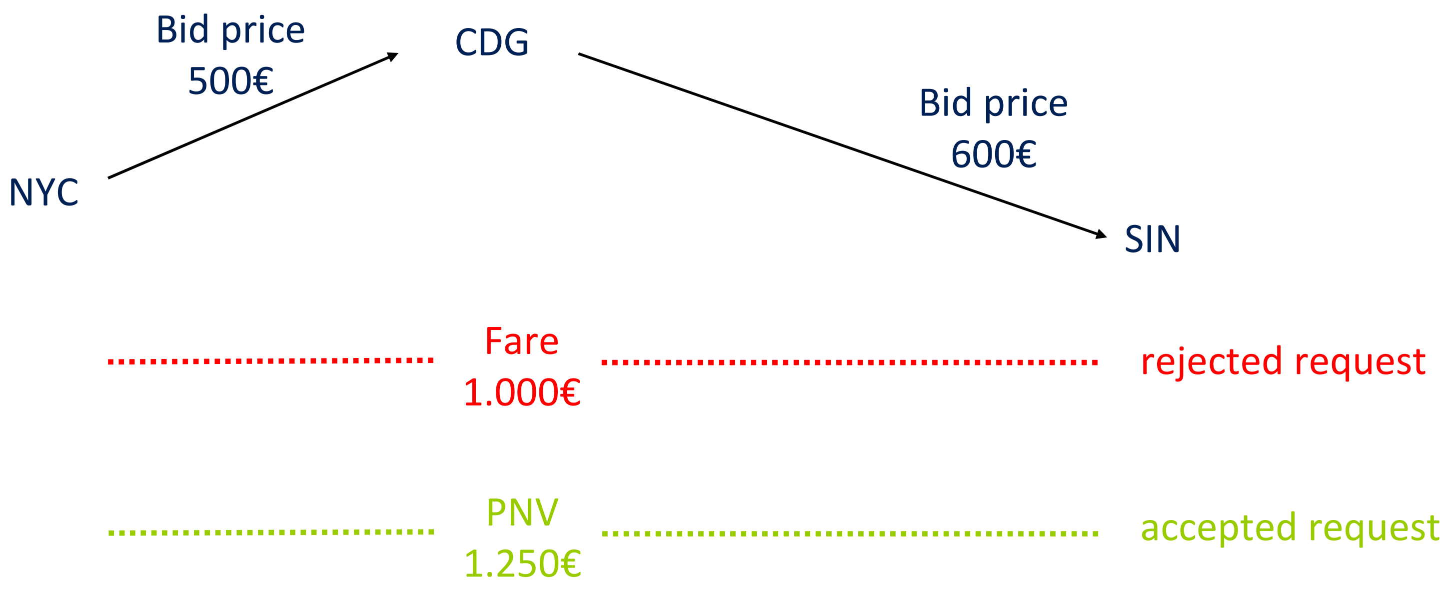
↓

1319 possible Origin-Destination paths

⇨ To which paths should seats be offered and by which quantity?

O&D control parameters

Introduction to the bid price



The bid-price is a financial quantity: it represents the minimum value required to book the next sellable seat on any given flight

Revenue Management in an airline

Revenue Management is one part in the sales process

Network	Deciding airline schedule: destinations, frequencies, timetable
Pricing	Defining fares and restriction
Distribution sales	Developing markets and sales
Revenue Mngt	Optimizing revenue obtained from the seat inventory

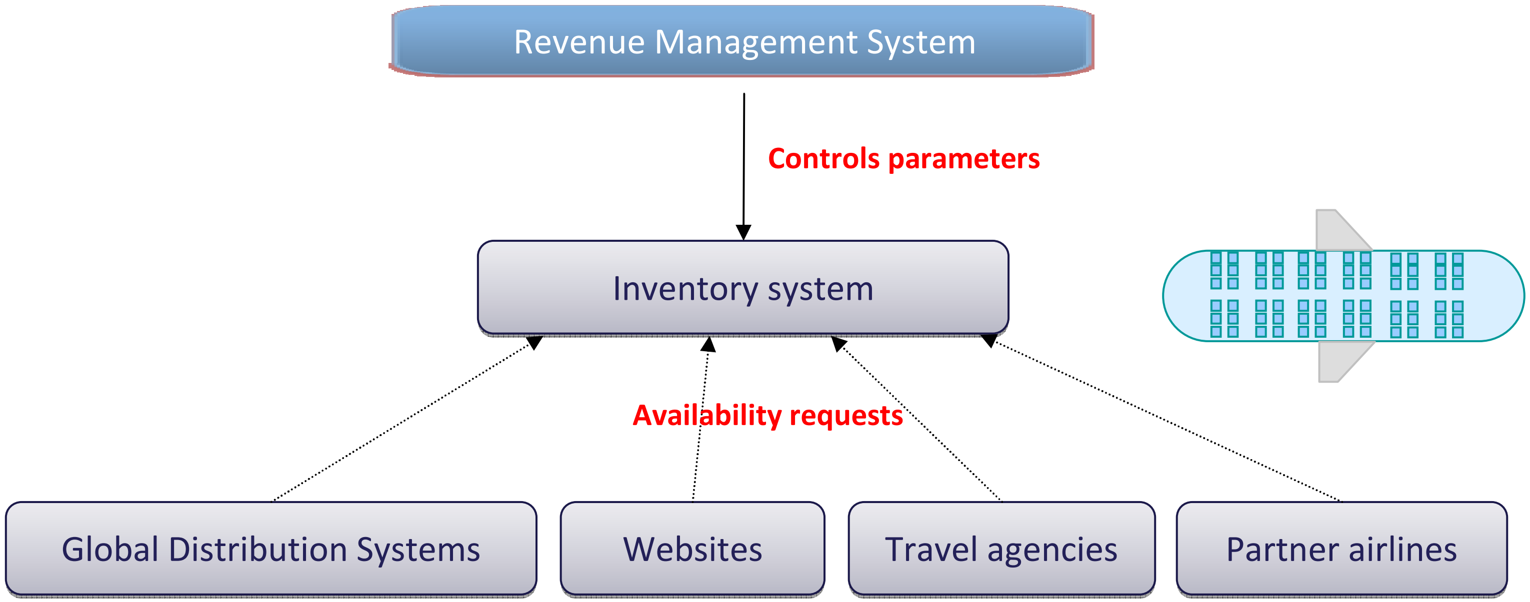
Reservation and inventory systems

Complex systems are needed to answer customer requests in real-time

Customers book through reservation systems (like www.airfrance.fr)

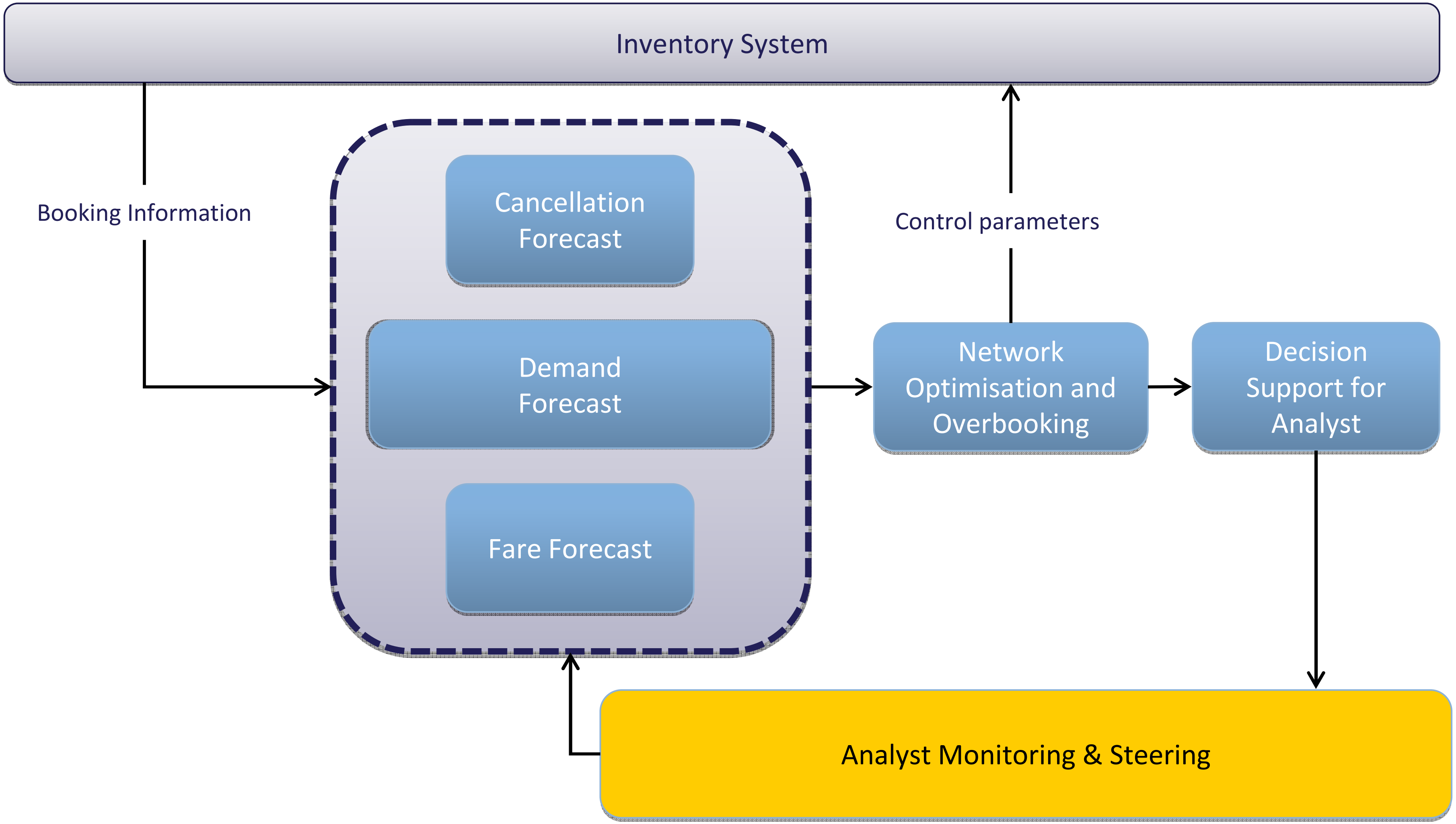
- These **reservation systems** must offer fares in a split second to every request

A centralized inventory system answers availability requests



A Revenue Management System in an airline

A quick overview



Optimization in Revenue Management

- The basic techniques behind the screens

Introduction to optimization for Revenue Management

More revenue is obtained by saving seats for high-value passengers

Demand = 40

Product Y « Business » 223 €
- No restrictions

Demand = 200

Product N « Economy » 72 €
- Saturday night stay

A319 « La Navette »
Capacity = 142 seats

Without RM: « 1st come 1st served »
Revenue = **10 224 €** (142x72)

Simple RM: « Protect 40 Y seats »
Revenue = **16 264 €** (40x223 + 102x72)

But in real life demand is not deterministic…

In this example demand is assumed to be deterministic which means that realized demand is exactly as forecasted

Traditional Revenue Management

Demand is assumed to be perfectly segmented

Sensitivity to restrictions:
- → Advance Purchase
- → Non Refundable Fare
- → Week-end Stay…

=> independant demand model

Segmentation


Classe Y (1200 €) – no restr.


Classe M (800 €) – book > 21 j


Classe K (500 €) – no refund + book > 21 j

	Demand Forecast
Class Y	10
Class M	13
Class K	25

If product is not available anymore, demand does not report on other choices

Demand distribution can be Poisson or Gaussian…

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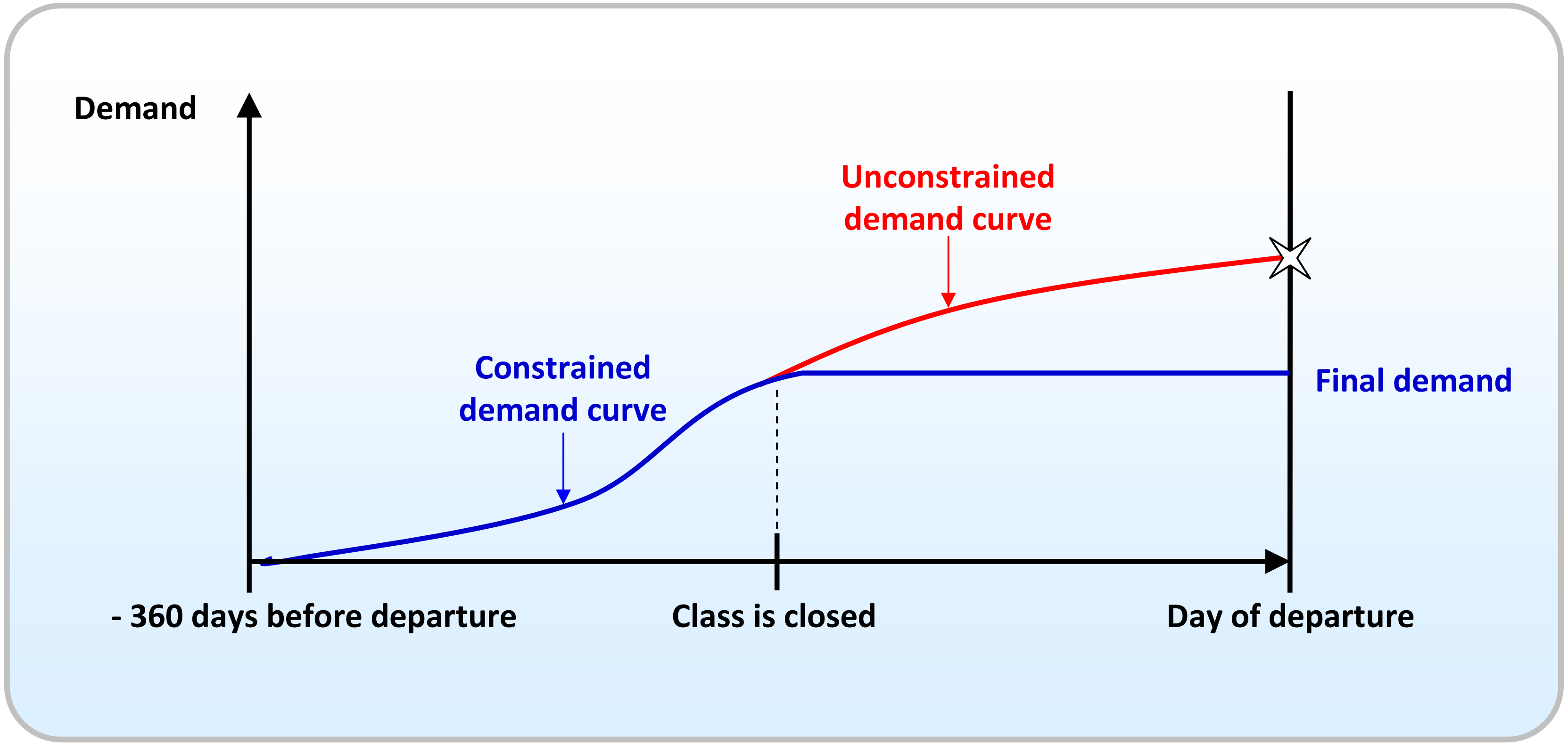
Demand unconstraining

Demand observation is incomplete because of class closures

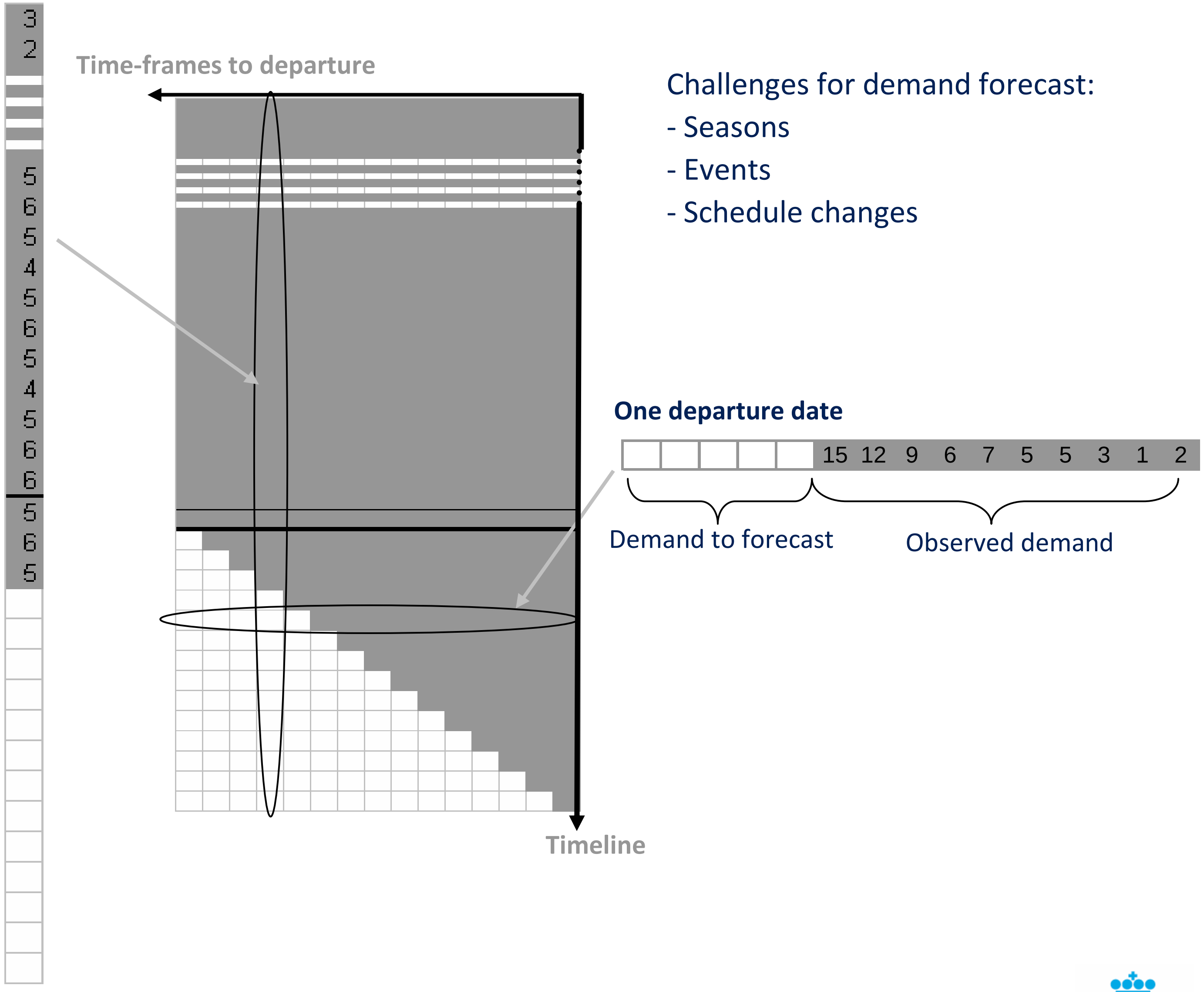


Demand forecast

Forecasts rely on past observations and bookings

Challenges for demand forecast:
- Seasons
- Events
- Schedule changes

One Timeframe, all departure dates



Optimization - static single-resource models

Main assumptions

- Demands for different classes are independent
- Demand for different classes arrives in non-overlapping intervals
- Demand arrives in order of increasing class prices
- Demand for given class does not depend on availability of other classes
- No group bookings

Littlewood's model

Optimal solution for two classes

Littlewood hypothesis

- Static single resources hypothesis
- Capacity is C
- There are two products with associated prices $p_1 > p_2$ and demands D_1 & D_2

Littlewood's model

- Expected revenue of seat x if we sell it to class 1 is: $p_1 \, . \, P(D_1 \geq x)$
- So the protection level y_1 for class 1 should be such that:
  - $p_2 < p_1 \, . \, P(D_1 \geq y_1)$
  - $p_1 \, . \, P(D_1 \geq (y_1 + 1)) \leq p_2$

Littlewood, Forecasting and control of passengers bookings, AGIFORS Annual Symposium Proceedings, vol. 12, 1972

EMSRa heuristic

Computing seat protections using Littlewood's model

- Class 1 from class 2 (S_{12})
- Class 1 from class 3 (S_{13}) & class 2 from class 3 (S_{23})
- Class 1 from class N (S_{1N}) & … & class N-1 from class N (S_{N-1N})

Sum protections and deduce booking limits

- BL_1 = capacity
- BL_2 = capacity - S_{12}
- BL_3 = capacity - (S_{13}+S_{23})
- BL_N = capacity - ($\sum_{(i<N)} S_{iN}$)

EMSRa is a heuristic

- It ignores the statistical averaging effect obtained by aggregating demand across classes

Belobaba, Air Travel Demand and Airline Seat Inventory Management, MIT. PhD thesis, 1987

EMSRb Heuristic

Define joint classes

- Class 1 mean m_1 with standard deviation σ_1 and fare F_1
- Classes 1 to 2 $m_{1..2} = m_1 + m_2$; $\sigma_{1..2} = \sqrt{(\sigma^2_1 + \sigma^2_2)}$; $F_{1..2} = (m_1F_1+m_2F_2)/m_{12}$
- Classes 1 to N-1 $m_{1..N-1} = \Sigma_{(i<N)} m_i$; $\sigma_{1..N-1} = \sqrt{(\Sigma_{(i<N)} \sigma^2_1)}$; $F_{1..N-1} = (\Sigma_{(i<N)} m_iF_i)/ m_{1..N-1}$

Computing seat protections using Littlewood's model

- Classes 1 to k-1 from class k ($S_{1..k-1/k}$)

Deduce booking limits

- BL_k = capacity – $S_{1..k-1/k}$

EMSRb is also a heuristic

- The weighted average revenue is an approximation

Belobaba, Application of a Probabilistic Decision Model To Airline Seat Inventory Control, Operations Research, Vol. 37, 1989

Optimization - network model

Deterministic LP Formulation

Definition of variables

X_j	allocation of capacity for O&D fare class j
r_j	price for fare class j
d_j	mean demand for fare class j
c_k	capacity of leg k
$\partial_{j,k}$	1 if O&D fare class j uses leg k

Writing down the Linear Program

- Objective: maximize revenue
- Constraints: capacity and demand constraints

$$\text{Max} \quad \sum_j r_j * X_j$$

$$\text{s.t.} \quad \begin{cases} \sum_{j \in k} \partial_{j,k} X_j \le c_k, & \forall k \\ 0 \le X_j \le d_j, & \forall j \end{cases}$$

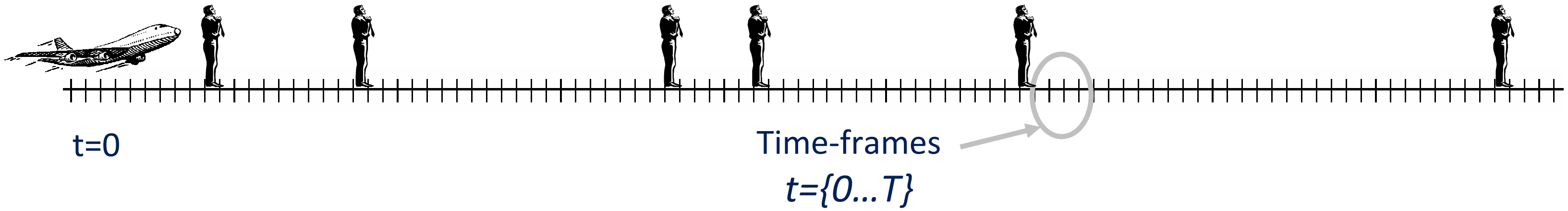
The bid-prices are the dual values corresponding to the capacity contraints

BPs can be used as control or to decompose the problem at leg level

Williamson, Airline Network Seat Inventory Control: Methodologies and Revenue Impacts, MIT. PhD thesis, 1992

Dynamic Programming models for the single-resource problem

The decision are taken on very small time-frames



Time to departure is divided in many small time-frames

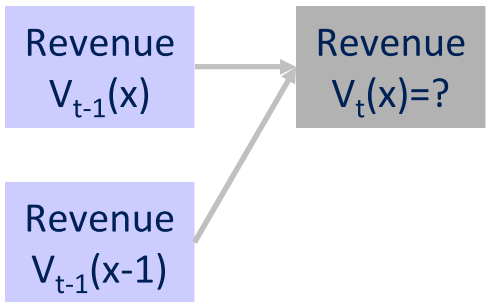
- such that the probability of having more than one request per time frame is negligible

The maximal expected revenue can be computed for each time-frame

- Say $\mathbf{V_t(x)}$ is the maximal revenue the company can expect when there are **x** seats remaining in time frame **t**
- As shown next, if $V_{t-1}(x)$ and $V_{t-1}(x-1)$ are known, it is possible to compute $V_t(x)$. It is therefore possible to recursively compute $V_t(x)$ for any given (t,x).

$V_0(x)=0$

$V_t(0)=0$



Subramanian, Stidham and Lautenbacher, Airline Yield Mgmt with Ovbkg, Cancellations, and No-Shows, Transportation Science, Vol. 33, 1999

Dynamic Programming models for the single-resource problem

Optimal decision at time t can be obtained based on future optimal decisions

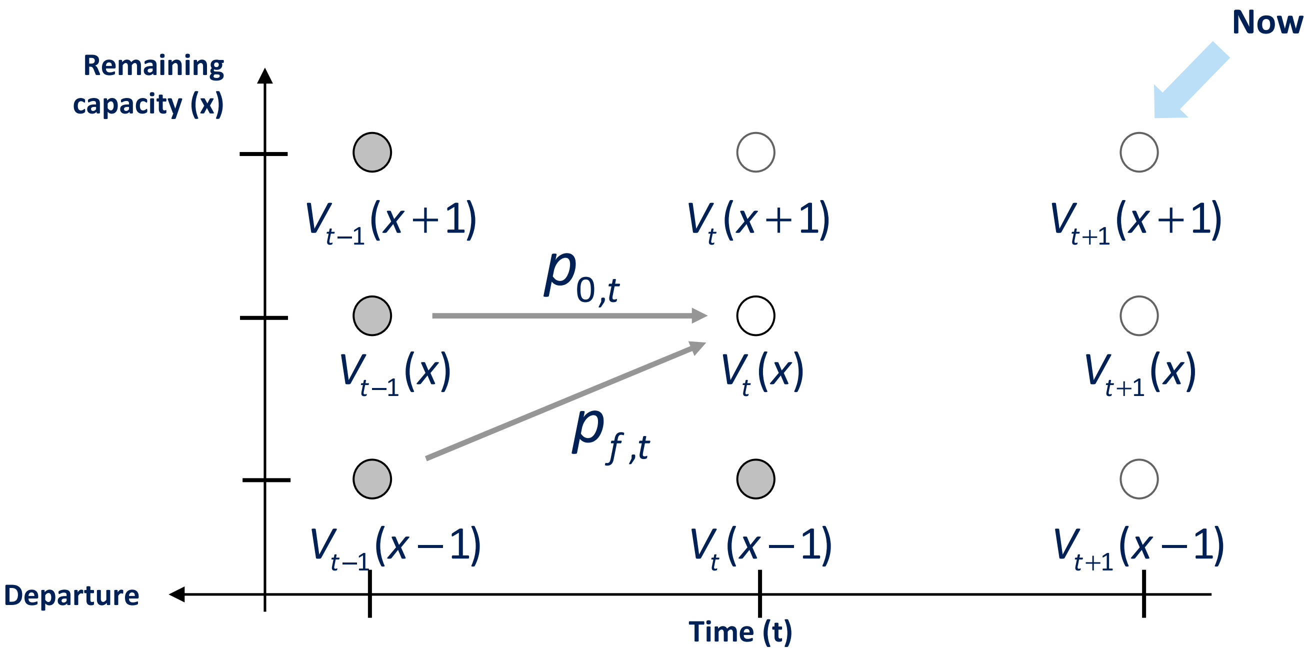
The Bellman equation

Fare Class	P_j Prob. of request in FC i, TF t	Revenue if request is accepted
1	$P_{1,t}$	$V_{t-1}(x-1) + r_1$
:	$P_{j,t}$	$V_{t-1}(x-1) + r_j$
f	$P_{f,t}$	$V_{t-1}(x-1) + r_f$
0	$P_{0,t}$	$V_{t-1}(x)$

$$V_t(x) = \sum_{f=1}^{N} p_{f,t} \max\left\{V_{t-1}(x-1) + r_f ; V_{t-1}(x)\right\} + p_{0,t} V_{t-1}(x)$$

Dynamic Programming models for the single-resource problem

A bid-price can be computed for each remaining capacity and time



$$V(x,t)-V(x-1,t)$$

↓

Marginal revenue of seat x at time t.

↓

The 'bid price' asked for this seat.

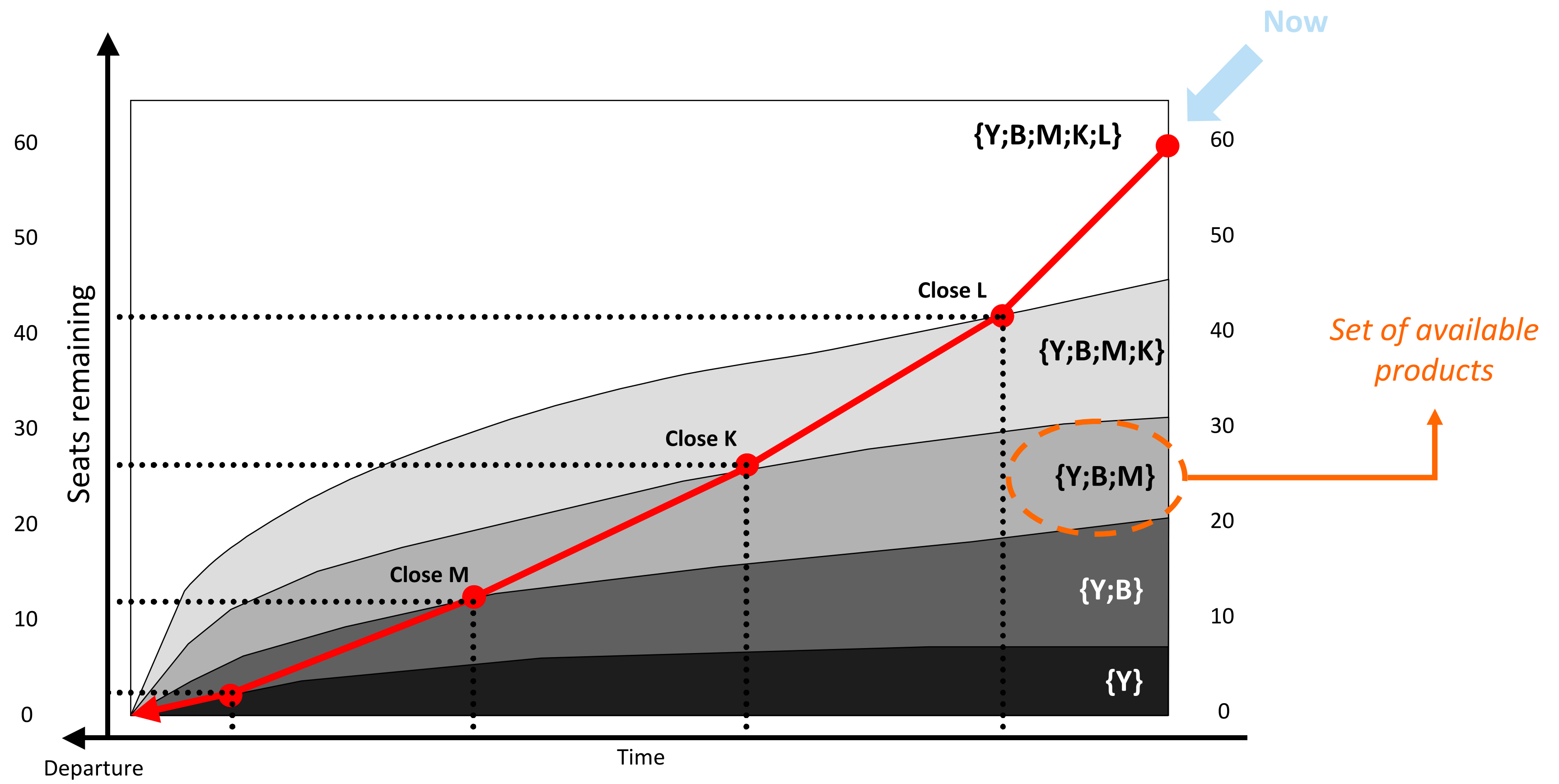
$$V_t(x)=\sum_{f=1}^{N} p_{f,t}\max\{V_{t-1}(x-1)+r_f;V_{t-1}(x)\}+p_{0,t}V_{t-1}(x)$$

- $p_{0,t}$: *Probability of no purchase & no arrival*
- $p_{f,t}$: *Probability of request in class f at time t*
- r_f: *Revenue for class f*

Van Ryzin, The Theory and Practice of Revenue Management, 2004

Dynamic Programming models for the single-resource problem

The obtained bid-price map can be used to decide on class availability



Dynamic Programming models for the network problem

The curse of dimensionality

Exact problems are hard to solve at network level…

$$V(X,t)=\sum_{f=1}^{N} p_{f,t}\max\{V(X-A_f,t-1)+r_f;V(X,t-1)\}+p_{0,t}V(X,t-1)$$

Vector of remaining capacities on each flight leg (X)

Flight leg utilization of O&D product f (A_f)

…so decomposition heuristics are used instead.

1200 aircraft movements / day (2000 cabins)
300 seats per cabin
500 timeframes
Problem size = 500 * 300^{2000} states

Adelman, Dynamic bid-prices in revenue management, Operations Research, Vol. 55(4), 2007
Kunnumkal, Topaloglu, A new DP decomposition method for the network RM pb with customer choice behavior, P. and O. Mgmt , Vol. 19, 2010

Conclusion

Thank you for your attention

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