Mixing Dynamic Programming and Spatial Decomposition Methods

Application to the decentralized management of urban microgrids

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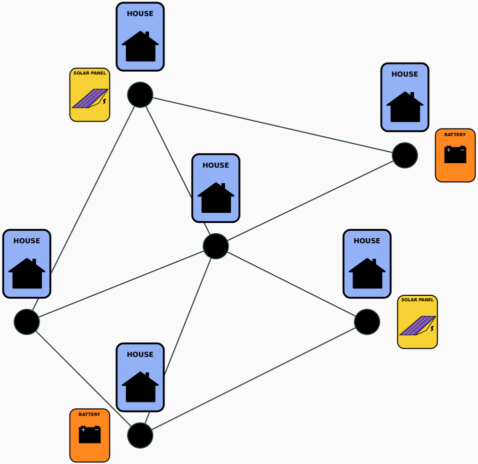
Motivation

We consider a *peer-to-peer* urban microgrid where houses exchange energy

We formulate it as a large-scale stochastic optimization problem



Question: **how to manage it in an (sub)optimal manner?**

Motivation

We will see that, for large district microgid, e.g.

- 48 buildings
- 16 batteries
- 30 edges network

methods mixing spatial and temporal decomposition (price and resource) give better results than the standard SDDP algorithm

- in terms of CPU time: $\times 3$ faster
- in terms of cost gap: 1.5% better

Network	SDDP CPU time	Price CPU time	SDDP cost value	Price cost value
48-Nodes	453'	128'	3550 ± 2.3	3490 ± 2.3

Lecture outline

Optimization upper and lower bounds by decomposition

Nodal decomposition of a network optimization problem

Numerical results on urban microgrids of increasing size

Conclusion

Optimization upper and lower bounds by decomposition

Abstract problem

We consider the following optimization problem

$$
\begin{aligned}
V^\sharp = \inf_{u^1, \cdots, u^N} \; & \sum_{i=1}^{N} J^i(u^i) \\
\text{s.t.} \; & u^i \in \mathbb{U}^i_{\mathrm{ad}} \; , \; \; i = 1, \dots, N \\
& \underbrace{\big(\Theta^1(u^1), \cdots, \Theta^N(u^N)\big) \in S}_{\text{coupling constraint}}
\end{aligned}
$$

with

- $u^i \in \mathbb{U}^i$ be a local decision variable
- $J^i : \mathbb{U}^i \to \mathbb{R}, \; i \in [\![1, N]\!]$ be a local objective
- $\mathbb{U}^i_{\mathrm{ad}}$ be a subset of $\mathbb{U}^i$
- $\Theta^i : \mathbb{U}^i \to \mathcal{C}^i$ be a local constraint mapping
- S be a subset of $\mathcal{C} = \mathcal{C}^1 \times \cdots \times \mathcal{C}^N$

We denote by S^o the polar cone of S

$$
S^o = \big\{ p \in \mathcal{C}^\star \mid \langle p \, , r \rangle \leq 0 \quad \forall r \in S \big\}
$$

Price and resource value functions

For each $i \in [\![1, N]\!]$, we define

- the local price value function

$$\underline{V}^i[p^i] = \inf_{u^i} \; J^i(u^i) + \left\langle p^i \, , \Theta^i(u^i) \right\rangle \, , \;\; \forall p^i \in (\mathcal{C}^i)^\star$$

- the local resource value function

$$\overline{V}^i[r^i] = \inf_{u^i} \; J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i \, , \;\; \forall r^i \in \mathcal{C}^i$$

Theorem

For any

- *admissible price* $p = (p^1, \cdots, p^N) \in S^o$
- *admissible resource* $r = (r^1, \cdots, r^N) \in S$

$$\sum_{i=1}^N \underline{V}^i[p^i] \;\leq\; V^\sharp \;\leq\; \sum_{i=1}^N \overline{V}^i[r^i]$$

Application to multistage stochastic optimization

We consider a large scale stochastic optimal control problem

$$V_0^\sharp(x_0) = \inf_{\boldsymbol{X},\boldsymbol{U}} \mathbb{E}\Big[\sum_{i=1}^{N}\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i)\Big]$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1})\ , \quad \boldsymbol{X}_0^i = x_0^i$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t)$$

$$\big(\Theta_t^1(\boldsymbol{X}_t^1, \boldsymbol{U}_t^1), \cdots, \Theta_t^N(\boldsymbol{X}_t^N, \boldsymbol{U}_t^N)\big) \in S_t$$

with

- $\boldsymbol{W} = (\boldsymbol{W}_1, \cdots, \boldsymbol{W}_T)$: **global** white noise process
- $\boldsymbol{X}^i = (\boldsymbol{X}_0^i, \cdots, \boldsymbol{X}_T^i)$: i-th local state process
- $\boldsymbol{U}^i = (\boldsymbol{U}_0^i, \cdots, \boldsymbol{U}_{T-1}^i)$: i-th local control process
- $g_t^i : \mathbb{X}_t^i \times \mathbb{U}_t^i \times \mathbb{W}_t \to \mathbb{X}_{t+1}^i$: i-th local dynamics
- $\Theta_t^i : \mathbb{X}_t^i \times \mathbb{U}_t^i \to \mathcal{C}^i$: i-th local coupling function
- $L_t^i : \mathbb{X}_t^i \times \mathbb{U}_t^i \times \mathbb{W}_t \to \mathbb{R}$: i-th local instantaneous cost
- $K^i : \mathbb{X}_T^i \to \mathbb{R}$: i-th local final cost

Application to multistage stochastic optimization

We consider a large scale stochastic optimal control problem

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V_0^\sharp(x_0) = \inf_{\boldsymbol{X},\boldsymbol{U}} \ & \mathbb{E}\Big[\sum_{i=1}^{N}\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i)\Big] \\
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- $K^i : \mathbb{X}_T^i \to \mathbb{R}$: i-th local final cost

Bounds for the global problem by spatial decomposition

Theorem

For any admissible price process $\boldsymbol{P} = (\boldsymbol{P}^1, \cdots, \boldsymbol{P}^N) \in S^o$
and any admissible resource process $\boldsymbol{R} = (\boldsymbol{R}^1, \cdots, \boldsymbol{R}^N) \in S$

$$\sum_{i=1}^{N} \underline{V}_0^i[\boldsymbol{P}^i](x_0^i) \;\leq\; V_0^\sharp(x_0) \;\leq\; \sum_{i=1}^{N} \overline{V}_0^i[\boldsymbol{R}^i](x_0^i)$$

with, for each $i \in [\![1, N]\!]$, the price local value function

$$\underline{V}_0^i[\boldsymbol{P}^i](x_0^i) = \inf_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + \langle \boldsymbol{P}_t^i, \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i)\rangle + K^i(\boldsymbol{X}_T^i)\Big]$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) \,, \quad \boldsymbol{X}_0^i = x_0^i$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t)$$

and the resource local value function

$$\overline{V}_0^i[\boldsymbol{R}^i](x_0^i) = \inf_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i)\Big]$$

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$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t)$$

$$\Theta_t^i(x_t^i, \boldsymbol{U}_t^i) = \boldsymbol{R}_t^i$$

Mix of spatial and temporal decompositions

1. To obtain bounds of the optimal value $V_0^\sharp(x_0)$, we have performed two spatial decompositions to compute
   - a collection $\left\{\underline{V}_0^i[\boldsymbol{P}^i](x_0^i)\right\}_{i\in[\![1,N]\!]}$ of price local value functions
   - a collection $\left\{\overline{V}_0^i[\boldsymbol{R}^i](x_0^i)\right\}_{i\in[\![1,N]\!]}$ of resource local value functions

   The computation of these local values can be performed in parallel

2. To compute each local value, we perform temporal decomposition based on Dynamic Programming (DP). For each i, we will obtain
   - a sequence $\left\{\underline{V}_t^i[\boldsymbol{P}^i]\right\}_{t\in[\![0,T]\!]}$ of price local value functions
   - a sequence $\left\{\overline{V}_t^i[\boldsymbol{R}^i]\right\}_{t\in[\![0,T]\!]}$ of resource local value functions

   The computation of these local values functions is done sequentially

To be able to use Dynamic Programming, we need to carefully choose the coordination processes $\boldsymbol{P}$ and $\boldsymbol{R}$ so that the dimension of the state in the local price and resource sub-problems remains tractable for DP.

Computing local value functions by time decomposition

We only deal with deterministic price and resource processes

No additional state variables in the sub-problems

Computing local value functions by time decomposition

We only deal with deterministic price and resource processes

No additional state variables in the sub-problems

Price decomposition

- Fix a deterministic admissible price $p = (p^1, \cdots, p^N) \in S^o$
- For each i, compute $\underline{V}_0^i[p^i](x_0^i)$ by Dynamic Programming (state x_t^i)

$$\underline{V}_t^i[p^i](x_t^i) = \inf_{u_t^i} \mathbb{E}\big[L_t(x_t^i, u_t^i, \boldsymbol{W}_{t+1}) + \langle p_t^i \,, \Theta_t^i(x_t^i, u_t^i) \rangle + \underline{V}_{t+1}^i[p^i](g_t^i(x_t^i, u_t^i, \boldsymbol{W}_{t+1}))\big]$$

- Return the sequence $\{\underline{V}_t^i[p^i]\}_{i \in [\![1,N]\!]}$

Resource decomposition

- Fix a deterministic resource $r = (r^1, \cdots, r^N) \in S$
- For each i, compute $\overline{V}_0^i[r^i](x_0^i)$ by Dynamic Programming (state x_t^i)

$$\overline{V}_t^i[r^i](x_t^i) = \inf_{u_t^i} \mathbb{E}\big[L_t(x_t^i, u_t^i, \boldsymbol{W}_{t+1}) + \overline{V}_{t+1}^i[r^i](g_t^i(x_t^i, u_t^i, \boldsymbol{W}_{t+1}))\big] \quad \text{s.t. } \Theta_t^i(x_t^i, u_t^i) = r_t^i$$

- Return the sequence $\{\overline{V}_t^i[r^i]\}_{i \in [\![1,N]\!]}$

Bounds improvement by optimization

For any admissible deterministic price process $p \in S^o$ and resource process $r \in S$, we have computed lower and upper bounds for the problem optimal value:

$$\sum_{i=1}^{N} \underline{V}_0^i[p^i](x_0^i) \ \leq \ V_0^\sharp(x_0) \ \leq \ \sum_{i=1}^{N} \overline{V}_0^i[r^i](x_0^i)$$

Bounds improvement by optimization

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To obtain tighter bounds, we maximise the lower bound w.r.t. the price and minimise the upper bound w.r.t. the resource:

$$\sup_{p \in S^o} \sum_{i=1}^{N} \underline{V}_0^i[p^i](x_0^i) \;\leq\; V_0^\sharp(x_0) \;\leq\; \inf_{r \in S} \sum_{i=1}^{N} \overline{V}_0^i[r^i](x_0^i)$$

Being deterministic variables, the optimal price and resource processes $\widehat{p}$ and $\widehat{r}$ can be computed using gradient-like algorithms.

Admissible control policies

Once the "best" value functions $\underline{V}_t^i[\widehat{p}^i]$ and $\overline{V}_t^i[\widehat{r}^i]$ have been obtained, it becomes possible to devise global admissible policies

- global price policy $\{\underline{\pi}_t\}_{t\in[\![0,T-1]\!]}$

$$\underline{\pi}_t(x_t^1,\cdots,x_t^N) \in \underset{u_t^1,\cdots,u_t^N}{\arg\min}\ \mathbb{E}\Big[\sum_{i=1}^N L_t^i(x_t^i,u_t^i,\mathbf{W}_{t+1}) + \sum_{i=1}^N \underline{V}_{t+1}^i[\widehat{p}^i](\mathbf{X}_{t+1}^i)\Big]$$
$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(x_t^i,u_t^i,\mathbf{W}_{t+1}),\ \forall i\in[\![1,N]\!]$$
$$\big(\Theta_t(x_t^1,u_t^1),\cdots,\Theta_t(x_t^N,u_t^N)\big)\in S_t$$

- global resource policy $\{\overline{\pi}_t\}_{t\in[\![0,T-1]\!]}$

$$\overline{\pi}_t(x_t^1,\cdots,x_t^N) \in \underset{u_t^1,\cdots,u_t^N}{\arg\min}\ \mathbb{E}\Big[\sum_{i=1}^N L_t^i(x_t^i,u_t^i,\mathbf{W}_{t+1}) + \sum_{i=1}^N \overline{V}_{t+1}^i[\widehat{r}^i](\mathbf{X}_{t+1}^i)\Big]$$
$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(x_t^i,u_t^i,\mathbf{W}_{t+1}),\ \forall i\in[\![1,N]\!]$$
$$\big(\Theta_t(x_t^1,u_t^1),\cdots,\Theta_t(x_t^N,u_t^N)\big)\in S_t$$

Progress status

- First, we have obtained lower and upper bounds
  for a global optimization problem with coupling constraints
  thanks to two spatial decomposition schemes
  - price decomposition
  - resource decomposition

- Second, we have computed the lower and upper bounds
  by dynamic programming (temporal decomposition)

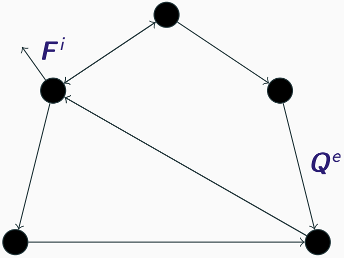
- Using the price and resource Bellman value functions,
  we have devised two online policies for the global system

- Now, we apply these decomposition schemes to
  a large-scale network problem

Nodal decomposition of a network optimization problem

Network and flows

Directed graph $G = (\mathcal{V}, \mathcal{E})$



- $\boldsymbol{Q}_t^e$ flow through edge e,
- $\boldsymbol{F}_t^i$ flow imported at node i

Let A be the *node-edge* incidence matrix

Each node corresponds to a building with its own devices (battery, solar panel. . .)

At each time $t \in [\![0, T-1]\!]$, the Kirchhoff current law couples node and edge flows

$$A\boldsymbol{Q}_t + \boldsymbol{F}_t = 0$$

Optimization problem at a given node

At each node $i \in \mathcal{V}$, given a node flow process $\boldsymbol{F}^i$, we minimize the cost

$$J^i_{\mathcal{V}}(\boldsymbol{F}^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\Big[\sum_{t=0}^{T-1} L^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1}) + K^i(\boldsymbol{X}^i_T)\Big]$$

subject to, for all $t \in [\![0, T-1]\!]$

i) nodal dynamics constraints (for battery and hot water tank)

$$\boldsymbol{X}^i_{t+1} = g^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1})$$

ii) non-anticipativity constraints (future remains unknown)

$$\sigma(\boldsymbol{U}^i_t) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_{t+1})$$

iii) nodal load balance equations (production + import = demand)

$$\Delta^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{F}^i_t, \boldsymbol{W}^i_{t+1}) = 0$$

Some differences with the previous stochastic optimization problem

- Hasard-Decision setting
- Local noise $\boldsymbol{W}^i_t$ in the formulation of problem at node i,
- Global noise $\boldsymbol{W}_t = (\boldsymbol{W}^1_{t+1}, \ldots, \boldsymbol{W}^N_{t+1})$ in the non -anticipativity constraint

Transportation cost and global optimization problem

We define the network cost as the sum over time and edge of the costs of flows $\boldsymbol{Q}_t^e$ through the edges of the network

$$J_{\mathcal{E}}(\boldsymbol{Q}) = \mathbb{E}\left(\sum_{t=0}^{T-1}\sum_{e\in\mathcal{E}} l_t^e(\boldsymbol{Q}_t^e)\right)$$

This transportation cost is additive in space and in time!

The global optimization problem is obtained by gathering all costs

$$V_0^\sharp \;=\; \min_{\boldsymbol{F},\boldsymbol{Q}} \;\sum_{i\in\mathcal{V}} J_{\mathcal{V}}^i(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q})$$

$$\text{s.t. } A\boldsymbol{Q} + \boldsymbol{F} = 0$$

Price and resource decompositions

- Price problem:

$$\underline{V}_0[p] = \min_{\boldsymbol{F},\boldsymbol{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) + \langle p \,, A\boldsymbol{Q} + \boldsymbol{F} \rangle$$
$$= \sum_{i\in\mathcal{V}} \Big(\min_{\boldsymbol{F}_i} \; J^i_{\mathcal{V}}(\boldsymbol{F}^i) + \langle p^i \,, \boldsymbol{F}^i \rangle \Big) \; + \; \Big(\min_{\boldsymbol{Q}} \; J_{\mathcal{E}}(\boldsymbol{Q}) + \langle A^\top p \,, \boldsymbol{Q} \rangle \Big)$$

- Resource problem:

$$\overline{V}_0[r] = \min_{\boldsymbol{F},\boldsymbol{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) \quad \text{s.t.} \quad Ar + \boldsymbol{F} = 0 \,, \;\; \boldsymbol{Q} = r$$
$$= \sum_{i\in\mathcal{V}} \Big(\min_{\boldsymbol{F}_i} \; J^i_{\mathcal{V}}(\boldsymbol{F}^i) \;\text{ s.t. }\; \boldsymbol{F}^i = -(Ar)^i \Big) \; + \; \Big(\min_{\boldsymbol{Q}} \; J_{\mathcal{E}}(\boldsymbol{Q}) \;\text{ s.t. }\; \boldsymbol{Q} = r \Big)$$

Objective

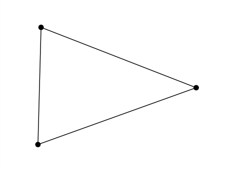
Find deterministic processes $\widehat{p}$ and $\widehat{r}$ with a gap as small as possible

$$\max_{p} \; \underline{V}_0[p] \;\; \leq \;\; V_0^\sharp \;\; \leq \;\; \min_{r} \; \overline{V}_0[r]$$

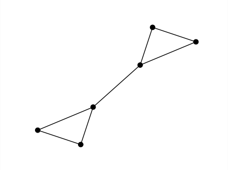
Numerical results on urban microgrids of increasing size

Different urban configurations

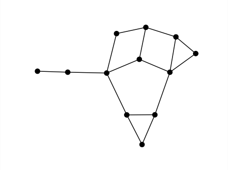
3-Nodes

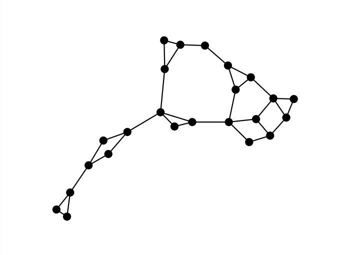


6-Nodes

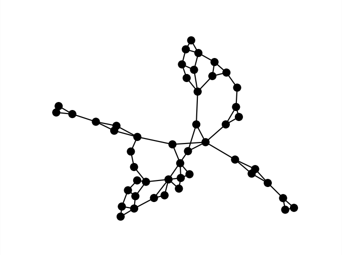


12-Nodes





24-Nodes



48-Nodes

Problem settings

- One day horizon with a 15mn time step: $T = 96$
- Weather corresponds to a sunny day in Paris *(June 28th, 2015)*
- We mix three kinds of buildings
  1. battery + electrical hot water tank
  2. solar panel + electrical hot water tank
  3. electrical hot water tank

  and suppose that all consumers are commoners sharing their devices

Algorithms inventory

Price decomposition

Price decomposition: maximization w.r.t. a deterministic price p

- Each nodal subproblem solved by SDDP (quick convergence)
- Maximisation w.r.t. p by Quasi-Newton (BFGS) method

$$p^{(k+1)} = p^{(k)} + \rho^{(k)} H^{(k)} \nabla \underline{V}(p^{(k)})$$

- Oracle $\nabla \underline{V}(p)$ estimated by Monte Carlo ($N^{scen} = 1,000$)

Resource decomposition

Similar to price decomposition

Global SDDP

We use the SDDP algorithm as reference method

- Noises $\boldsymbol{W}_t^1, \cdots, \boldsymbol{W}_t^N$ are independent node by node:
  total support size is $|\mathrm{supp}(\boldsymbol{W}_t^i)|^N$. We need to resample the noise!
- Convergence once gap between UB and LB is lower than 1%

Exact upper and lower bounds on the global problem

	Network	**3-Nodes**	**6-Nodes**	**12-Nodes**	**24-Nodes**	**48-Nodes**
State dim.	$\lvert\mathbb{X}\rvert$	4	8	16	32	64
Global SDDP	time	1'	3'	10'	79'	453'
Global SDDP	LB	225.2	455.9	889.7	1752.8	3310.3
Price	time	6'	14'	29'	41'	128'
Price	LB	213.7	447.3	896.7	1787.0	3396.4
Resource	time	3'	7'	22'	49'	91'
Resource	UB	253.9	527.3	1053.7	2105.4	4016.6

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For the **48-Nodes** microgrid,

- price decomposition gives a better exact lower bound than global SDDP

$$
\begin{array}{ccccccc}
\underline{V}_0[sddp] & \leq & \underline{V}_0[price] & \leq & V^\sharp & \leq & \overline{V}_0[resource] \\
3310.3 & \leq & 3396.4 & \leq & V^\sharp & \leq & 4016.6
\end{array}
$$

- price decomposition is more than 3 times faster than global SDDP

Policy evaluation by Monte Carlo simulation (1,000 scenarios)

	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
SDDP policy	226 ± 0.6	471 ± 0.8	936 ± 1.1	1859 ± 1.6	3550 ± 2.3
Price policy	228 ± 0.6	464 ± 0.8	923 ± 1.2	1839 ± 1.6	3490 ± 2.3
Gap	+0.9 %	-1.5%	-1.4%	-1.1%	-1.7%
Resource policy	229 ± 0.6	471 ± 0.8	931 ± 1.1	1856 ± 1.6	3503 ± 2.2
Gap	+1.3 %	0.0%	-0.5%	-0.2%	-1.2%

All the cost values above are statistical upper bounds of $V^\sharp$

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Resource policy	229 ± 0.6	471 ± 0.8	931 ± 1.1	1856 ± 1.6	3503 ± 2.2
Gap	+1.3 %	0.0%	-0.5%	-0.2%	-1.2%

All the cost values above are statistical upper bounds of $V^\sharp$

For the **48-Nodes** microgrid,

- price policy beats global SDDP policy and resource policy

$$
\begin{array}{ccccccc}
V^\sharp & \leq & C[price] & \leq & C[resource] & \leq & C[sddp] \\
V^\sharp & \leq & 3490 & \leq & 3503 & \leq & 3550
\end{array}
$$

- the exact upper bound given by resource decomposition is not so tight

$$
\begin{array}{ccccccc}
\underline{V}_0[price] & \leq & V^\sharp & \leq & C[price] & \leq & \overline{V}_0[resource] \\
3396.4 & \leq & V^\sharp & \leq & 3490 & \leq & 4016.6 \\
\text{gap} & & & & <3\% & & >18\%
\end{array}
$$

Conclusion

Conclusion

- We have two algorithms that decompose spatially and temporally a large-scale optimization problem under coupling constraints

- On this case study, price decomposition beats global SDDP for large instances ($\geq$ 24 nodes)
  - in time (more than twice faster)
  - in precision (more than 1% better)

- Can we obtain tighter bounds? *(especially for resource decomposition. . .)*
  If we select properly price $\boldsymbol{P}$ and resource $\boldsymbol{R}$ processes among the class of Markovian processes (instead of deterministic ones) we can obtain "better" nodal value functions (with an extended local state)

Further details in

François Pacaud (2018). "Decentralized Optimization Methods for Efficient Energy Management under Stochasticity". PhD Thesis, Université Paris Est.