ICIAM 2003, Sydney, Australia, 7-11 July 2003

Optimal strategies of dispersion for coral fish larvae: a Stochastic Dynamic Programming approach to estimate self-recruitment

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Plan

1. **Introduction**
    - (a) *Why studying larvae dispersion?*
    - (b) *Why focusing upon auto-recruitment?*
    - (c) *Why modelling?*

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Plan

1. **Introduction**
    - *(a) Why studying larvae dispersion?*
    - *(b) Why focusing upon auto-recruitment?*
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2. **Description of the models**
    - *(a) Problem characteristics*
    - *(b) Theoretical framework*
    - *(c) Parametrization*

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3. **Simulations**
4. **Perspectives**

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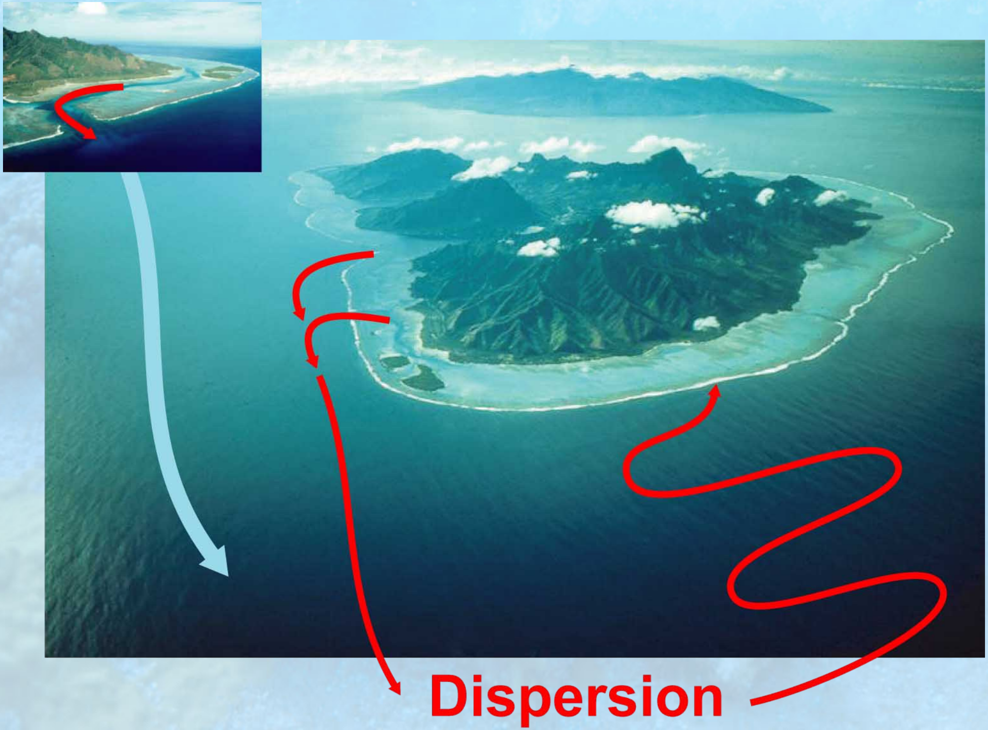
Introduction



Dispersion

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Introduction

Why studying larvae dispersion?

- Debate concerning the demographic limiting factors for populations: before or after recrutment?

Introduction

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- Debate concerning the demographic limiting factors for populations: before or after recrutment?
- The dispersion at the larva stade delimitates the populations and determines their connectivity: importance for gestion and conservation purposes

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Introduction

Why studying larvae dispersion?

- Debate concerning the demographic limiting factors for populations: before or after recrutment?
- The dispersion at the larva stade delimitates the populations and determines their connectivity: importance for gestion and conservation purposes
- The omnipresence of larvae dispersion for marine organisms is an evolutionary puzzle.

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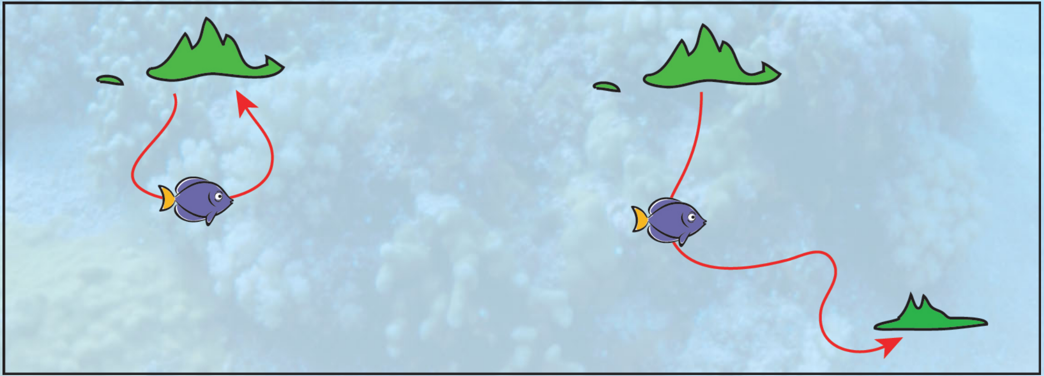
Introduction

Why focusing upon auto-recruitment?

Def. `Auto-recrutment`:

Recrutment of an organism on its natal place



⇒ retention phenomena (active or passive).

Introduction

Why focusing upon auto-recruitment?

Difficulties in studying dispersion:

- **Wide spatial range**, and thus strong larvae dilution
- **Very low survival rate:** difficulty with capture-recapture techniques

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Introduction

Why focusing upon auto-recruitment?

Difficulties in studying dispersion:

- **Wide spatial range**, and thus strong larvae dilution
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Avdantages in studying auto-recrutment:

- **Shortest** spatial range (retention in the vicinity of the island)
- Marquage-capture-recapture **possible** (Jones et al., 1999)
- Estimation of auto-recrutment by population **genetics**.

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Introduction

Interest of modelling

- Well developed current models

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Introduction

Interest of modelling

- Well developed current models
- Less costly than planktonic study.

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Introduction

Interest of modelling

- Well developed current models
- Less costly than planktonic study.
- Contribution to decision-making (conservation purposes).

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Introduction

A brief history of larvae dispersion modelling

First models:

Dight et al., 1990, Black et al., 1991.

- Focus on **current** studies
- **Passive** particles advection/diffusion

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Introduction

A brief history of larvae dispersion modelling

First models:

Dight et al., 1990, Black et al., 1991.

- Focus on **current** studies
- **Passive** particles advection/diffusion

Recent models:

Wolanski et al. (1997), Armsworth (2000), Armsworth (2001), Armsworth et al. (2001).

- Currents but **active** particles
- Introduction of **biological** characteristics of particles (energy budget...)

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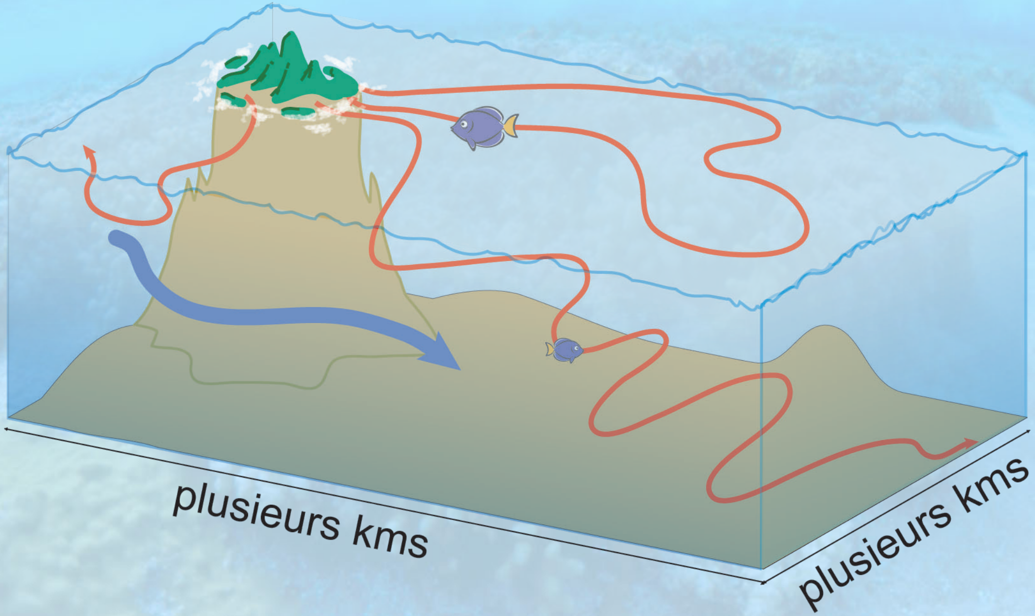
Introduction

Modelling



Modelling **trajectories** in a given **environment**

Introduction

Specificities of the modelling approach

- **Wider** temporal and spatial scales
- More **complete** description of the environment
- Larvae are no longer passive in a given environment, but **active** in exploiting it.

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The model

Problem characteristics

- Isolated island (auto-recrutment)

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The model

Problem characteristics

- Isolated island (auto-recrutment)
- A larva born in the island and maximizing its return probability at a given horizon

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The model

Problem characteristics

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- A larva born in the island and maximizing its return probability at a given horizon
- Comportmental alternatives during dispersion ⇒ **choices and trade-offs** (active larva)

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Optimality

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The model

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Optimality

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The model

Problem characteristics

Def. `Optimal`:
Which maximizes auto-recrutment probability.

Def. `Optimal trajectory`:
Random state trajectory for which the sequence of decisions is optimal (in expectation)

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The model

Theoretical framework

Time Discrete. Time step $=$ 6h. Temporal horizon fixed $= T$.

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The model

Theoretical framework

Time Discrete. Time step = 6h. Temporal horizon fixed = T.

State Energetic ressources level ($\theta \in [0, \theta_{max}]$) + Position ($x \in [0, x_{max}]$ or bounded (x,y,z))

The model

Theoretical framework

Time Discrete. Time step = 6h. Temporal horizon fixed = T.

State Energetic ressources level ($\theta \in [0, \theta_{max}]$) + Position ($x \in [0, x_{max}]$ or bounded (x,y,z))

Environment To each spatial position is associated

- a probability of dying by predation
- a feeding probability
- a current vector (intensity, direction)

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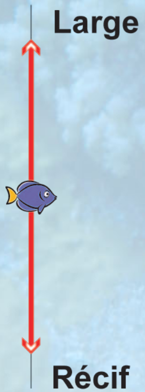
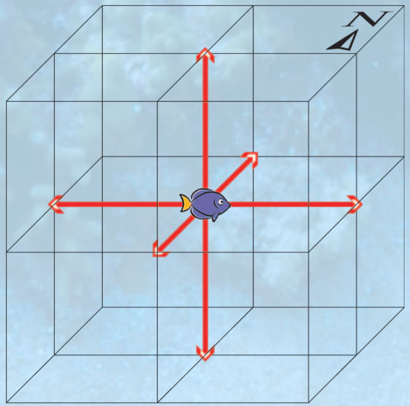
The model

Theoretical framework

Controlled dynamics Larva **trade-off** between two comportmental alternatives

- getting food and thus energy but at the price of swimming in a random direction
- swimming directionally but at the price of spending energy.





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The model

Transitions

Stochastic dynamic model: transition probabilities and matrices are indexed by the **control**

$$
\begin{array}{cc|cccc|cccc|cccc|}
\text{Energy} & & \multicolumn{4}{c|}{0} & \multicolumn{4}{c|}{1} & \multicolumn{4}{c}{2} \\
\text{Position} & & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 2 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 3 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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 & 3 & 0 & 0 & 0 & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{p} \\
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 & 3 & 0 & 0 & 0 & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{p} \\
\end{array}
= \mathcal{M}^0
$$

$$
\begin{array}{cc|cccc|cccc|cccc|}
\text{Energy} & & \multicolumn{4}{c|}{0} & \multicolumn{4}{c|}{1} & \multicolumn{4}{c}{2} \\
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 & 3 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 & 0 & \mathbf{1-p+p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & \mathbf{p} & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 2 & 0 & \mathbf{p} & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 3 & 0 & 0 & \mathbf{p} & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
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 & 2 & 0 & 0 & \mathbf{1-p} & 0 & 0 & \mathbf{p} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 3 & 0 & 0 & 0 & \mathbf{1-p} & 0 & 0 & \mathbf{p} & 0 & 0 & 0 & 0 & 0 \\
\end{array}
= \mathcal{M}^1
$$

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The model

Optimization

Optimization criterion Maximization of return probability at the horizon.

The model

Optimization

Optimization criterion Maximization of return probability at the horizon.

Zero instantaneous gains:

$$\forall t = 0, ..., T-1, \quad L(\theta, x, u, t) = 0 \tag{1}$$

Final gain

$$\Phi(\theta, x) = \mathbf{1}_{\{x=0\}} \tag{2}$$

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The model

Optimization

Optimization criterion Maximization of return probability at the horizon.

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$$\forall t = 0, ..., T-1, \quad L(\theta, x, u, t) = 0 \tag{1}$$

Final gain

$$\Phi(\theta, x) = \mathbf{1}_{\{x=0\}} \tag{2}$$

The optimization problem is

$$\max_{u_0,\dots,u_{T-1}} \mathbb{E}\left(\sum_{t=0}^{T-1} L(\theta_t, x_t, u_t, t) + \Phi(\theta_T, x_T)\right) = \max_{u_0,\dots,u_{T-1}} \mathbb{E}(\mathbf{1}_{\{x_T=0\}}) \tag{3}$$

The model

Stochastic dynamic programming

Bellman equation:

$$\begin{cases} V(\theta, x, T) & = & \Phi(\theta_T, x_T, T) \\ V(\theta, x, t) & = & \max_u \left[L(\theta, x, u, t) + (\mathcal{M}^u V(\cdot, t+1))(\theta, x) \right] \\ u^{\#}(\theta, x, t) & \in & \arg\max \left[L(\theta, x, u, t) + (\mathcal{M}^u V(\cdot, t+1))(\theta, x) \right] \end{cases} \tag{4}$$

Gives both

the **optimal feedbacks** $(u^{\#}(\theta, x, t))$

and the **maximal auto-recrutment probability**.

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The model

Stochastic dynamic programming

$$\begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\\1\\0\\0\\0\end{pmatrix}$$

$$\begin{array}{cc|cccc|cccc|cccc|}
\text{Energy} & & \multicolumn{4}{c|}{0} & \multicolumn{4}{c|}{1} & \multicolumn{4}{c|}{2} \\
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 & 2 & 0 & 0 & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{p} \\
 & 3 & 0 & 0 & 0 & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{p} \\
\hline
2 & 0 & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{p} & 0 & 0 \\
 & 1 & 0 & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{p} & 0 \\
 & 2 & 0 & 0 & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{p} \\
 & 3 & 0 & 0 & 0 & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{p}
\end{array}
=
\begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\\boxed{0}\\0\\0\end{pmatrix}$$

$$= \mathcal{M}^0$$

$$\begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\\1\\0\\0\\0\end{pmatrix}$$

$$\begin{array}{|cccc|cccc|cccc|}
\multicolumn{4}{c|}{0} & \multicolumn{4}{c|}{1} & \multicolumn{4}{c|}{2} \\
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\
\hline
\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\mathbf{1-p+p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{p} & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbf{p} & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbf{p} & \mathbf{1-p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\mathbf{1-p} & 0 & 0 & 0 & \mathbf{p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbf{1-p} & 0 & 0 & \mathbf{p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbf{1-p} & 0 & 0 & \mathbf{p} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{1-p} & 0 & 0 & \mathbf{p} & 0 & 0 & 0 & 0 & 0
\end{array}
=
\begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\0\\\mathbf{p}\\\boxed{\mathbf{p}}\\0\\0\end{pmatrix}$$

$$= \mathcal{M}^1$$

$\Rightarrow$ **optimal feedback**

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The model

Stochastic dynamic programming

By (4) and (3), one notices that

$$\begin{aligned} V(\theta, x, 0) &= \max_{u_0,\dots,u_{T-1}} \mathbb{E}\left(\sum_{t=0}^{T-1} L(\theta_t, x_t, u_t, t) + \Phi(\theta_T, x_T) \right) && (5) \\ &= \max_{u_0,\dots,u_{T-1}} \mathbb{E}(\mathbf{1}_{\{x_T=0\}}) && (6) \\ &= \max_{u_0,\dots,u_{T-1}} \mathbb{P}(x_T = 0) && (7) \end{aligned}$$

$\Rightarrow$ **maximal auto-recrutment probability**.

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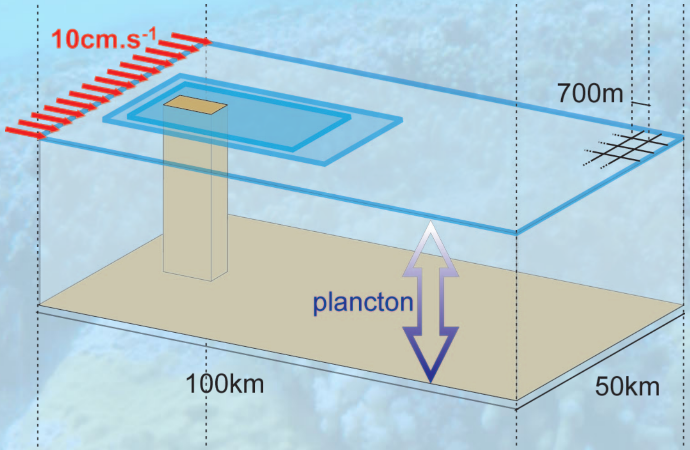
The model

Parameters



Environment

- **Current:** unidirectional and uniform
- **Predators et plankton** "reef effect" and "island mass effect"
- **Plankton** Daily vertical migration

The model

Parameters

Larva

- **Demersal** eggs
- Pelagic stage = **20 days**
- **3 development stages**. Swimming speed = 3, 10, 30 cm.s^{-1} ; energy consumption = yolk sac, then 1 unit by time step.
- **Energy** ⇒ 1 day without feeding before dying

The model

Biological realism and numerical consequences

Transition matrices of very **large size**:

$$100 \times 100 \times 3 \times 6 = 180\ 000 \text{ states}$$

$$\text{matrix} = 180\ 000 \times 180\ 000 \leftrightarrow 130\text{MB RAM}$$

The model

Biological realism and numerical consequences

Transition matrices of very **large size**:

$$100 \times 100 \times 3 \times 6 = 180\ 000 \text{ states}$$
$$\text{matrix} = 180\ 000 \times 180\ 000 \leftrightarrow 130\text{MB RAM}$$

Two problems:

1. matrices declaration
2. RAM

The model

Biological realism and numerical consequences

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Two problems:

1. matrices declaration
2. RAM

Two solutions:

1. C langage
2. sparse matrices

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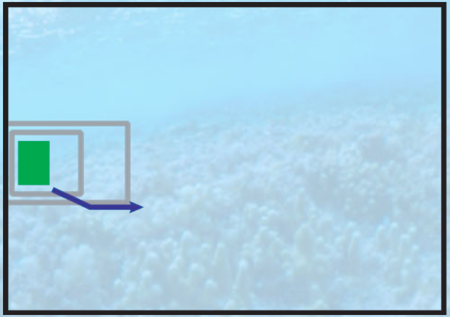
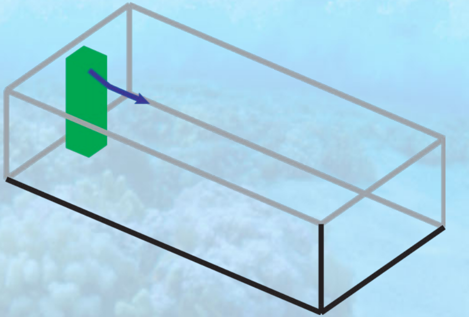
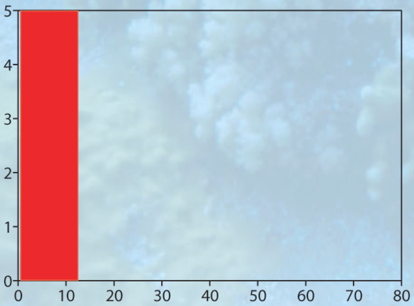
Simulations







Avoids high predation zones

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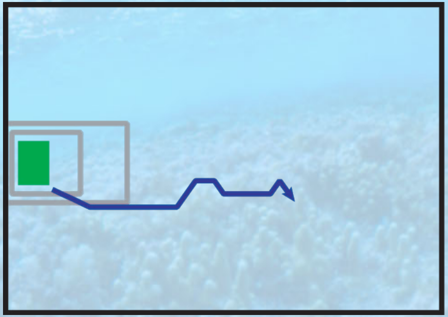
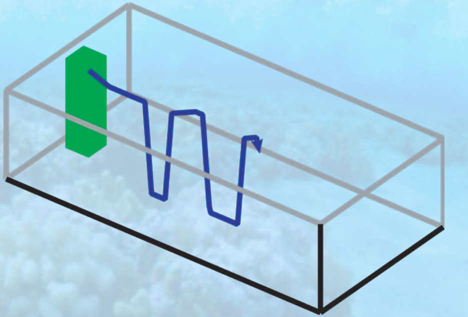
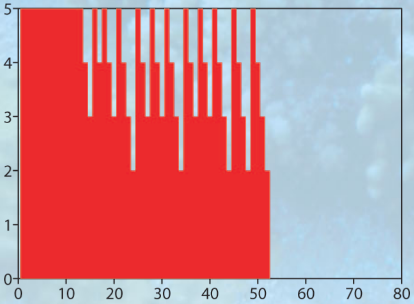
Simulations







Maximizes the probability of finding food

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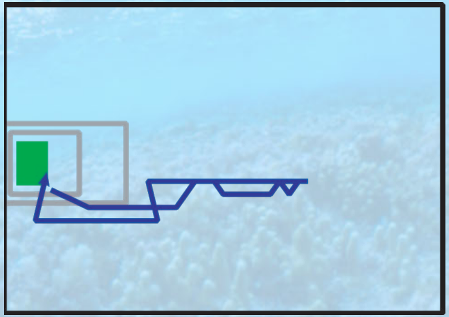
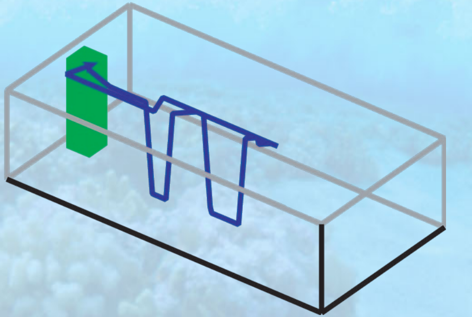
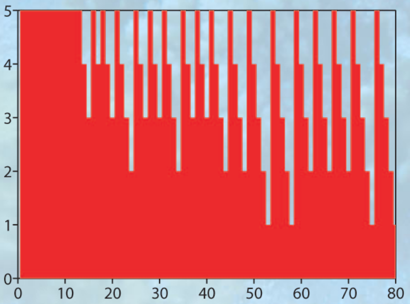
Simulations







Fights against current when efficient + avoids high predation zones

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Discussion

Model specificities

- Optimization = **rather new** theoretical framework (exception: Armsworth 2001)

- **Widening** of temporal and spatial scales

Discussion

Model specificities

- Optimization = **rather new** theoretical framework (exception: Armsworth 2001)
- **Widening** of temporal and spatial scales
- Focus on the larvae abilities: comportmental **trade-offs** ($\nleftrightarrow$ courants)

Discussion

Model specificities

- Optimization = **rather new** theoretical framework (exception: Armsworth 2001)

- **Widening** of temporal and spatial scales

- Focus on the larvae abilities: comportmental **trade-offs** ($\nleftrightarrow$ courants)

- **Richer description** of the environment (currents + predation and plankton)

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Discussion

Why optimization?

Evolutionary argument High mortality at the larva pre-reproductive stage =
high evolutive pressure on survival

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Discussion

Why optimization?

Evolutionary argument High mortality at the larva pre-reproductive stage =
high evolutive pressure on survival

Probabilistic argument Upper bound for maximal auto-recrutment probability.

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Perspectives

- Introduction of **finer** functions to describe the island and reef effects, and the evolution of the larva swimming abilities.

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Perspectives

- Introduction of **finer** functions to describe the island and reef effects, and the evolution of the larva swimming abilities.
- Introduction of a **second** island (auto-recrutment versus dispersion)

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Perspectives

- Introduction of **finer** functions to describe the island and reef effects, and the evolution of the larva swimming abilities.
- Introduction of a **second** island (auto-recrutment versus dispersion)
- Better description of the currents via a **hydrodynamics** model

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Perspectives

- Introduction of **finer** functions to describe the island and reef effects, and the evolution of the larva swimming abilities.
- Introduction of a **second** island (auto-recrutment versus dispersion)
- Better description of the currents via a **hydrodynamics** model
- **Sensitivity analysis**.

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