**A mathematical approach to viable management of fisheries and biodiversity through protected areas: the Abore reef reserve case**

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**To begin with. . .**

#### A quite honest talk on

the multiple difficulties of <sup>a</sup> multidisciplinary work with multiple contributors, multiple species, multiple objectives!

or

a MISSION IMPOSSIBLE from

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#### **to. . .**

// // DATA AND PARAMETER IDENTIFICATION // CORAL //

```
cor max=0.8;
// maximum covering = 80 %
cormin=0.05;// minimal covering = 5 %
```
 $1/$  R c tested from 1.001 to 1.01 by dichotomy R\_c=1.002;  $K_c = cor_max * R_c / (R_c - 1)$  ;

```
function Xp=logistic(t,X)
Xp= X : * (R c * (1 - X/K c)) ;// on force les Densités à être positives
endfunction
```

```
time=0:(9*365);
```
### **Scientific context**

*Appel à propositions de recherche biodiversité et changement global* Institut français de la biodiversité Ministère de l'Ecologie et du Développement durable

Accepted project (2004): **Modèles pour une gestion durable de la biodiversité sous incertitude et dynamique globales**

#### **Problem statement**

- Protected area: a relevant tool for sustainable managemen<sup>t</sup> of renewable resource?
- What is a **protected area (PA) effect**?
	- on *stocks*?
	- on *catches*?
- Which **sustainable management** through PA?
	- *Size*
	- *Placement*

# **Modelling requirements**

#### • **The context**:

- *Population dynamics*: nonlinearity, complexity, age-structured, spatial
- *Decision*: size and location of the PA
- *Uncertainties*: catches, stocks, processes, etc.
- **The issues:**
	- *Multi-criteria*
		- *Ecology:* conservation
		- *Economy:* fishing income
	- *Intergenerational equity*: both short and long term horizon

# **PLAN**

- 1. The Abore reef reserve, New Caledonia
- 2. A dynamic state model
- 3. The difficulties of parameter estimation from data
- 4. Measuring the reserve effect by stochastic viability
- 5. Simulations (skipped because not ye<sup>t</sup> discussed with biologists)
- 6. Discussion

#### **Some facts and figures**

*Assessment of the Impact of Removing Marine Reserve Status on Demersal and Benthic Fish Communities : <sup>a</sup> Comprehensive Approach*

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#### **The Noumea lagoon, New Caledonia**

The Noumea lagoon, located in South-Western New Caledonia, South Pacific (Figure 1) is <sup>a</sup> large coral reef ecosystem where several marine reserves were established in the 1980's in view of protecting the coral reef ecosystem from damage due to fishing and other human activities.

#### **The Abore reef reserve**

The Abore Reef reserve is located on <sup>a</sup> 25 km long barrier reef representing an area of ca. 15 000 ha.

Fishing was banned from the whole reef from 1990 to 1993, and allowed again on 2/3 of the reef from August 1993 for <sup>a</sup> fishing experiment in the perspective of adaptive management.

This opening was monitored by the Natural Resource Department of the South Province and by LERVEM (New Caledonia University)

#### **Expected reserve effects**

The main effects expected from the establishment of reserves are

- *increased abundances and biomasses* of spawning stocks and recruitment inside the protected area and in surrounding areas through spillover
- *rebuilding of ecosystems* through *protection of habitat* from fishing gears

# **A strong fishing pressure**

In the Pacific islands, the reef and lagoon subsistence fisheries represen<sup>t</sup> about 80 % of total coastal catch. Right after the reserve opening in August 1993:

- the number of boats and fish yield during the 2 weeks after the opening reached the levels previously observed for <sup>a</sup> whole year;
- monitoring of fishing effort and catch rates showed that, in the open area, *benefits from the 1990-1993 closure were dissipated within <sup>a</sup> few weeks*.

#### **Fishing experiment**

The whole reef was finally closed to fishing from August 1995.

In 1995, 2/3 of the reef had been closed for 3 years (1990-1993) and fished for 2 years (1993-1995), while the remaining had been permanently closed during the 5 years (1990-1995).

The area open to fishing from August 1993 is the impact area (area B in Figure 2), while the reference area has been permanently closed (area A in Figure 2).

#### **Survey / Before After Control Impact design**

A survey was conducted in July 1993 and July 1995, respectively right before the opening and before the final closure.

A scientific evaluation of how reserve is likely to affect fish community, e.g. increased densities, larger fish, modified interspecific relationships.

# **A glimpse at complexity and uncertainty**

The experimental design rests on <sup>a</sup> stratification of the reef into three morphological zones : reef flat, inner slope and lagoon (Figure 2), delineated on aerial photographs.

The structure and functioning of communities are often poorly known, in particular because coral reef ecosystems exhibit <sup>a</sup> very high diversity of fish species, generally linked with live coral cover.

For demersal and benthic species, spatial distributions of populations mostly depend on habitat preferences.

# **A glimpse at biodiversity: species richness**

A total of **374 species** were identified during the survey. As a consequence of the high diversity of the reef community, the number of species observed at <sup>a</sup> given transect often exceeded 100, most species being encountered only at <sup>a</sup> few stations.

The criteria used for partitioning the fish community into species groups were *mobility, taxonomy and feeding habits*.

#### **Four groups of species defined for mobility**

• *territorial species* living in <sup>a</sup> very restricted range (usually less than  $10 \text{ m}^2$ );

• *sedentary species* with <sup>a</sup> restricted range between ten and <sup>a</sup> few hundred square meters;

• *weakly mobile species* not restricted to <sup>a</sup> specific range(up to several thousands  $m^2$ );

• *highly mobile species* usually foraging over very large areas; they are not restricted to a given reef over a short period of time.

These categories form <sup>a</sup> continuum, and some species were difficult to assign to <sup>a</sup> given category.

### **Taxonomy: 9 over 41 families**

Species belonging to 41 families were recorded during the surveys. Species in <sup>a</sup> given family were likely to be more similar in terms of trophic, morphologic and demographic features than species belonging to different families.

Only 9 families were retained for the analysis : *Acanthuridae, Chaetodontidae, Labridae, Lethrinidae, Lutjanidae, Pomacentridae, Scaridae, Serranidae, Siganidae and others*. These were selected either because they were important to fisheries, or because they were encountered at <sup>a</sup> large number of stations, and with non negligible abundances.

#### **Feeding habits / diet**

Species at <sup>a</sup> high trophic level were generally those targeted by fishermen, and were thus likely to be sensitive to the reserve status.

Feeding habits were expressed in percentage of food types in diet. Food types were categorized as *nekton, macroinvertebrates, macroalgae, microinvertebrates, microalgae, zooplankton, other plankton, coral and detritus*.

The analysis of species diets yielded 7 clusters, each cluster forming <sup>a</sup> *trophic group* (Table 2). Note that in each group, the mean diet included several food items. Groups were named on the basis of their mean diet composition.

#### **Densities**

Total fish density per transect ranged between 0.95 and 114 individuals/m<sup>2</sup>, with a mean of 7.6 ind/m<sup>2</sup>.

The largest densities observed occurred at one or two stations with large concentrations of Clupeidae. When this family was excluded from computations, total density per transect over the two years dropped to 2.9 ind/m<sup>2</sup>, with a mean of 4.0 ind/m<sup>2</sup> in1993 and 1.7 ind/m<sup>2</sup> in 1995.

# **A brief summary**

• **Area:** 15 000 ha

• **Biodiversity:** some 374 species ; 41 families ;

7 trophic groups ; few functional groups but <sup>a</sup> lot of species within each group

• **Catch pressure:** very strong ; the biomass surplus accumulated in 3 years (from 1990 to 1993) has been fished in. . . 15 days

• **A climatic impact on coral:** the coral is <sup>a</sup> refuge for (small) fishes ; affected by cyclones destructions • **Issues:**

- Is there <sup>a</sup> *reserve effect*?
- How does *climate change* modify the previous assertions?

#### A DYNAMIC STATE MODEL

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# **Highly delicate modelling options**

- Restriction to . . . 4 *trophic groups* <sup>+</sup> habitat
- One "typical" species is selected within each group

**These basic options lead to very difficult discussions between marine biologists and mathematicians**

#### **State: densities**

- •• **Coral/Habitat**  $x_0(t)$ : percentage of covering;
- **Piscivors**  $x_1(t)$ : predators of fishes, target of fishermen (length 77 cm on average);
- **Macrocarnivores**  $x_2(t)$ : predators of macroinvertebrates and of <sup>a</sup> few fishes (length 38 cm on average);
- **Herbivors**  $x_3(t)$ : some are targets of fishermen (length from 24 to 39 cm);
- $\bullet$ • Other fishes (small)  $x_4(t)$ : sedentary and territorial organisms, microcarnivores (17 cm), coralivores (16 cm), zooplanctonophages (13 cm);

# **"Typical" species: <sup>a</sup> highly delicate choice. . .**

- *big grouper*
- *small grouper*
- *parro<sup>t</sup> fish*
- *damsel fish*

#### **A logistic habitat covering growth model**

$$
x_0(t+1) = x_0(t) \left( R_0 \left( 1 - \frac{x_0(t)}{K_0} \right) \right)
$$

The time unit  $\Delta t = 1$  is to be fixed later. •  $R_0$  is the intrinsic growth rate (for low covering). •  $K_0$  is related to the so called *carrying capacity*  $x_0^{\sharp}$ solution of

$$
1 = R_0 \bigg( 1 - \frac{x_0^{\sharp}}{K_0} \bigg)
$$

Cyclonic events occur randomly with probability p at every time step and bring the coral covering to 30 % of its last value.

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#### **A** *Lotka-Volterra* **model**

$$
x_1(t+1) = x_1(t) \left(R_1 + \alpha_2^1(x_0(t))(E_1^1 - 1)x_1(t) + E_2^1\alpha_2^1(x_0(t))x_2(t) + \dots + E_4^1\alpha_4^1(x_0(t))\right)
$$
  
\n
$$
x_2(t+1) = x_2(t) \left(R_2 - \alpha_2^1(x_0(t))x_1(t) + \alpha_2^2(x_0(t))(E_2^2 - 1)x_2(t) + \dots + E_4^2\alpha_4^2(x_0(t))x_4(t)\right)
$$
  
\n
$$
\vdots
$$

$$
x_4(t+1) = x_4(t) \Bigg(R_4 - \alpha_4^1(x_0(t))x_1(t) - \alpha_4^2(x_0(t))x_2(t) - \dots + \alpha_4^4(x_0(t))(E_4^4 - 1)x_4(t)\Bigg)
$$

The time unit  $\Delta t = 1$  is to be fixed later.

#### **Details for trophic group 2**

. . .

$$
\frac{x_2(t+1)}{x_2(t)} = R_2
$$

 $-\alpha_2^1(x_0(t))x_1(t)$ 

 $-\alpha_2^2(x_0(t))x_2(t)$ 

 intrinsic growth rate predator 1's catches rate predator 2's catches rate  $+ E_2^2 \alpha_2^2(x_0(t))x_2(t)$  conversion of predator 2's catches rate of prey 2

 $+ E_A^2 \alpha_4^2(x_0(t))x_4(t)$  conversion of predator 2' catches rate of prey 4

- Intrinsic growth rate  $R_k$  includes mortality and recruitment of trophic group  $k$  due to external trophic groups.
- ••  $\alpha_j^i(x_0)x_i$  is the proportion of prey j captured by  $x_i$  predators  $i$  in one time unit. It depends on coral covering through <sup>a</sup> refuge mechanism:

$$
\alpha_j^i(x_0) = \gamma(x_0)\widehat{\alpha}_j^i = e^{(\beta - \lambda x_0)}\widehat{\alpha}_j^i
$$

- • $\bullet$   $E_j^i$  stands for the conversion factor of one unit of prey  $j$  density into growth of predator  $i$ .
- The matrices  $\alpha$  and  $E$  are upper triangular.

To summarize the dynamics of the trophic groups, excep<sup>t</sup> the coral, we write in <sup>a</sup> matrix form

 $x_k(t+1) = x_k(t) (R_k+\widetilde{S})$  $(x_0(t))x(t))_{k}\,,\quad k=1,\dots,N\,,$ with



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#### THE DIFFICULTIES OF PARAMETER ESTIMATION FROM DATA

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#### **The coral data**

$$
x_0(t+1) = \begin{cases} x_0(t) \left( R_0 \left( 1 - \frac{x_0(t)}{K_0} \right) \right) & \text{prob.} \ (1-p) \\ 0.3 \times x_0(t) & \text{prob. } p \end{cases}
$$

We identify the maximal value of 80 % with the carrying capacity  $x_0^{\sharp}\stackrel{{\rm def}}{=} \frac{R_0-1}{R_0}K_0=0.8.$ After a cyclonic event, the coral grows by 10 % <sup>a</sup> year but not linearly: it takes 8 to 10 years to reach the initial covering. Probability occurence p corresponds to 1 cyclone every 5 to 6 years.

# **The case of coral** Simulations give  $R_0=1.002$  for  $\Delta t=1$  day.



#### **Characteristic figures of the organisms**

The  $\emph{diet composition}$   $\emph{Stom}^i_j$  is the proportion of prey  $j$  in the stomach of predator  $i$ :

 $\text{Stom}^i_j = \frac{biomass\,of\,prey\,j}{total\,biomass\,of\,preys\,in\,stomach\,ofi}$ 

The matrix Stom is upper triangular.

Theoretically, all lines sum up to 1 in the matrix Stom  $(\sum_{preys} \frac{1}{i}^{S} \text{tom}_j^i = 1)$  but, as the predators do not only eat organisms mentioned in this model, the sums of lines may be less than 1.

# **Characteristic figures of the organisms**

The *stomachal capacity*  $B_k$  is the maximum biomass that the stomach of the organism  $k$  can contain: we suppose that it represents 30 % of the organism mass.

The *average mass*  $W_k$  *of the organism*  $k$  *is supposed* to be given by

 $W = \begin{pmatrix} 0.5 & 0.5 & 0.7 & 0.1 \end{pmatrix}$  kg

with typical species *big grouper small grouper parro<sup>t</sup> fish damsel fish*

#### **Field data: abundances, densities**

In 1995, *outside the Abore reserve, hence under fishing pressure*, the mean data were (except for coral)

 $\overline{(x_1 \quad x_2 \quad x_3 \quad x_4) = (0.04 \quad 0.48 \quad 1.17 \quad 0.49) \text{ ind/m}^2}$ 

In the sequel, we shall assume that this vector represents densities at equilibrium, denoted by  $(x_1^\star$   $x_2^\star$   $x_3^\star$   $x_4^\star).$ 

Problem of coherency because data should not be under fishing pressure. This is <sup>a</sup> consequence of frequent changes in modelling options.
# **Diet composition estimation**

The diet composition is evaluated to be  $\mathrm{Stom} =$ 



The lines should sum up to 1, but the predators eat other organisms like invertebrates, zooplankton, algae, etc. Line 2: macrocarnivores eat mostly invertebrates Lines 3: herbivors eat mostly algae Lines 4: other fishes (small) eat mostly zooplankton or coral

Piscivors and macrocarnivors eat fish. . . *but the group is unknown*, so that the above matrix is the fruit of different assumptions:

- piscivors' diet is composed of 77 % fish, of all species, especially small ones, with canibalism
- macrocarnivors' diet is composed of 10 % fish, the rest being mostly invertebrates
- *the proportions of diet composition by group is supposed to be the ambient proportions* (perfect mixing and "opportunistic" behaviour assumptions)

# **A "stomachal cycle"**

Suppose that the densities  $x_i^*$  are at equilibrium. During <sup>a</sup> "stomachal cycle", we have

- 1. Stom<sup>2</sup>; proportion of biomass of prey  $\overline{j}$  in the stomach of predator  $i.$
- 2. Stom<sup> $\ell_i \times B_i$ : biomass of prey j in the stomach of</sup>  $predator i.$  (should be a volume times a volumic mass)
- 3. Stom<sup> $i<sub>i</sub> \times B_i \times x_i^*$ : biomass of preys j in  $x_i^*$ </sup> stomachs of predator  $i.$

#### **Identification of** α  $\stackrel{i}{j}$

# During a time unit, we hav e

- $1.~~\alpha$  $\overset{i}{j}\mathcal{x}$  $\stackrel{\star}{j}\stackrel{\star}{x}$  $\underset{i}{\stackrel{\star}{i}}$ : number of preys  $j$  caught by  $x$  $\star_i$ predators i;
- $2. \, \, \alpha$  $\overset{i}{j}\mathcal{x}$  $\stackrel{\star}{j}\stackrel{\star}{x}$  $\star_i$  $\times W_j$ : biomass of preys j in x  $_{i}^{\star}$  stomachs of predator  $i.$

If *the time unit coincides with one "stomachal cycle"*:

$$
\text{Stom}_{j}^{i} B_{i} x_{i}^{\star} = \alpha_{j}^{i} x_{j}^{\star} x_{i}^{\star} W_{j} \iff \alpha_{j}^{i} = \frac{1}{x_{j}^{\star}} \frac{\text{Stom}_{j}^{i} B_{i}}{W_{j}}
$$

 $\Delta t=1$  day

# **Identification of** <sup>α</sup>



# **Identification of conversion factors**  $E^i_j$

 $E^{i}_{j} = \frac{number\; of\; individuals\; i\; produced}{number\; of\; individuals\; j\; consumed} \, .$ Figures in the litterature (Arias Gonzales) indicate that

$$
\widetilde{E}^i_j = \frac{biomass \ i\ produced}{biomass \ j\ consumed} \leq 0.3
$$

with values of 0.12, 0.13 for piscivores and macrocarnivores, whatever the prey:

> $\bm E$  $\widetilde{\bm \Gamma^i}$  $j^i_j = 0.125 \, , \quad j = 1, 2 \, , \ i = 1, \ldots, 4 \, .$

We deduce that

 $\,E$  $\widetilde{E}^i_j =$  $number~of~individuals~i~produced~\times W_i$  $number~of~individuals~j~consumed~\times W_j$ 

and thus

!<br>•

!<br>•

!<br>•

!<br>•

<sup>E</sup>ij <sup>=</sup> <sup>E</sup><sup>e</sup>ij <sup>×</sup> <sup>W</sup><sup>j</sup> <sup>W</sup>i <sup>≈</sup> <sup>0</sup>.125W<sup>j</sup> <sup>W</sup><sup>i</sup> <sup>=</sup> 0.125 0.125 0.175 0.025 ! 0. 0.125 0.175 0.025 ! 0. 0. 0. 0. ! 0. 0. 0. 0. !

#### **Identification of** S  $\widetilde{\phantom{m}}$



# **Identification of growth rates**

By writing that the densities  $x_i^\star$  are at equilibrium, we obtain the intrinsic growth rates

$$
R_i = 1 - \sum_{j=1}^4 \widetilde{S}_i^j x_j^{\star}, \quad i = 1, \dots, 4
$$

! 0.975 ! ! 1.007 ! ! 1.008 ! !1.054 !

# **Trajectories (1)**



# **Trajectories (2)**



#### A MATHEMATICAL MEASURE OF THE PROTECTED AREA EFFECT

# **Catches**

•• **Catches** on the piscivors  $x_1(t)$ , macrocarnivors  $x_2(t)$  and herbivors  $x_3(t)$ :

$$
C_i = e_i x_i \quad \text{with} \quad e_4 = 0
$$

• **Uncertainty scenarios** for *exploitation rates*  $e_i(\omega(t)) \in \left[\overline{e}_i - \sigma_i, \overline{e}_i + \overline{\sigma_i}\right] \subset \left[0, 1\right]$ 

# **Exploited dynamics**

$$
x_0(t+1) = (1 - \theta^{-1} \theta(\omega(t))) \quad x_0(t) \left( R_0 \left( 1 - \frac{x_0(t)}{K_0} \right) \right)
$$
  
+ 
$$
\theta(\omega(t)) \quad 0.3 \times x_0(t)
$$
  

$$
\theta = 1: \text{ cyclone.} \quad 30 \text{ Yoreduction}
$$
  

$$
x_i(t) = x_i^e(t) \left( R + S(x_0(t), \omega(t)) x^e(t) \right)_i
$$
  

$$
x_i^e(t+1) = x_i(t) (1 - e_i(\omega(t))) \quad \text{catches}
$$

# **Exploited dynamics with reserve**

- Model not spatially explicit: scalar 1 <sup>−</sup> A measures the *size of protected area PA*
	- $(A = 0:$  full reserve;  $A = 1:$  no reserve)
- **A fixed proportion** A <sup>∈</sup> [0, 1] **is open to harvesting** so that catches are given by  $C_i(t) = e_i(\omega(t))Ax_i(t)$
- **Exploited dynamics with <sup>a</sup> protected area**: idem but  $x^e \rightarrow x^A$  and

 $x_i^A(t+1)=x_i(t)(1-Ae_i(\omega(t)))\,,\quad i=1,\ldots,4$ 

# **Sources of uncertainties**

Scenarios  $\omega(0), \omega(1), \ldots$  comprise uncertainties from • cyclonic events impacting coral:

 $\theta(\omega(t)) \sim \text{Bernoulli}(1, p)$ 

• catch effort:

 $e_i(\omega(t)) \sim \text{Uniform}[\overline{e}_i - \sigma_i, \overline{e}_i + \sigma_i]$ 

Assuming statistical independence, this gives <sup>a</sup> **probability**  $\mathbb P$  on scenarios  $\omega \in \Omega$ 

**Catches economic value** • **Utility function of catches:**

$$
U\bigg(C_1(t),C_2(t),C_3(t)\bigg)
$$

• **Substitutable and essential factors:**

$$
U(c_1, c_2, c_3) = c_1^{0.5} \times c_2^{0.5} \times c_3^{0.5}
$$

# **Stochastic intertemporal decision**

- **Optimal discounted utility approach**:  $\max_A$  ${}_A\,\mathbb{E}_{\omega}\left[\sum_{t=0}^T\rho^t U\bigg(C_1(t),C_2(t),C_3(t)\bigg)\right].$
- **Maximin approach**:
	- $\max_A$  $\mathbb{E}_{\omega}\left[\min_{t=0,...,T}U\bigg(C_1(t),C_2(t),C_3(t)\bigg)\right].$
- **Viability and effectiveness approaches**:

**Conservation requirements: existence values**

• **Implicit Conservation with viability**:  $U(x(t)) \geq U(C(t)) \geq U_{\mathfrak{b}_1}$  $\implies$ 

 $x_1(t) > 0$ ,  $x_2(t) > 0$ ,  $x_3(t) > 0$ 

• **Explicit conservation**:

 $x_3(t) \geq x_{3,b}$ 

#### **A stochastic effectiveness analysis**

**An indicator of sustainability at confidence level**  $\beta \in [0,1]$  is the *sustainable kernel*  $\text{Sust}_{\beta}(A) \stackrel{\text{def}}{=}$  $\left\{x(0) \middle| \mathbb{P}_{\omega}\left(U\left(C_1(t), C_2(t)\right) \geq U_b \text{ for all times }\right) \geq \beta\right\}$ 

It consists of initial states  $x(0)=(x_0(0),\ldots,x_4(0))$ such that the probability that <sup>a</sup> random trajectory starting from  $x(0)$  provides a utility from catches greater than  $U_b$  is at least  $\beta$ .

# **Global reserve effect definition**

A particular case is the  $\mathbf{robust}\ \mathbf{kernel}\ \mathbf{Sust}_1 = \mathbf{Sust}\ \mathbf{:}$ whatever the scenarios, a minimal utility  $U_\flat$  is ensured.

We say that **global reserve effect** holds if there exists  $A < 1$  such that  $\operatorname{Sust}_{\beta}(1) \subsetneq \operatorname{Sust}_{\beta}(A)$ (Global reserve effect holds true if the kernel is enlarged when fishing is restricted)

#### **Optimal sustainable reserve size**

Given an *initial condition*  $x = x(0)$ :

- **Maximal guaranteed use values** is, for size  $A\in[0,1]$ :  $U^{\star}_{\mathfrak{b}}(A, x, \beta) \stackrel{\text{def}}{=} \overline{\max(U_{\mathfrak{b}}}, x \in \text{Sust}_{\beta}(A))$ The largest utility which can be ensured with probability  $\beta$ .
- **Optimal reserve size** is  $A_{\beta}^{\star}(x) \stackrel{\text{def}}{=} \arg \max_{A \in [0,1]} U_{\flat}^{\star}(A, x, \beta)$ The corresponding reserve size

**A reserve effect measure**

- **Reserve effect index:**  $\overline{\text{PAI}_{\beta}(x)} \stackrel{\text{def}}{=} U_{\flat}^{\star}(\overline{A_{\beta}^{\star}(x),x)} - U_{\flat}^{\star}(1,x)$ Compares difference of utility witout and with (optimal) reserve
- **Definition:** reserve effect holds true for state x if  $\text{PAI}_{\beta}(x) > 0$

# **Contribution values**

- **Contribution of trophic group** i: compare the *sustainability kernel* Sust in two cases
	- 1. Sust<sub>i</sub> without *i*, namely  $x_i(t) \equiv 0$
	- 2. Sust with i, namely any  $x_i(t) > 0$
- **Contribution index**: the difference  $I_i \stackrel{{\mathrm {\footnotesize def}}}{=} \operatorname{Sust}\backslash\operatorname{Sust}_i$
- A particular case:  $Sust_i = \emptyset$

# CONCLUSIONS

# **Conclusion**

- **Methodology**: invariance, co-viability
	- The role of constraints
	- Multi-criteria: conservation and efficiency
	- **-**- Intergenerational equity
- **Reserve problem**:
	- Formalization of reserve effect
- **Aboré reserve**: <sup>a</sup> model
	- with calibrated trophic interactions
	- account for the climatic change through coral

# **Perspectives**

- **Robustness with respect to parameter changes**
- **Indicator of refuge function of coral**
- **Spatially explicit**
- **Biodiversity measures**
- **Non cooperative harvesting agents**
- **Age structure**

#### **To end with. . .**

# NEVER PREPARE A TALK FOR A CONFERENCE IN SEPTEMBER AT THE OTHER END OF THE WORLD WHEN YOUR SCIENTIFIC PARTNERS ARE IN AUGUST VACATIONS!

# **Contribution value for herbivors**

If herbivors  $x_3$  collapse, the whole ecosystem can disappear?

**Conjecture 1** *If*  $x_3 = 0$ *, the* sustainability *kernel is empty:*  $Sust_{3}(A) = \emptyset$ 

 $\Longrightarrow$  **Strong contribution** value of  $x_3$  $I_3 = {\rm Sust} \backslash {\rm Sust}_3 = {\rm Sust}$ 

# **A co-viability analysis without global changes**

- **Assumption 1:** *No damage* for coral: p <sup>=</sup> 0. Coral at equilibrium  $80\%$
- **Assumption 2:** Robust approach: Confident rate  $\beta=1$
- **A reserve effect with moderate harvesting: Conjecture 2** *There exists <sup>a</sup> fishing effort threshold* <sup>e</sup>] *such that*
	- *For any*  $\overline{e} \in ]0; e^{\sharp}]$ , a PA effect.
	- For any  $\overline{e} > e^{\sharp}$ , no PA effect.



Figure 4: Reserve Effect Index  $PAI(u)$  at  $x(0)$  =  $(0.26; 1.7; 6.84; 0.8)$ . Uncertainty  $\sigma = 5\%$ .

# **A reserve effect with moderate harvesting**



(a) Guaranteed utility of captures  $U_{\flat}^{\star}(A)$ 

Figure 5: A reserve effect at  $x(0)$  $(0.26; 1.7; 6$  mathematical ppps ach toviable management offisheries and biodiversity mrough protected dreas: the Abore reef reserve case – p. 68

# **A reserve effect with moderate harvesting**



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(a)  $x(t)$  without reserve MPA= 0% (b)  $x(t)$  with optimal MPA = 94

Figure 6: A reserve effect at  $x(0)$  $(0.26:1.7:$   $\alpha$  mathematical approach toviable management offisheries and biodiversitythrough protected areas:the Abore reef reserve case – p. 69

# **No reserve effect for large exploitation rates**



(a) Guaranteed utility of captures  $U_{\flat}^{\star}(A)$ 

Figure 7: No reserve effect at  $x_0$  $(0.26; 1.7; 6$  mSechatic DppSach to fistogename energy to sect of the protected areas the Abore  $80$  Me case – p. 70

# **No reserve effect for large exploitation rate**



Figure 8: No reserve effect at  $x_0$  $(0.26:1.7:$   $\&$  mathematical ppp oach to vable management offisheries and biodiversity through protected areas:the Abore certa (eye) case – p. 71

**A co-viability analysis with certain climatic changes**

- **Assumption:** *Certain damage* for coral: p <sup>=</sup> 1. • **A stronger reserve effect !!!! Conjecture <sup>3</sup>** *There exists <sup>a</sup> threshold* <sup>e</sup>] *such that*
	- *For any*  $\overline{e} \in ]0; e^{\sharp}]$ , a PA effect.
	- *For any*  $\overline{e} > e^{\sharp}$ , no PA effect.


Figure 9: With climatic change: Reserve Effect Index  $PAI(u).$ 

A mathematical approach toviable managemen<sup>t</sup> offisheries and biodiversitythrough protected areas:the Abore reef reserve case – p. 73