Two-Timescale Decision-Hazard-Decision Formulation for Storage Usage Values Calculation in Energy Systems Under Uncertainty

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Le réseau de transport d'électricité

École des Ponts ParisTech

Cepel, Rio de Janeiro
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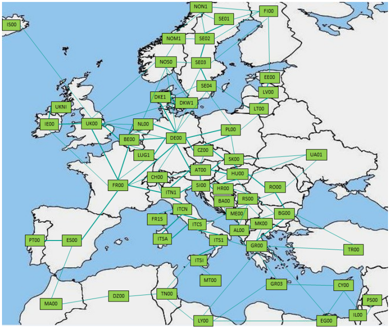
A context of large scale prospective studies



- As the French transmission system operator, RTE conducts prospective studies on energy transition
- Penetration of renewable energy will require deploying a large number of storage facilities
- As a result, there is an increasing interest in usage value calculation for stored energy

Motivation

- The calculation of usage value for storage can be formulated as the result of stochastic multistage optimization problem with two timescales:
  - hourly controls and constraints
  - weekly planning of the decisions
- The current approach is weekly hazard-decision or weekly anticipative planning
  - delicate when units outages cannot be anticipated
- We introduce a new information structure: decision-hazard-decision

Outline

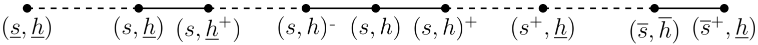
1. Prospective study problem as a stochastic multistage optimization problem in a two-timescale timeline
2. Current practice: hazard-decision
3. Exploring a new approach: decision-hazard-decision
4. Conclusions and future work
5. Additional material

Outline

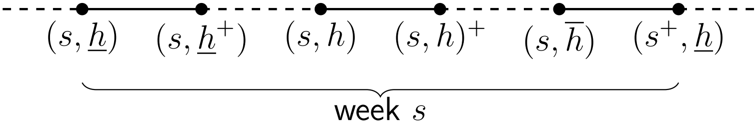
Two-timescale timeline

52 weeks	$\mathbb{S}$	$\underline{s} \prec \cdots \prec s^- \prec s \prec s^+ \prec \cdots \prec \overline{s}$
168 hours	$\mathbb{H}$	$\underline{h} \prec \cdots \prec h^- \prec h \prec h^+ \prec \cdots \prec \overline{h}$

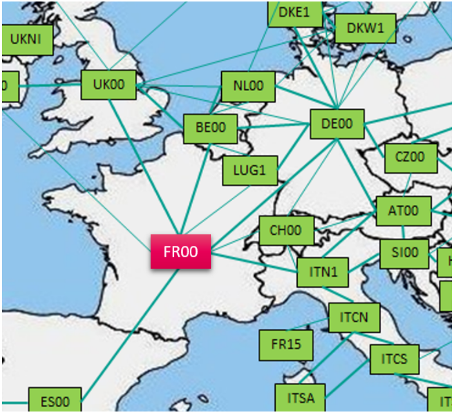
- Complete timeline $\overline{\mathbb{S} \times \mathbb{H}} = \mathbb{S} \times \mathbb{H} \cup \{(\overline{s}^+, \underline{h})\}$



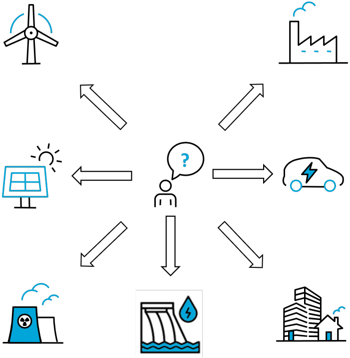
- Zooming on the week s



We focus on one node



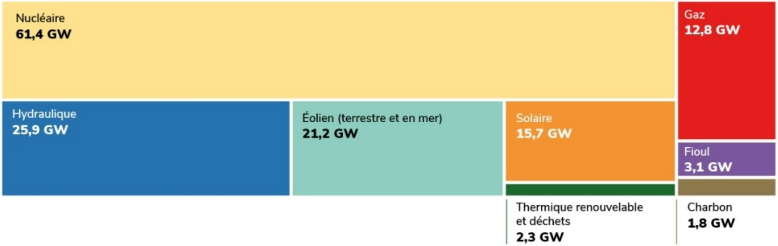
One-node system description



We consider a one-node system composed of:

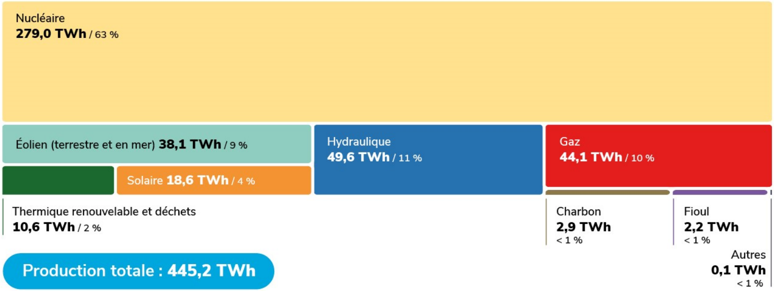
- one storage unit (aggregated dam)
- dispatchable units
- sources of uncertainties:
  - fatal production
  - demand
  - inflows
  - dispatchable unit's availability

French electrical production system (capacity)



RTE - Electricity balance 2022

French electrical production system (production)



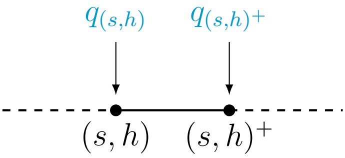
RTE - Electricity balance 2022

Hourly variables definition

Level of stock

The (scalar) variable q
denotes the level of stock in the storage

level of stock

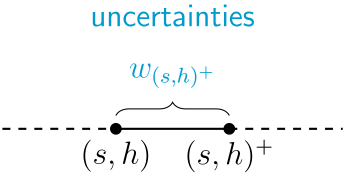


"Lac France": the aggregation of all French dams

Hourly variables definition

Uncertain variables

The (vector) variable w
denotes the uncertainties in the system



Residual demand
Inflows
Dispatchable unit's availability (one for each unit or cluster)

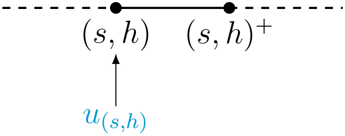
Physical decision variables: planning or recourse?

- Depending on the information structure modelling choice, we will classify the physical decision variables
  - either as planning decisions $u_{(s,h)}$
  - or as recourse decisions $v_{(s,h)}$
- In the current practice, there are only planning decisions $u_{(s,h)}$ and therefore no recourse decisions $v_{(s,h)}$
- In the decision-hazard-decision framework that we propose, there will be both planning decisions $u_{(s,h)}$ and recourse decisions $v_{(s,h)}$

Hourly variables definition

Nonanticipative or planning controls

The (vector) variable u
denotes the nonanticipative controls



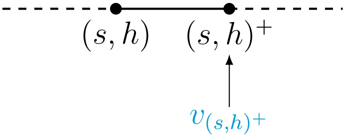
nonanticipative or planning controls
(decision $u_{(s,h)}$ before hazard $w_{(s,h)^+}$)

The planning controls are the switch-on/off decisions for the slow dispatchable units (nuclear plants)

Hourly variables definition

Recourse controls

The (vector) variable v denotes recourse controls:
corrective decisions made once the uncertainties are known

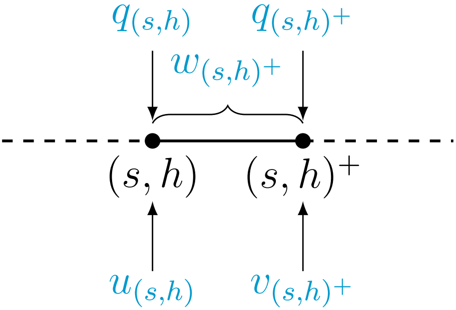


recourse controls

(hazard $w_{(s,h)^+}$ followed by decision $v_{(s,h)^+}$)

The recourse controls are the switch-on/off decisions for the fast dispatchable units (gas, fuel) and all the power modulation (nuclear, gas, fuel, storage output . . .)

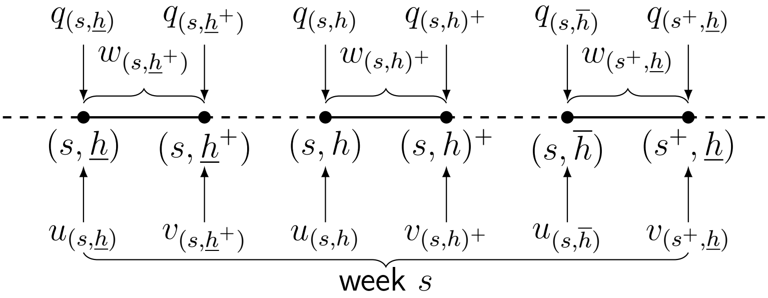
Overview of the hourly interval



At the beginning of the hour one makes the decision $u_{(s,h)}$*, then the uncertainties* $w_{(s,h^+)}$ *(demand, inflows, availability) materialize during the hour, then finally one makes the corrective decision (recourse)* $v_{(s,h)^+}$

Compact notation for weekly variables

For the week s: 1 week = 168 hours



weekly:
- planning $u_{[\![s[\![} = \big(u_{(s,\underline{h})}, u_{(s,\underline{h}^+)}, \dots, u_{(s,h)}, \dots, u_{(s,\overline{h})}\big)$
- uncertainty $w_{]\![s]\!]} = \big(w_{(s,\underline{h}^+)}, \dots, w_{(s,h)^+}, \dots, w_{(s,\overline{h})}, w_{(s^+,\underline{h})}\big)$
- recourse $v_{]\![s]\!]} = \big(v_{(s,\underline{h}^+)}, \dots, v_{(s,h)^+}, \dots, v_{(s,\overline{h})}, v_{(s^+,\underline{h})}\big)$

Where do we stand?

We have introduced a two-timescale timeline
and different variables indexed by its elements

- Stock's level $q_{(s,h)}$
- Planning decisions $u_{(s,h)}$
- Recourse decisions $v_{(s,h)}$
- Uncertainties $w_{(s,h)}$

and the corresponding compact weekly notation
$u_{[\![s[\![}$, $v_{]\!]s]\!]}$ and $w_{]\!]s]\!]}$

Outline

1. Prospective study problem as a stochastic multistage optimization problem in a two-timescale timeline
   - Timeline and variables description
   - Stochastic multistage optimization problem formulation
2. Current practice: hazard-decision
3. Exploring a new approach: decision-hazard-decision
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Yearly prospective problem with hourly variables

$$\inf_{\mathbf{U},\mathbf{V}} \mathbb{E}\overbrace{\left[\sum_{s\in\mathbb{S}}\sum_{h\in\mathbb{H}} L_{(s,h)}(\mathbf{Q}_{(s,h)}, \mathbf{U}_{(s,h)}, \mathbf{W}_{(s,h)^+}, \mathbf{V}_{(s,h)^+}) + K(\mathbf{Q}_{(\overline{s}^+,\underline{h})})\right]}^{\text{Expected intertemporal cost}}$$

s.t. $\quad \forall (s,h) \in \mathbb{S} \times \mathbb{H}$

$\mathbf{Q}_{(\underline{s},\underline{h})} = \mathbf{W}_{(\underline{s},\underline{h})}$ (initial condition)

$\mathbf{Q}_{(s,h)^+} = f_{(s,h)}(\mathbf{Q}_{(s,h)}, \mathbf{U}_{(s,h)}, \mathbf{W}_{(s,h)^+}, \mathbf{V}_{(s,h)^+})$ (stock dynamics)

with information constraints over both controls $\mathbf{U}_{(s,h)}$ and $\mathbf{V}_{(s,h)^+}$

$$\underbrace{L_{(s,h)}(q_{(s,h)}, u_{(s,h)}, w_{(s,h)^+}, v_{(s,h)^+})}_{\text{Instantaneous cost function}} = \underbrace{\delta\big(g(u_{(s,h)}, w_{(s,h)^+}, v_{(s,h)^+}) = 0\big)}_{\text{energy balance constraint}}$$

$$+ \underbrace{\delta_{[\underline{q},\overline{q}]}\Big(f_{(s,h)}\big(q_{(s,h)}, u_{(s,h)}, w_{(s,h)^+}, v_{(s,h)^+}\big)\Big)}_{\text{bounds constraint}}$$

$$+ \underbrace{C^u(u_{(s,h)})}_{\substack{\text{cost function for}\\ \text{planning control } u}} + \underbrace{C^v(v_{(s,h)^+})}_{\substack{\text{cost function for}\\ \text{recourse control } v}}$$

Yearly prospective problem equivalent formulation with weekly variables (compact notation)

Written equivalently with compact notation as

$$\inf_{\mathbf{U},\mathbf{V}} \mathbb{E}\left[\sum_{s\in\mathbb{S}} L_s(\mathbf{Q}_{(s,\underline{h})}, \mathbf{U}_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, \mathbf{V}_{]\!]s]\!]}) + K(\mathbf{Q}_{(\overline{s}^+,\underline{h})})\right]$$

s.t.

$$\mathbf{Q}_{(\underline{s},\underline{h})} = \mathbf{W}_{(\underline{s},\underline{h})}$$

$$\mathbf{Q}_{(s^+,\underline{h})} = f_s(\mathbf{Q}_{(s,\underline{h})}, \mathbf{U}_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, \mathbf{V}_{]\!]s]\!]}) \ , \ \ \forall s \in \mathbb{S}$$

with information constraints over planning controls $\mathbf{U}_{[\![s[\![}$ and recourse controls $\mathbf{V}_{]\!]s]\!]}$

- L_s is the weekly composition of the hourly cost
- f_s is the weekly composition of the hourly dynamics

Yearly prospective problem equivalent formulation with weekly variables (compact notation)

Written equivalently with compact notation as

$$\inf_{\mathbf{U},\mathbf{V}} \mathbb{E}\left[\sum_{s\in\mathbb{S}} L_s(\mathbf{Q}_{(s,\underline{h})}, \mathbf{U}_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, \mathbf{V}_{]\!]s]\!]}) + K(\mathbf{Q}_{(\overline{s}^+,\underline{h})})\right]$$

s.t.

$$\mathbf{Q}_{(\underline{s},\underline{h})} = \mathbf{W}_{(\underline{s},\underline{h})}$$

$$\mathbf{Q}_{(s^+,\underline{h})} = f_s(\mathbf{Q}_{(s,\underline{h})}, \mathbf{U}_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, \mathbf{V}_{]\!]s]\!]}) \ , \ \ \forall s \in \mathbb{S}$$

with information constraints over planning controls $\mathbf{U}_{[\![s[\![}$ and recourse controls $\mathbf{V}_{]\!]s]\!]}$

- *52 time steps but the planning and recourse controls, and the uncertainties are vectors of dimension 168*
- *We keep the hourly constraints*

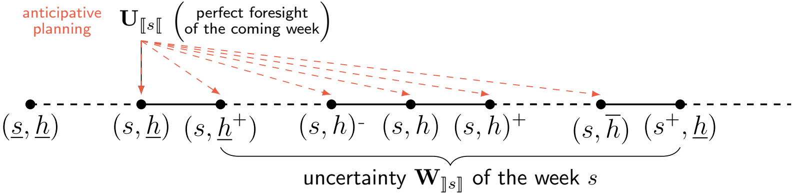
Where do we stand?

- We have described the timeline and variables
- We have formulated the problem
  under two equivalent forms:
  hourly variables *versus* (compact) weekly variables
- We have not specified the information constraints
- We now detail the current practice
  for information modelling

Outline

Information Structures

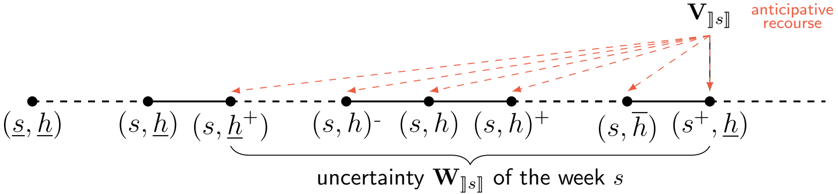
Weekly Hazard-Decision I



$$B_s^{\mathsf{HD}}(x_s) = \mathbb{E}\left[\inf_{u_{[\![s[\![} \in \mathbb{U}_{[\![s[\![}} L_s\big(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}\big) + B_{s^+}^{\mathsf{HD}}\big(f_s(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]})\big) \right]$$

Information Structures

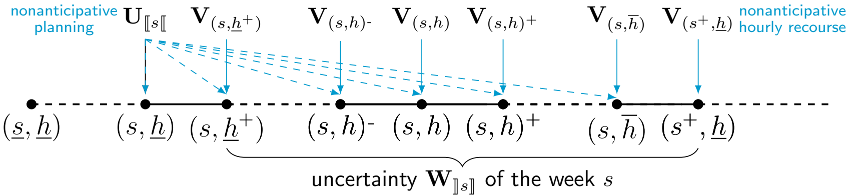
Weekly Hazard-Decision II



$$B_s^{\mathsf{HD}}(x_s) = \mathbb{E}\left[\inf_{v_{\rrbracket s \rrbracket} \in \mathbb{V}_{\rrbracket s \rrbracket}} L_s\big(x_s, v_{\rrbracket s \rrbracket}, \mathbf{W}_{\rrbracket s \rrbracket}\big) + B_{s^+}^{\mathsf{HD}}\big(f_s(x_s, v_{\rrbracket s \rrbracket}, \mathbf{W}_{\rrbracket s \rrbracket})\big) \right]$$

Information Structures

Weekly Planning - Hourly Recourse

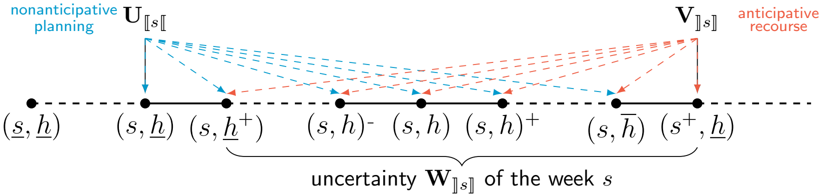


$$B_s^{\text{WP-HR}}(x_s) =$$

$$\inf_{u_{[\![s[\![}} \mathbb{E}\Bigg[\inf_{v_{(s,\underline{h}^+)}} \mathbb{E}\Bigg[\dots \inf_{v_{(s,\overline{h})}} \mathbb{E}\Bigg[\inf_{v_{(s^+,\underline{h})}} \Big(\overbrace{L_s}^{L_s(x_s,u_{[\![s[\![},w_{]\!]s]\!]},v_{]\!]s]\!]})} + B_{s^+}^{\text{WP-HR}}(\overbrace{x_{s^+}}^{f_s(x_s,u_{[\![s[\![},w_{]\!]s]\!]},v_{]\!]s]\!]})})\Big) \mid w_{(s,\underline{h}^+)}, w_{(s,\underline{h}^+)^+},\dots, w_{(s,\overline{h})}\Bigg] \cdots \mid w_{(s,\underline{h}^+)}\Bigg]\Bigg]$$

Information Structures

Weekly Planning - Weekly Recourse



$$B_s^{\text{WP-WR}}(x_s) = \inf_{u_{[\![s[\![}} \mathbb{E}\left[\inf_{v_{]\!]s]\!]}} \left\{ L_s(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]}) + B_{s^+}^{\text{WP-WR}}\big(f_s(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]})\big) \right\}\right]$$

Outline

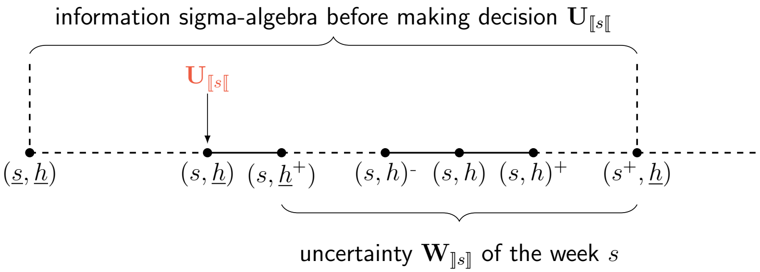
Weekly hazard-decision information structure

In the weekly hazard-decision framework,

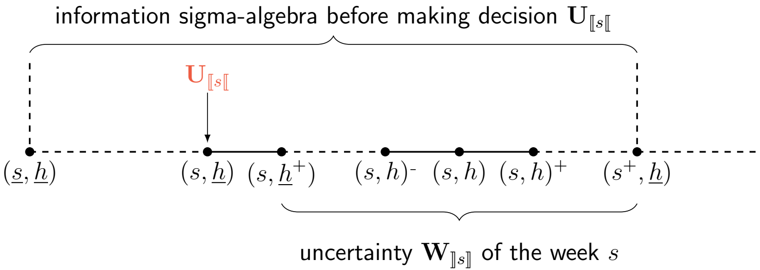
- one makes a decision $\mathbf{U}_{[\![s[\![}$ at the beginning of the week (switch on/off and power modulation of all units at the 168 hours of the coming week)
- but with perfect foresight of the uncertainties (demand, inflows and availabilities) of the coming week

Because of perfect foresight,
one does not need the recourse variables $\mathbf{V}_{]\!]s]\!]}$

Weekly hazard-decision information structure



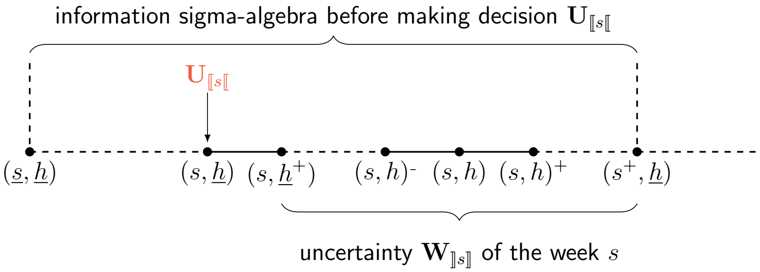
Weekly hazard-decision information structure



No need for recourse controls $\mathbf{V}_{]\!]s]\!]}$

All the physical decision variables are considered as planning decisions $\mathbf{U}_{[\![s[\![}$

Weekly hazard-decision information structure



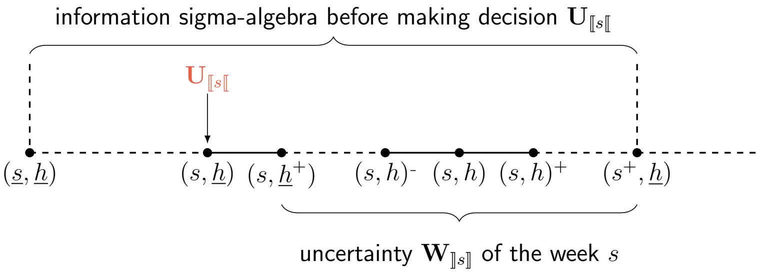
- For all hour $(s, h) \in \mathbb{S} \times \mathbb{H}$,

$$\sigma(\mathbf{U}_{(s,h)}) \subseteq \sigma\big(\mathbf{W}_{(\underline{s},\underline{h})}, \dots, \mathbf{W}_{(s,\underline{h}^+)}, \dots, \mathbf{W}_{(s^+,\underline{h})}\big)$$

- Or equivalently, for all week $s \in \mathbb{S}$,

$$\sigma(\mathbf{U}_{[\![s[\![}) \subseteq \sigma\big(\mathbf{W}_{(\underline{s},\underline{h})}, \mathbf{W}_{]\!]\underline{s}]\!]}, \dots, \mathbf{W}_{]\!]s]\!]}\big)$$

Weekly hazard-decision information structure



- The uncertainties are anticipated in weekly blocks
- When the decision is made at the beginning of the week, the demand, the inflow and the availability of the dipatchable units are considered known at every hour of the coming week

How do we value the storage?

Outline

Bellman equations for weekly hazard-decision

- Defining the weekly state $x_s = q_{(s,\underline{h})}$ (stock in the storage) we write the weekly Bellman equations

$$B^{\mathsf{HD}}_{\overline{s}^+}(x_{\overline{s}^+}) = K(x)$$

$$B^{\mathsf{HD}}_{s}(x_s) = \mathbb{E}\Bigg[\inf_{u_{[\![s[\![} \in \mathbb{U}_{[\![s[\![}} L_s\big(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}\big) + B^{\mathsf{HD}}_{s^+}\big(f_s(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]})\big)\Bigg]$$

Every week s, the Bellman function $B^{\mathsf{HD}}_s(x_s)$
gives the value of the storage x_s at the beginning of the week

Bellman equations for weekly hazard-decision

- Defining the weekly state $x_s = q_{(s,\underline{h})}$ (stock in the storage) we write the weekly Bellman equations

$$B^{\mathsf{HD}}_{\overline{s}^+}(x_{\overline{s}^+}) = K(x)$$

$$B^{\mathsf{HD}}_s(x_s) = \mathbb{E}\left[\inf_{u_{[\![s[\![} \in \mathbb{U}_{[\![s[\![}} L_s\big(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}\big) + B^{\mathsf{HD}}_{s^+}\big(f_s(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]})\big)\right]$$

- If the sequence $\big(\mathbf{W}_{]\!]\underline{s}]\!]}, \ldots, \mathbf{W}_{]\!]s]\!]}, \ldots, \mathbf{W}_{]\!]\overline{s}]\!]}\big)$ of uncertainties are (weekly) independent, the weekly Bellman equations lead to optimality
- Within the week, the hourly uncertainties $\mathbf{W}_{]\!]s]\!]} = \big(\mathbf{W}_{(s,\underline{h}^+)}, \ldots, \mathbf{W}_{(s,h)^+}, \ldots, \mathbf{W}_{(s^+,\underline{h})}\big)$ are not assumed to be independent

Bellman equations for weekly hazard-decision

- Defining the weekly state $x_s = q_{(s,\underline{h})}$ (stock in the storage) we write the weekly Bellman equations

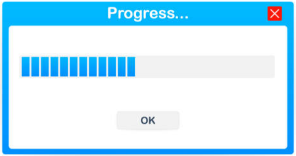
$$B^{\mathsf{HD}}_{\overline{s}^+}(x_{\overline{s}^+}) = K(x)$$

$$B^{\mathsf{HD}}_{s}(x_s) = \mathbb{E}\left[\inf_{u_{[\![s[\![} \in \mathbb{U}_{[\![s[\![}} L_s\big(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}\big) + B^{\mathsf{HD}}_{s^+}\big(f_s(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]})\big)\right]$$

The expectation is computed as a sum over the N uncertainties scenarios

$$\mathbb{E}\left[\ldots\right] = \sum_{N} \left[\text{deterministic problem along scenario}\right]$$

Computing Bellman functions



French node modelling with clusters

- A thermal cluster is composed of homogeneous production units, for instance all nuclear units with the same physical and economic parameters.
- We aggregate all the units inside a thermal cluster
- With each cluster, we associate two decision variables,
  - one scalar ($\mathbb{R}$) for power modulation
  - one integer ($\mathbb{N}$) for switch on/off decision

Numerical resolution of Bellman equations for weekly hazard-decision

- We solve as many weekly independent and deterministic problems as the number N of uncertainties scenarios
- If we consider the french node with 12 thermal clusters (with minimum power constraints)

	Integer variables	**Binary** variables	**Continuous** variables	**Comments**
Dynamic programming state	-	-	$\mathbb{R}^1$	Level of stock at the beginning of the week
Thermal controls	$(\mathbb{N}^{168})^{12}$	-	$(\mathbb{R}^{168})^{12}$	Switch on/off and power modulation
Storage controls	-	-	$(\mathbb{R}^{168})^2$	Pumping and turbing
Storage level	-	-	$(\mathbb{R}^{168})^1$	Level of stock
Slack variables	-	-	$(\mathbb{R}^{168})^2$	ENS and energy spillage
Uncertainties	$(\mathbb{N}^{168})^{12}$	-	$(\mathbb{R}^{168})^2$	Demand, inflows and availabilities

Where do we stand?

- Bellman functions are a tool to compute usage values of storages

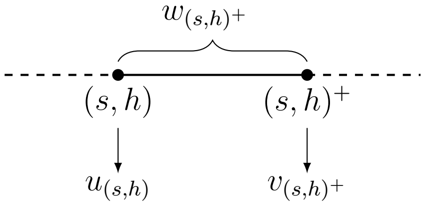
$$\text{storage value} = B_s(x_s)$$
$$\text{usage value} = -\frac{d}{dx_s} B_s(x_s)$$

- The weekly hazard-decision structure assumes that the weekly uncertainties are known in advance
    - Not bad when considering uncertainties with available accurate forecast
    - Delicate for the units outages: dispatchable units (nuclear, thermal)
    - This is why we turn to decision-hazard-decision structure

Outline

Why decision-hazard-decision

We need recourse controls in addition to the nonanticipative controls to satisfy the equality constraints



$$\overbrace{g\Big(\underbrace{u_{(s,h)}}_{\text{planning}},\ \underbrace{w_{(s,h)^+}}_{\substack{\text{forthcoming}\\ \text{uncertainty}}},\ \underbrace{v_{(s,h)^+}}_{\substack{\text{recourse}\\ \text{control}}}\Big)}^{\text{Production} = \text{Demand}} = 0$$

Therefore, we study the decision-hazard-decision formulation

Outline

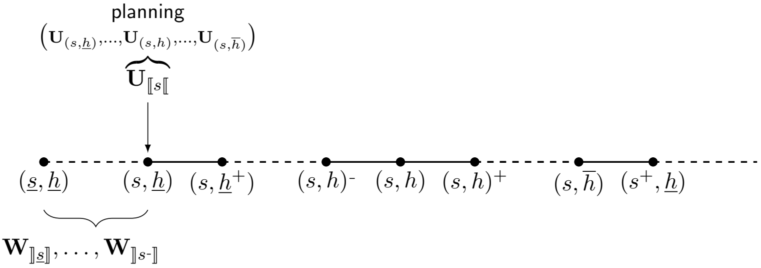
Weekly planning-weekly recourse

In the weekly planning-weekly recourse framework,

- first, one makes a nonanticipative decision $\mathbf{U}_{[\![s[\![}$ (switch on/off of the slow units at the 168 hours of the coming week) at the beginning of the week, knowing only the past
- then, the uncertainties $\mathbf{W}_{]\!]s]\!]}$ (demand, inflows and availabilities) materialize during the coming week
- finally, the corrective recourse decisions $\mathbf{V}_{]\!]s]\!]}$ (switch on/off of fast units and power modulation of all units for the 168 hours) are made

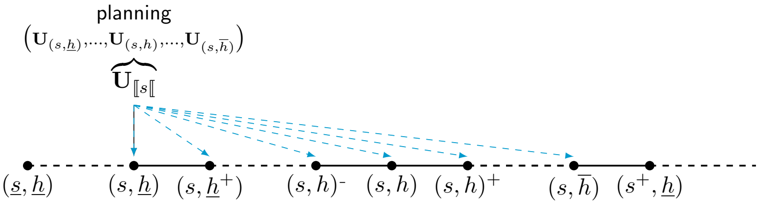
Weekly planning-weekly recourse

At the beginning of the week the vector of
nonanticipative or planning decisions
is made knowing only the past uncertainties



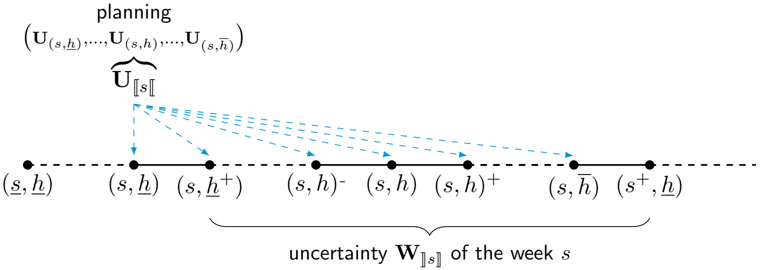
Weekly planning-weekly recourse

The 168 hours planning at the beginning of the week
has an impact on the 168 hourly balances within the week



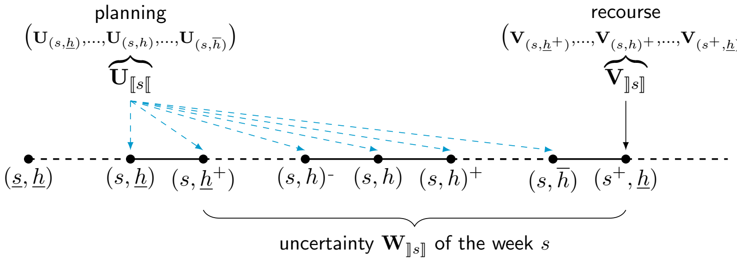
Weekly planning-weekly recourse

The weekly block of 168 uncertainties materialize



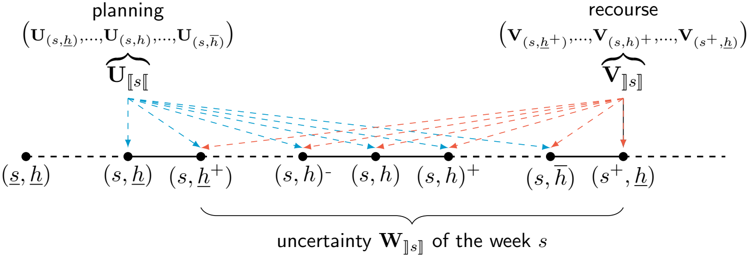
Weekly planning-weekly recourse

The vector of recourse or corrective decisions
is made knowing all the uncertainties for the week

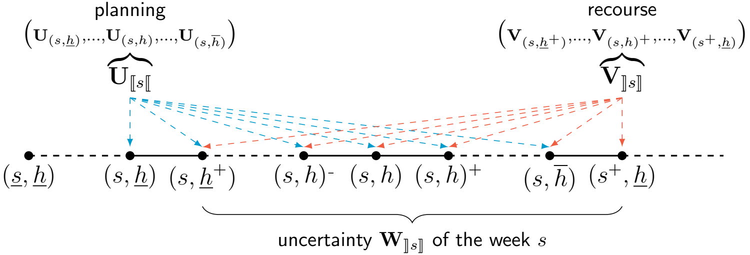


Weekly planning-weekly recourse

The 168 hours recourse has an impact on the hourly balances within the week

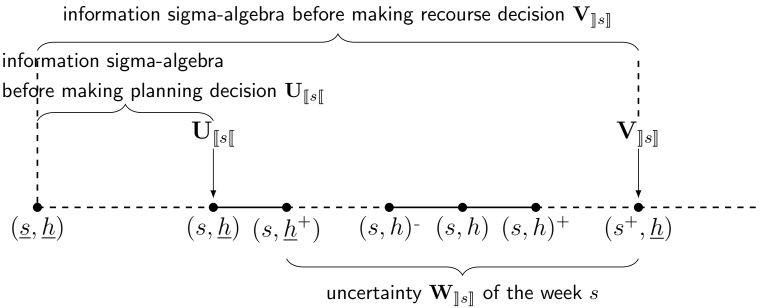


Weekly planning-weekly recourse



- The arrows from left to right represent NONANTICIPATIVITY
- The arrows from right to left represent ANTICIPATIVITY

Weekly planning - weekly recourse



- For all week $s \in \mathbb{S}$

weekly planning $\sigma(\mathbf{U}_{\llbracket s \llbracket}) \subseteq \sigma(\mathbf{W}_{(\underline{s},\underline{h})}, \ldots, \mathbf{W}_{\rrbracket s^- \rrbracket})$

weekly recourse $\sigma(\mathbf{V}_{\rrbracket s \rrbracket}) \subseteq \sigma(\mathbf{W}_{(\underline{s},\underline{h})}, \ldots, \mathbf{W}_{\rrbracket s^- \rrbracket}, \mathbf{W}_{\rrbracket s \rrbracket})$

Outline

Bellman equations for weekly planning-weekly recourse

- Defining the weekly state $x_s = q_{(s,\underline{h})}$ (stock in the storage) we write the weekly Bellman equations

$$
\begin{aligned}
B_{\overline{s}+}^{\text{WP-WR}}(x_{\overline{s}+}) &= K(x_{\overline{s}+}) \\
B_{s}^{\text{WP-WR}}(x_s) &= \inf_{u_{\llbracket s \llbracket}} \mathbb{E}\Bigg[\inf_{v_{\rrbracket s \rrbracket}} \Big\{ L_s(x_s, u_{\llbracket s \llbracket}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket}) \\
&\qquad + B_{s+}^{\text{WP-WR}}\big(f_s(x_s, u_{\llbracket s \llbracket}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket})\big) \Big\} \Bigg]
\end{aligned}
$$

Every week s, the Bellman function $B_s^{\text{HD}}(x_s)$
gives the value of the storage x_s at the beginning of the week

Bellman equations for weekly planning-weekly recourse

- The difference with the weekly hazard-decision framework is that

$$\overbrace{\mathbb{E} \underbrace{\inf_{u_{\llbracket s \llbracket}}}_{\text{slow fast}}}^{\text{weekly hazard-decision}} \quad \text{versus} \quad \overbrace{\underbrace{\inf_{u_{\llbracket s \llbracket}}}_{\text{slow}} \mathbb{E} \underbrace{\inf_{v_{\rrbracket s \rrbracket}}}_{\text{fast}}}^{\text{weekly planning-weekly recourse}}$$

- Beware that the physical decision variables are
  - all classified as $u_{\llbracket s \llbracket}$ in the weekly hazard-decision framework but,
  - they are separated between $u_{\llbracket s \llbracket}$ and $v_{\rrbracket s \rrbracket}$ in the weekly planning - weekly recourse framework

Bellman equations for weekly planning-weekly recourse

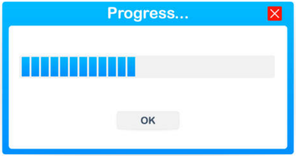
- Defining the weekly state $x_s = q_{(s,\underline{h})}$ (stock in the storage) we write the weekly Bellman equations

$$
\begin{aligned}
B^{\text{WP-WR}}_{\overline{s}^+}(x_{\overline{s}^+}) &= K(x_{\overline{s}^+}) \\
B^{\text{WP-WR}}_{s}(x_s) &= \inf_{u_{[\![s[\![}} \mathbb{E}\Big[\inf_{v_{]\!]s]\!]}} \Big\{ L_s(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]}) \\
&\qquad\qquad + B^{\text{WP-WR}}_{s^+}\big(f_s(x_s, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]})\big)\Big\}\Big]
\end{aligned}
$$

- If the sequence $\big(\mathbf{W}_{]\!]\underline{s}]\!]}, \dots, \mathbf{W}_{]\!]s]\!]}, \dots, \mathbf{W}_{]\!]\overline{s}]\!]}\big)$ of uncertainties are (weekly) independent, the weekly Bellman equations lead to optimality

- Within the week, the hourly uncertainties $\mathbf{W}_{]\!]s]\!]} = \big(\mathbf{W}_{(s,\underline{h}^+)}, \dots, \mathbf{W}_{(s,h)^+}, \dots, \mathbf{W}_{(s^+,\underline{h})}\big)$ are not assumed to be independent

Computing Bellman functions in decision-hazard-decision



Numerical resolution of Bellman equations for weekly planning-weekly recourse

The expectation is computed as a sum over the (N) uncertainties scenarios

$$\inf_{u_{[\![s[\![}} \mathbb{E}\left[\inf_{v_{]\!]s]\!]}} \ldots\right] = \inf_{u_{[\![s[\![}} \sum_{N} \left[\inf_{v_{]\!]s]\!]} \text{function of}(u_{[\![s[\![}, v_{]\!]s]\!]})\right]$$

French node modelling adaptation for weekly planning - weekly recourse

- Switch on/off decisions for 4 slow (nuclear) clusters can be represented
  - either by 4 integers in $\mathbb{N}^4$
  - or by 60 binary in $\{0, 1\}^{60}$ as there are 60 independent units inside the 4 clusters
- Due to the information structure peculiarity, we choose the second option even if the decision set is larger

Numerical resolution of Bellman equations for weekly planning-weekly recourse

For the French node with 12 thermal clusters the decisions are now separated between planning and recourse

Planning u:

- Switch on/off decisions for 4 slow (nuclear) clusters: independent decisions for 60 units in total

Recourse v:

- Switch on/off decisions for 8 fast (thermal) clusters: aggregated decisions within each cluster
- Power modulation for nuclear and thermal clusters
- Storage output (pumping and turbining)
- Energy not served and spillage variables

Numerical resolution of Bellman equations for weekly planning-weekly recourse

- We cannot solve the deterministic weekly problems independently
- For the french node considering the minimum power constraints, and N scenarios of uncertainties to compute the expectation as a sum, we have

	Integer variables	**Binary** variables	**Continuous** variables	**Comments**
Dynamic programming state	-	-	$\mathbb{R}^1$	Level of stock at the beginning of the week
Thermal controls planning	-	$(\{0,1\}^{168})^{60}$	-	Switch on/off of units in planning
Thermal controls recourse	$\left((\mathbb{N}^{168})^8\right)^N$	-	$\left((\mathbb{R}^{168})^{60+8}\right)^N$	Switch on/off recourse and power modulation
Storage controls	-	-	$\left((\mathbb{R}^{168})^2\right)^N$	Pumping and turbing
Storage level	-	-	$\left((\mathbb{R}^{168})^1\right)^N$	Level of stock
Slack variables	-	-	$\left((\mathbb{R}^{168})^2\right)^N$	ENS and energy spillage
Uncertainties	$\left((\mathbb{N}^{168})^{12}\right)^N$	-	$\left((\mathbb{R}^{168})^2\right)^N$	Availabilities, inflows and residual demand

Numerical results for a small study case

We consider a small electrical system
to conduct the numerical study

- 3 thermal clusters with 1 unit each
  (instead of 12 clusters for the French node)
  - Planning u: base unit, semi-base unit
  - Recourse v: peak unit
- $N = 20$ scenarios

	Integer variables	**Binary** variables	**Continuous** variables
Dynamic programming state	-	-	$\mathbb{R}^1$
Thermal controls planning	-	$(\{0,1\}^{168})^2$	-
Thermal controls recourse	$\left((\mathbb{N}^{168})^1\right)^{20}$	-	$\left((\mathbb{R}^{168})^{2+1}\right)^{20}$
Storage controls	-	-	$\left((\mathbb{R}^{168})^2\right)^{20}$
Storage level	-	-	$\left((\mathbb{R}^{168})^1\right)^{20}$
Slack variables	-	-	$\left((\mathbb{R}^{168})^2\right)^{20}$
Uncertainties	$\left((\mathbb{N}^{168})^3\right)^{20}$	-	$\left((\mathbb{R}^{168})^2\right)^{20}$

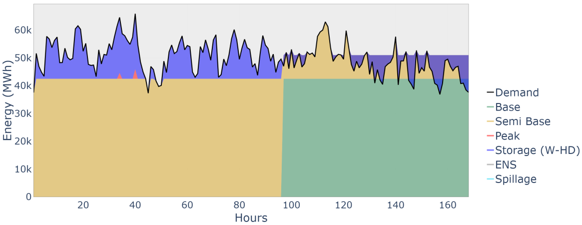
Usage value comparison

$$\text{UV}^{\text{W-HD}} = -\frac{d}{dx} B_s^{\text{W-HD}}(x) \qquad \text{UV}^{\text{WP-WR}} = -\frac{d}{dx} B_s^{\text{WP-WR}}(x)$$

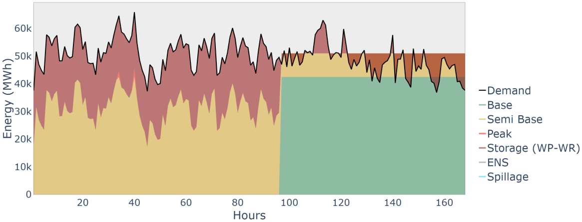
Dispatch comparison

As a consequence of the merit order change, we observe different allocation of the production means

Weekly hazard-decision



Weekly planning - weekly recourse



Dispatch

- Weekly hazard-decision:

The price obtained from the weekly hazard-decision formulation is higher than the semi-base price: hence the storage is the marginal production mean

- Weekly planning - weekly recourse:

The price obtained from the weekly planning - weekly recourse formulation is lower than the semi-base price: hence the semi-base unit is the marginal production mean

Intuition

- With no anticipation, one makes conservative planning decisions to be ready for the coming uncertainties
- As a consequence, there is potential available production that makes the energy in storages less value
- Therefore, there is less interest in refilling the storage

Difference in the storage level trajectory

Outline

From the study case we conclude that...

- Different information structures give different Bellman functions
- Different Bellman functions give different usage values
- Different usage values give different merit orders
- Different merit orders give different energy allocations, hence difference storage trajectories

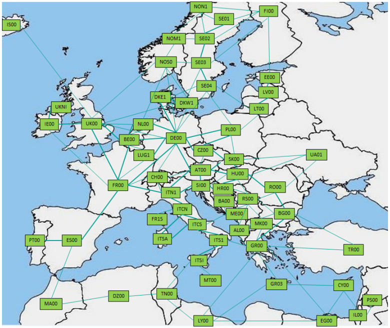
Summing up

- Current practice with perfect foresight
- We have studied a new information structure
  - weekly planning-weekly recourse
- The Bellman functions corresponding to the different information structures are ordered as follows

$$\underbrace{B_s^{\text{WP-WR}}}_{\substack{\text{weekly planning} \\ \text{weekly recourse}}} \geq \underbrace{B_s^{\text{HD}}}_{\substack{\text{current practice} \\ \text{weekly anticipative} \\ \text{planning}}}$$

- Currently working on how to compute solutions of non-classical Bellman equations for a real scale problem

Future work



- Extend to multiple nodes with multiple storages
- Spatial decomposition techniques mixed with stochastic dynamic programming

Thank you, questions?

Le réseau
de transport
d'électricité

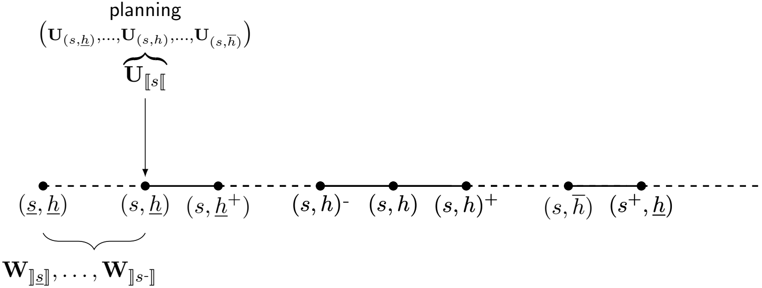
École des Ponts
ParisTech

Outline

1. Prospective study problem as a stochastic multistage optimization problem in a two-timescale timeline
2. Current practice: hazard-decision
3. Exploring a new approach: decision-hazard-decision
4. Conclusions and future work
5. Additional material

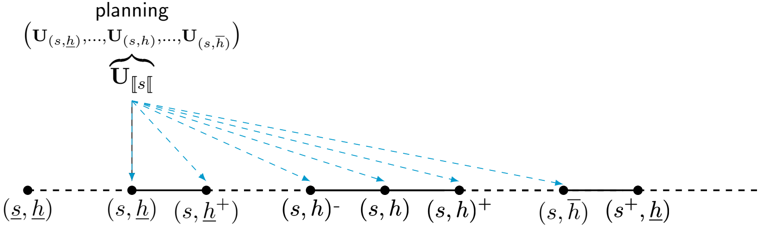
Weekly planning-hourly recourse

At the beginning of the week the vector of nonanticipative or planning decisions is made knowing only the past uncertainties



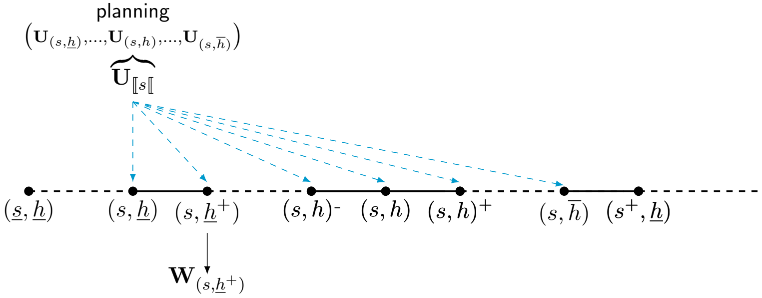
Weekly planning-hourly recourse

The vectorial planning at the beginning
of the week has an impact in the hourly balances within the week



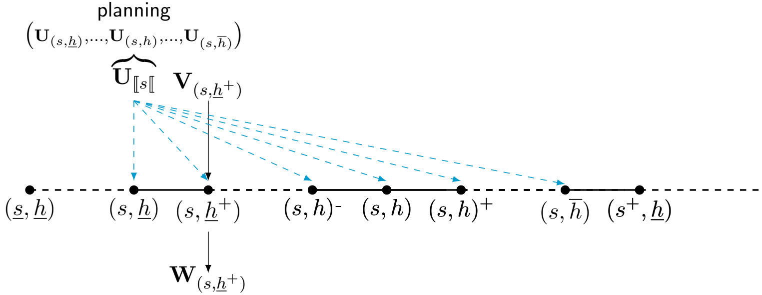
Weekly planning-hourly recourse

The first hourly uncertainty materialize



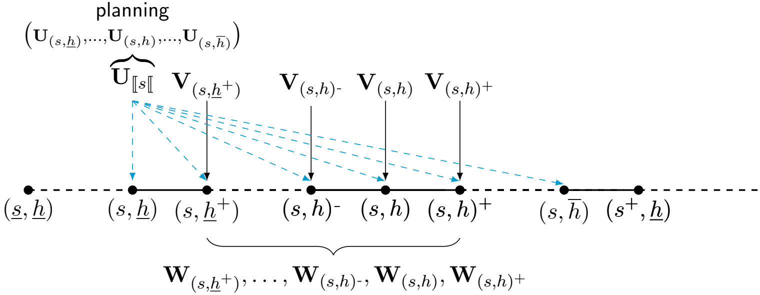
Weekly planning-hourly recourse

The first recourse decision is made



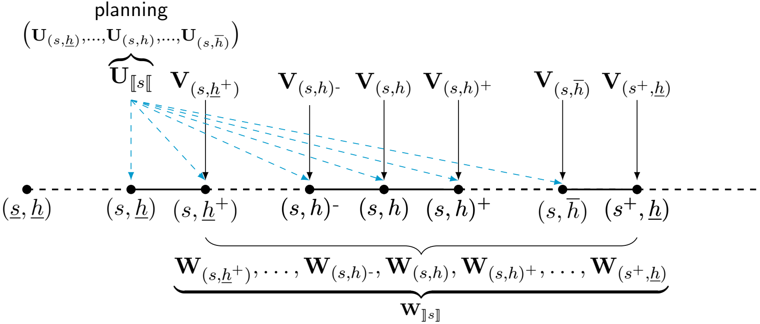
Weekly planning-hourly recourse

Then, hour by hour the uncertainties are disclosed the hourly recourse decisions are made knowing only the past uncertainties

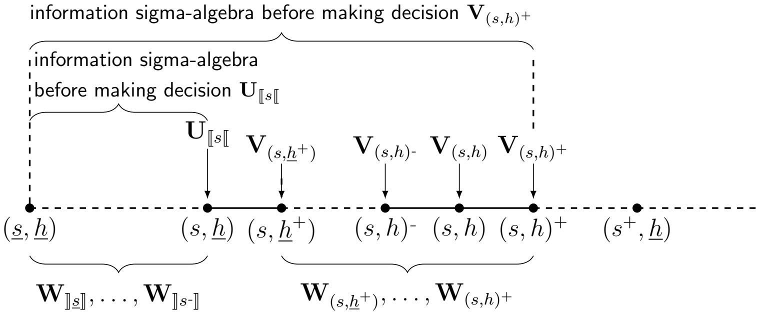


Weekly planning-hourly recourse

At the end of the week,
when all the uncertainties are known,
the last recourse decision is made



Weekly planning-hourly recourse



- For all week $s \in \mathbb{S}$,
  $\sigma\big(\mathbf{U}_{[\![s[\![}\big) \subset \sigma\big(\mathbf{W}_{(\underline{s},\underline{h})}, \mathbf{W}_{]\!]\underline{s}]\!]}, \ldots, \mathbf{W}_{]\!]s^-]\!]}\big)$
- For all hour $(s,h) \in \mathbb{S} \times \mathbb{H}$
  $\sigma\big(\mathbf{V}_{(s,h)^+}\big) \subset \sigma\big(\mathbf{W}_{(\underline{s},\underline{h})}, \mathbf{W}_{]\!]\underline{s}]\!]}, \ldots, \mathbf{W}_{]\!]s^-]\!]}, \mathbf{W}_{(s,\underline{h}^+)}, \ldots, \mathbf{W}_{(s,h)^+}\big)$

Bellman equations for weekly planning-hourly recourse

- Defining the weekly state $x_s = q_{(s,\underline{h})}$ (stock in the storage) we write the weekly Bellman equations

$$B^{\text{WP-HR}}_{\overline{s}^+}(x_{\overline{s}^+}) = K(x_{\overline{s}^+})$$

$$B^{\text{WP-HR}}_{s}(x_s) =$$

$$\inf_{u_{[\![s[\![}} \mathbb{E}\Bigg[\inf_{v_{(s,\underline{h}^+)}} \mathbb{E}\Bigg[\dots \inf_{v_{(s,\overline{h})}} \mathbb{E}\Bigg[\inf_{v_{(s^+,\underline{h})}} \Big(\overbrace{L_s}^{\overset{L_s(x_s,u_{[\![s[\![},w_{]\!]s]\!]},v_{]\!]s]\!]})}{\uparrow}} + B^{\text{WP-HR}}_{s^+}(\overbrace{x_{s^+}}^{\overset{f_s(x_s,u_{[\![s[\![},w_{]\!]s]\!]},v_{]\!]s]\!]})}{\uparrow}})\Big) \mid w_{(s,\underline{h}^+)}, w_{(s,\underline{h}^+)^+}, \dots, w_{(s,\overline{h})}\Bigg] \cdots \mid w_{(s,\underline{h}^+)}\Bigg]\Bigg]$$

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Bellman equations for weekly planning-hourly recourse

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$$
B^{\text{WP-HR}}_{\overline{s}^+}(x_{\overline{s}^+}) = K(x_{\overline{s}^+})
$$

$$
B^{\text{WP-HR}}_{s}(x_s) =
$$

$$
\underbrace{\inf_{u_{[\![s[\![}} \mathbb{E}\Bigg[\underbrace{\inf_{v_{(s,\underline{h}^+)}} \mathbb{E}\Bigg[\dots \underbrace{\inf_{v_{(s,\overline{h})}} \mathbb{E}\Big[\underbrace{\inf_{v_{(s^+,\underline{h})}} \Big(\overbrace{L_s}^{\substack{L_s(x_s,u_{[\![s[\![},w_{]\!]s]\!]},v_{]\!]s]\!]}) \\ \uparrow}} + B^{\text{WP-HR}}_{s^+}\big(\overbrace{x_{s^+}}^{\substack{f_s(x_s,u_{[\![s[\![},w_{]\!]s]\!]},v_{]\!]s]\!]}) \\ \uparrow}}\big)\Big)}_{\text{last recourse}} \mid w_{(s,\underline{h}^+)}, w_{(s,\underline{h}^+)+}, \dots, w_{(s,\overline{h})}\Big]}_{\text{last but one recourse} \\ \vdots} \dots \mid w_{(s,\underline{h}^+)}\Bigg]}_{\text{first recourse}}\Bigg]}_{\text{planning}}
$$

Bellman equations for weekly planning-hourly recourse

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- If the sequence $\big(\mathbf{W}_{]\!]\underline{s}]\!]}, \dots, \mathbf{W}_{]\!]s]\!]}, \dots, \mathbf{W}_{]\!]\overline{s}]\!]}\big)$ of uncertainties are (weekly) independent, the weekly Bellman equations lead to optimality

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