KMUTT Department of Mathematics
King Mongkut's University of Technoloty Thonburi
Bangkok, Thailand

21–29 December 2015

Stochastic and Dynamic Optimization. Optimal Energy Allocation in Micro-Grids.

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Eligibility/Pre-requisites.

- Mathematical skills. Computer skills.
- Continuous optimization: linear programming, convexity, duality, first-order optimality conditions. [Ber96]
- Probability calculus: probability space, probability, random variable, distribution law of a random variable, indicator function, mathematical expectation, independence, law of large numbers. [Fel68, Bre93, Pit93]
- Software Scicoslab to be installed (else, install software Scilab)

Learning outcomes. After the course the student should be able to

- design mathematical models for energy storage and delivery of renewable energies, especially in micro-grids, and formulate cost-minimization problems,
- use the scientific software Scicoslab and numerically solve small scale problems.

Course main content. The course mixes theoretical sessions, modeling exercises and computer sessions.

In introduction, we present examples of micro-grid and virtual power plant management — where the question of electrical storage is put, due to the need to answer a varying demand and to incorporate intermittent and highly variable renewable energies. We show how such problems can be formulated as dynamic stochastic optimization problems [LCCL14].

In a deterministic optimization problem, the values of all parameters are supposed known. What happens when this is no longer the case? And when some values are revealed during the stages of decision? We present stochastic optimization, at the same time as a frame to formulate problems under uncertainty, and as methods to solve them according to the formulation. More precisely, we present *stochastic programming* in two stages (and the resolution on scenario tree or by scenarios) and *stochastic control* in discrete time (and the resolution by *stochastic dynamic programming*).

We devote time to the *Stochastic Dual Dynamic Programming (SDDP)* algorithm, widely used in the world of the energy, which mixes dynamic programming and cutting plane algorithm. The SDDP approach seems especially adapted to micro-grid management issues.

Modeling exercises and computer sessions tackle issues like optimal economic dispatch of energy production units, storage/delivery optimization problem to buffer an intermittent and variable source of energy, dam optimal management with stochastic water inflows, battery optimal management with renewable energy inputs.

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Link course. `http://cermics.enpc.fr/~delara/ENSEIGNEMENT/KMUTT2015/`

Program

1 / Monday 21 December 2015 (9h00–12h00)

Introductory talks

To introduce the course, we present examples of micro-grid and virtual power plant management that make use of dynamic stochastic optimization

- *"Cooperating between academy and industry for the optimization of smart grids"*
- Work done by Francis Sourd (Sun'R) and Ariel Waserhole (Sun'R)
  "SunHydro: l'optimisation stochastique au cœur d'un projet collaboratif"
- Work done by Tristan Rigaut (Efficacity)
  "Energy optimization and climate control of a subway station microgrid"
- Work done by Romain Bonabe de Rougé (Efficacity), Valentin Foucher (student at École Polytechnique) and François Pacaud (Efficacity)
  "Optimal sizing and integration of a micro-combined heat and power generation with thermal and electrical storage in an individual residential housing"

2 / Monday 21 December 2015 (13h00–16h00)

Lecture and exercises

We present, under the form of exercises, examples of optimization problems under uncertainty: "the blood-testing problem", "the newsvendor problem" ("stock management problems").

3 / Tuesday 22 December 2015 (9h00–12h00)

Lecture and exercises

Recalls and exercises on probability calculus.
End of the newsvendor problem.

Computer session

Introduction to the scientific software Scicoslab. [CCN10]

4 / Tuesday 22 December 2015 (13h00–16h00)

Special KMUTT Talk

- *"Cooperating between academy and industry for the optimization of smart grids"*
- Work done by Francis Sourd (Sun'R) and Ariel Waserhole (Sun'R)
  "SunHydro: l'optimisation stochastique au cœur d'un projet collaboratif"
- Work done by Tristan Rigaut (Efficacity)
  "Energy optimization and climate control of a subway station microgrid"
- Work done by Romain Bonabe de Rougé (Efficacity), Valentin Foucher (student at École Polytechnique) and François Pacaud (Efficacity)
  "Optimal sizing and integration of a micro-combined heat and power generation with thermal and electrical storage in an individual residential housing"

5 / Wednesday 23 December 2015 (9h00–12h00)

Computer session

Introduction to the scientific software Scicoslab. [CCN10]

Computer exercise on the newsvendor problem.
(only Section 1)

6 / Wednesday 23 December 2015 (13h00–16h00)

Modelling exercise on static inventory problems

We begin by formulating a problem of optimal choice of a product quantity (energy, for instance) to satisfy a demand, with costs of purchase, of backorder and of holding. We show how to obtain a linear program, first in a determinist setting (demand known in advance), then in a probabilistic one with a finite number of scenarios of demand.

Modelling exercise on optimal energy allocation

We continue by an exercise of modelling. How can we mathematically formulate a problem of optimal allocation of power production units, at minimal cost, with guaranteed minimal production and guaranteed maximal pollution? We naturally obtain a linear program. We will see how the introduction of uncertainties (costs, renewable power production) modifies the problem formulation. It is the opportunity to touch the notions of *risk* and of *non anticipativity*.

Lecture

Two-stage stochastic programming on a scenario tree.
Non-anticipativity constraint along scenarios: tree representation.
[SDR09, KW12]

Computer session

Sizing of reserves for the balancing on an electric market. Two stage stochastic programming (linear optimization on a tree).
(From Question 1 to Question 4)

7 / Monday 28 December 2015 (9h00–12h00)

Computer session

Sizing of reserves for the balancing on an electric market. Two stage stochastic programming (convex quadratic optimization on a tree).
(From Question 5 to Question 7)

8 / Monday 28 December 2015 (13h00–16h00)

Lecture

Recalls and exercises on continuous optimization [Ber96].

- Recalls on convexity: convex sets, convex functions, strict and strong convexity (characterization by the Hessian in the smooth case), operations preserving convexity.
- Abstract formulation of a minimization problem: criterion, constraints. Sufficient conditions for the existence of a minimum (continuity and compacity/coercivity). Sufficient condition for the uniqueness of a minimum (strict convexity). Exercises with a quadratic objective function on an interval.
- Definition of a local minimizer; necessary condition in the differentiable case. Formulation of a minimization problem under explicit equality constraints. Necessary first-order optimality conditions in the regular/affine equality constraints case; Lagrangian, duality, multipliers. Sufficient first-order optimality conditions in the convex-affine case. Exercises.
- Saddle point. Existence of a saddle point for a continuous, convex-concave function displaying coercivity along two coordinate lines. Uzawa algorithm.

9 / Tuesday 29 December 2015 (9h00–12h00)

Lecture

Two-stage stochastic programming on a comb.
Non-anticipativity constraint along scenarios.
Scenario decomposition by Lagrangian relaxation. Progressive Hedging [RW91].

Computer session

Sizing of reserves for the balancing on an electric market. Two stage stochastic programming (linear and convex quadratic optimization on a comb).
(From Question 8 to Question 12)

10 / Tuesday 29 December 2015 (13h00–16h00)

Computer session

Sizing of reserves for the balancing on an electric market. Two stage stochastic programming (linear and convex quadratic optimization on a comb).
(From Question 8 to Question 12)

Lecture

Dynamical models of storage (battery models, dam models). Inventory problems.
The secretary problem.

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