Cours RTE 2022
Optimisation stochastique

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En guise de conclusion

Mélange des techniques de décomposition spatiale

et de décomposition par blocs temporels

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We start with a stochastic multistage optimization problem presenting a two time scales structure

$$\min_{(\mathbf{X}_{0:D+1}, \mathbf{U}_{0:D})} \mathbb{E}\bigg[\sum_{d=0}^{D} L_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d) + K(\mathbf{X}_{D+1})\bigg]$$

$$\text{s.t} \quad \mathbf{X}_0 = x_0 \ , \ \ \mathbf{X}_{d+1} = f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d)$$

$$\sigma(\mathbf{U}_{d,m}) \subset \sigma\Big(\mathbf{W}_{d',m'}, \, (d', m') \leq (d, m)\Big)$$

The decision $\mathbf{U}_d$ (resp. noise $\mathbf{W}_d$) at the slow time step d is a sequence of decisions (resp. noises) at the fast time scale

$$\mathbf{U}_d = (\mathbf{U}_{d,0}, \ldots, \mathbf{U}_{d,m}, \ldots, \mathbf{U}_{d,M})$$

$$\mathbf{W}_d = (\mathbf{W}_{d,0}, \ldots, \mathbf{W}_{d,m}, \ldots, \mathbf{W}_{d,M})$$

$\mathbf{X}_d$ being the state at the beginning of the slow time step d

We assume that the $\mathbf{W}_d$ are independent at the slow time scale

The problem is assumed to have a large spatial dimension

$$\min_{(\mathbf{X}_{0:D+1},\mathbf{U}_{0:D})} \mathbb{E}\Big[\sum_{i=1}^{N}\Big(\sum_{d=0}^{D} L_d^i(\mathbf{X}_d^i,\mathbf{U}_d^i,\mathbf{W}_d) + K^i(\mathbf{X}_{D+1}^i)\Big)\Big]$$

$$\text{s.t} \quad \mathbf{X}_0^i = x_0^i \ , \ \ \mathbf{X}_{d+1}^i = f_d^i(\mathbf{X}_d^i,\mathbf{U}_d^i,\mathbf{W}_d)$$

$$\sigma(\mathbf{U}_{d,m}^i) \subset \sigma\big(\mathbf{W}_{d',m'},\ (d',m') \leq (d,m)\big)$$

$$\sum_{i=1}^{N} \Theta_d^i(\mathbf{X}_d^i,\mathbf{U}_d^i,\mathbf{W}_d) = 0$$

The question is: how to decompose the problem both

- in the time dimension (time blocks)
- and in the space dimension (units)

If we first perform a time blocks decomposition (say by resources), we obtain a collection of relaxed deterministic intraday function L_d^{R}

$$L_d^{\mathrm{R}}(x_d^1, \dots, x_d^N, r_{d+1}^1, \dots, r_{d+1}^N) = \min_{\mathbf{U}_d} \mathbb{E}\left[\sum_{i=1}^N L_d^i(x_d^i, \mathbf{U}_d^i, \mathbf{W}_d)\right]$$

$$\text{s.t} \quad f_d^i(x_d^i, \mathbf{U}_d^i, \mathbf{W}_d^i) \geq r_{d+1}^i \ , \quad \forall i \in [\![1, N]\!]$$

$$\sigma(\mathbf{U}_{d,m}^i) \subset \sigma(\mathbf{W}_{d,0:m}) \ , \quad \forall i \in [\![1, N]\!]$$

$$\sum_{i=1}^N \Theta_d^i(\mathbf{X}_d^i, \mathbf{U}_d^i, \mathbf{W}_d) = 0$$

- This intraday problem seems to be a stochastic version of the deterministic problem solved by Antares when using targets
- The problem may be difficult to solve in a stochastic framework
- and has to be solved for every pair (x_d, r_{d+1})

If we start by time blocks decomposition. . . II

Once computed the resource intraday functions, we have to solve the associated Bellman equation

$$\overline{V}_d^{\mathrm{R}}(x_d) = \inf_{r_{d+1}} \; L_d^{\mathrm{R}}(x_d, r_{d+1}) + \overline{V}_{d+1}^{\mathrm{R}}(r_{d+1})$$

- This Bellman equation can not be solved directly by DP
  $\longrightarrow$ curse of dimensionality
- SDDP, or some decomposition tool, is needed

Now we start with the space decomposition

If we first perform a spatial decomposition (say by prices), we obtain a collection of subproblems indexed by the spatial index i

$$\min_{\mathbf{U}_d^i} \mathbb{E}\Big[\sum_{d=0}^{D} L_d^i(\mathbf{X}_d^i, \mathbf{U}_d^i, \mathbf{W}_d) + \big\langle \lambda_d \ , \Theta_d^i(\mathbf{X}_d^i, \mathbf{U}_d^i, \mathbf{W}_d) \big\rangle + K^i(\mathbf{X}_{D+1}^i) \Big]$$

$$\text{s.t} \quad f_d^i(\mathbf{X}_d^i, \mathbf{U}_d^i, \mathbf{W}_d) = \mathbf{X}_{d+1}^i$$

$$\sigma(\mathbf{U}_{d,m}^i) \subset \sigma\big(\mathbf{W}_{d',m'}, \ (d', m') \leq (d, m)\big)$$

- This subproblem involves a small dimension state
- and displays a two time scales structure, hence possible time blocks decomposition

In order to use time blocks decomposition with resources, one has to check that the monotonicity property holds true when introducing the duality terms $\big\langle \lambda_d \ , \Theta_d^i(\mathbf{X}_d^i, \mathbf{U}_d^i, \mathbf{W}_d) \big\rangle$ in the criterion. . .

Now we start with the space decomposition II

Performing a time blocks decomposition (say by resources) on the i-th subproblem leads to defining deterministic intraday functions $L_d^{\mathrm{R},i}$

$$L_d^{\mathrm{R},i}(x_d^i, r_{d+1}^i, \lambda_d) = \min_{\mathbf{U}_d^i} \mathbb{E}\Big[L_d^i(x_d^i, \mathbf{U}_d^i, \mathbf{W}_d) + \big\langle \lambda_d \, , \Theta_d^i(x_d^i, \mathbf{U}_d^i, \mathbf{W}_d)\big\rangle\Big]$$
$$\text{s.t} \quad f_d^i(x_d^i, \mathbf{U}_d^i, \mathbf{W}_d) \geq r_{d+1}^i$$
$$\sigma(\mathbf{U}_{d,m}^i) \subset \sigma\big(\mathbf{W}_{d,0:m}\big)$$

Once computed the intraday functions, we have to solve a Bellman equation associated to subsystem i

$$\overline{V}_d^{\mathrm{R},i}(x_d^i) = \min_{r_{d+1}^i} L_d^{\mathrm{R},i}(x_d^i, r_{d+1}^i, \lambda_d) + \overline{V}_{d+1}^{\mathrm{R},i}(r_{d+1}^i)$$

and then update the prices λ_d by a gradient-like method

- The computation of the intraday functions for all pair (x_d^i, r_{d+1}^i) is tractable, but has to be done for different values of the price λ_d
- The Bellman equation involves a low dimension state