Multi-Criteria Dynamic Decision Under Uncertainty: Management Strategy Evaluation, Stochastic Viability Analysis and Beyond

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Motivation

Many **natural resources management** problems (such as *fishery management*) are

- **dynamic** (*stocks population dynamics*)
- marked by **uncertainties** (*on recruitment* for instance).

Sustainability issues lead to take into account possibly **conflicting criteria**

- **ecological** (*spawning stock biomass preservation*)
- **economic** (*guaranteed income for fishermen*).

Outline of the presentation

MANAGEMENT STRATEGY EVALUATION FOR FISHERIES

Fisheries management issues

Primary considerations in fisheries management are

- sustainability of the resource base;
- economic viability;
- equity in access to the resource.

One of the reasons of management **failure** in fisheries is the **conflict** between ecological constraints and **social and economic priorities**, the latter often **having priority over resource conservation**.

Uncertainty management

Moreover, fisheries management issues are highly marked by uncertainty, let it be on stock evaluation, on the recruitment process, on catches, on ecosystemic effects, etc.

An important issue is thus to determine **management procedures**

- that give acceptable results with respect to the **sustainability objectives**
- while being **robust to uncertainties**.

Management Procedures

A Management Procedure (MP) is defined in Butterworth et al. (1997) as a **set of rules, which translate data from a fishery into a regulatory mechanism** (such as total allowable catches or maximum fishing effort).

According to Oliveira and Butterworth (2004), such MPs have been developed (though not always implemented) for a number of disparate fisheries since their development within the International Whaling Commission in the late 1980s.

Ranking Management Procedures?

Ideally, before defining the MP to be applied, one should **compare different potential MPs** and rank them with respect to their ability to keep the fishery sustainable in an uncertain environment.

The so-called Management Strategy Evaluation (MSE) denotes a class of **procedures based on simulation to compare alternative MPs**.

Management Strategy Evaluation (MSE)

As detailed in Sainsbury et al. (2000), the MSE approach consists in

- defining an **operational set of management objectives**,
- and **evaluating using simulations the performance of various alternative management strategies** with respect to the specified objectives, taking into account uncertainty in the modeled processes.

The method consists in

- **testing a particular MP in a great number of simulations** over a given time period, each simulation representing a plausible "state of nature" (scenario),
- and in **evaluating statistics** over the simulation results to summarize the performance of the particular MP and test robustness with respect to uncertainty.

Performance statistics, uncertainty management

As stated in (Geromont et al., 1999), performance statistics usually fall into three categories, with improved performance in one area generally leading to worse performance in at least one of the other two. These three relate to the general objectives of

- maximizing catch,
- minimizing risk to the resource,
- maximizing industrial stability.

The choice and design of performance statistics usually implicitly incorporates the way uncertainty is handled as well as how time periods are aggregated. For instance,

Catches. Expected catch = Mean [$\sum_t$ catches]

Risk to the resource. Probability that the biomass will drop below some pre-specified threshold = Proba [$\inf_t$ biomass $<$ threshold]

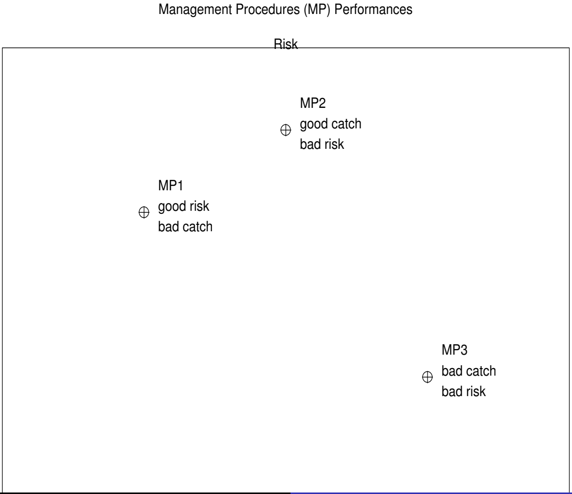
Industrial stability. Average of the absolute change in catch expressed as a proportion of average catch.

Trade-offs in selecting MPs

To the best of our knowledge, the ultimate phases of MSE, namely the **trade-offs analysis and the MPs ranking**, have the **less methodological content**.

In general, one is left with a difficult multi-criteria decision problem, where the selection “method” is, at best, visual.

Visual examination: trade-off curve



Lack of a "common currency"

Even if **MSE allows to clarify hidden or implicit assumptions in MP evaluations, and describes trade-offs** in management objectives, the last step of trade-offs analysis – which should permit MPs ranking – leaves the decision-maker(s) with clearer perspectives but without tools.

The difficulty comes from the **absence of a "common currency"** for conflicting performance measures which should be approved by all parties.

A STOCHASTIC VIABILITY ANALYSIS

Discrete-time dynamical system

Consider a **discrete-time control dynamical system**

$$x(t+1) = G\big(t, x(t), u(t), w(t)\big) , \quad t = t_0, \ldots, T ,$$

with

- **time** $t \in \{t_0, \ldots, T\} \subset \mathbb{N}$, (the time period $[t, t+1[$ may be a year, a month, etc.)
- **state** $x(t) \in \mathbb{X} := \mathbb{R}^n$, (biomasses, abundances, etc.)
- **control** $u(t) \in [u^\flat, u^\sharp] \subset \mathbb{U} := \mathbb{R}$, (catches or harvesting effort)
- **uncertainty** $w(t) \in \mathbb{W} := \mathbb{R}^q$, (recruitment or mortality uncertainties, climate fluctuations or trends, etc.)

Scenarios

We define

$$\Omega := \mathbb{W}^{T-t_0+1}$$

as the set of **scenarios**, the notation for a scenario being

$$w(\cdot) := \big(w(t_0), \dots, w(T)\big) \ .$$

Scenarios are **perturbations of the dynamics** (not of the observations or outputs); they include

- time-varying disturbances (trends, fluctuations, etc.): climate, temperature, recruitment, etc.
- fixed parameters.

State feedback

A **(state) feedback**

$$\mathfrak{u} : \mathbb{N} \times \mathbb{X} \to \mathbb{U}$$

is a **decision rule** which assigns a control $u = \mathfrak{u}(t, x) \in \mathbb{U}$ to any state x for any time t.

A feedback induces a sequence of controls by

$$u(t) = \mathfrak{u}\big(t, x(t)\big) \ .$$

Observation pattern: perfect information

Perfect information.

$$\boxed{\mathrm{y(t) = x(t)}}$$

Partial information.

$$y(t) = h(x(t))$$

Imperfect information.

$$y(t) = h(x(t), w(t))$$

Thus, we consider that an appropriate assessment method yields the state of the system, or at least the part necessary for a given MP.

Solution state map

Given a feedback $\mathfrak{u}$, a scenario $w(\cdot) \in \Omega$, an initial state $x_0 \in \mathbb{X}$ and an initial time t_0, the **solution state**

$$x_F[t_0, x_0, \mathfrak{u}, w(\cdot)](\cdot) = (x(t_0), x(t_0 + 1), \ldots, x(T))$$

is the state path solution of dynamics

$$\begin{cases} x(t+1) = G\big(t, x(t), u(t), w(t)\big) \text{ with } x(t_0) = x_0 \\ u(t) = \mathfrak{u}\big(t, x(t)\big) , \quad t = t_0, \ldots, T-1 . \end{cases}$$

The **solution control** $u_F[t_0, x_0, \mathfrak{u}, w(\cdot)](\cdot)$ is the associated decision path $u(\cdot) = \big(u(t_0), u(t_0+1), \ldots, u(T-1)\big)$ where $u(t) = \mathfrak{u}\big(t, x(t)\big)$.

Viability constraints defined by indicators

Consider K real-valued functions

$$\mathcal{C}_k : \mathbb{N} \times \mathbb{X} \times \mathbb{U} \times \mathbb{W} \to \mathbb{R}$$

for $k = 1, \ldots, K$, that we shall call **constraint functions**.

For instance,

- constraint = spawning stock biomass - limit biomass
- constraint = catches - threshold

Viable scenarios

For any feedback $\mathfrak{u}$, initial state x_0, and initial time t_0, let us define the set of **viable scenarios** by:

$$\Omega_{\mathfrak{u},t_0,x_0} := \left\{ w(\cdot) \in \Omega \;\middle|\; \begin{array}{l} x(t_0) = x_0 \\ x(t+1) = G\big(t, x(t), u(t), w(t)\big) \\ u(t) = \mathfrak{u}\big(t, x(t)\big) \\ \mathcal{C}_k\big(t, x(t), u(t), w(t)\big) \geq 0 \\ k = 1, \ldots, K \\ t = t_0, \ldots, T \end{array} \right\} .$$

Any viable scenario $w(\cdot)$ in $\Omega_{\mathfrak{u},t_0,x_0}$ is such that the state and control trajectory driven by the feedback $\mathfrak{u}$ satisfies the constraints.

Viability probability

Probability $\mathbb{P}$ on the set Ω of scenarios.

The **viability probability** associated to the initial time t_0, the initial state x_0 and the feedback $\mathfrak{u}$ is

$$\mathbb{P}\left[\Omega_{\mathfrak{u},t_0,x_0}\right] .$$

The **maximal viability probability** is

$$\Pi^\star(t_0, x_0) := \sup_{\mathfrak{u}} \mathbb{P}\left[\Omega_{\mathfrak{u},t_0,x_0}\right] .$$

An **optimal viable feedback** is a feedback $\mathfrak{u}^\star$ which **maximizes the probability of viable scenarios**:

$$\mathbb{P}\left[\Omega_{\mathfrak{u}^\star,t_0,x_0}\right] \geq \mathbb{P}\left[\Omega_{\mathfrak{u},t_0,x_0}\right] .$$

MSE AND VIABILITY APPROACH: COMPARISON AND EXTENSIONS

Management Modelling in the language of control theory

Management Modelling	Control Theory
uncertainties model	uncertainties model $\mathbb{W}$ scenarios model Ω scenarios probabilistic model $\mathbb{P}$
operating model	state control dynamical model G
observation model	observation or output model
assessment procedure	parameter/state estimation
Management Procedure (MP)	feedback or decision rule $\mathfrak{u}$

Table: Management Modelling in the language of control theory

Criterion (aggregation with respect to time)

A **criterion** π is a function

$$\begin{array}{rcl} \pi : \mathbb{X}^{T+1-t_0} \times \mathbb{U}^{T-t_0} \times \mathbb{W}^{T+1-t_0} & \to & \mathbb{R} \\ \big(x(\cdot), u(\cdot), w(\cdot)\big) & \mapsto & \pi\big(x(\cdot), u(\cdot), w(\cdot)\big) \end{array}$$

$\mathcal{I}\big(x(t), u(t), w(t)\big)$	single term
$\sum_{t=t_0}^{T-1} L\big(t, x(t), u(t), w(t)\big)$	aggregation with respect to time
$\prod_{t=t_0}^{T} \mathbf{1}_{\{x(t) \geq x^\flat\}} = \mathbf{1}_{\{\inf_{t=t_0,\ldots,T} x(t) \geq x^\flat\}}$	no trade-offs in time

Evaluation of the criterion

Given a decision rule $\mathfrak{u}$, the **evaluation of the criterion** is

$$\pi^{\mathfrak{u}}\big(t_0, x_0, w(\cdot)\big) := \pi\big(x_F[t_0, x_0, \mathfrak{u}, w(\cdot)](\cdot), u_F[t_0, x_0, \mathfrak{u}, w(\cdot)](\cdot), w(\cdot)\big) \ ,$$

that is, is the value of the criterion along the state and control trajectories $x_F[t_0, x_0, \mathfrak{u}, w(\cdot)](\cdot)$, $u_F[t_0, x_0, \mathfrak{u}, w(\cdot)](\cdot)$, driven by the feedback $\mathfrak{u}$.

Aggregation (with respect to scenarios)

Once evaluated, the criterion is **aggregated with respect to scenarios** in the form

$$\pi^{\mathfrak{u}}_{\mathbb{AG}}(t_0, x_0) := \mathbb{AG}_{w(\cdot)} \left[\pi^{\mathfrak{u}}\big(t_0, x_0, w(\cdot)\big) \right] ,$$

where the aggregation operator $\mathbb{AG}$ denotes

- either the **mean operator** $\mathbb{E}$ (mathematical expectation with respect to probability $\mathbb{P}$),
- or the $\inf_{w(\cdot)\in\Omega}$ operator (**worst case or robust** approach).

Management Strategy Evaluation in the language of control theory

Management Strategy Evaluation	Control Theory
summary performance statistics	criteria $\pi_1, \ldots, \pi_K$
testing MP by simulation	criteria evaluation $\pi_1^{\mathfrak{u}}(t_0, x_0, w(\cdot)), \ldots, \pi_K^{\mathfrak{u}}(t_0, x_0, w(\cdot))$ for various scenarios $w(\cdot)$
uncertainty management	aggregating with respect to scenarios $\pi_{1,\mathbb{AG}_1}^{\mathfrak{u}}(t_0, x_0), \ldots, \pi_{K,\mathbb{AG}_K}^{\mathfrak{u}}(t_0, x_0)$

Table: Management Strategy Evaluation in the language of control theory

Stochastic viability approach in the language of control theory

Stochastic Viability Approach	Control Theory
indicators and reference points	constraints $\mathcal{C}_1, \ldots, \mathcal{C}_K$
testing MP by simulation	constraints satisfaction evaluation $w(\cdot) \in \Omega_{\mathfrak{u},t_0,x_0}$ $\mathcal{C}_1 \geq 0, \ldots, \mathcal{C}_K \geq 0$ for various scenarios $w(\cdot)$ and for all times $t = t_0, \ldots, T$
uncertainty management	aggregating with respect to scenarios $\mathbb{P}\left[\Omega_{\mathfrak{u},t_0,x_0}\right]$

Table: Stochastic viability approach in the language of control theory

Trade-offs analysis and the viability approach

In the MSE framework, trade-offs are between aggregated statistics $\pi^{\mathfrak{u}}_{1,\mathbb{AG}_1}(t_0, x_0)$, ..., $\pi^{\mathfrak{u}}_{K,\mathbb{AG}_K}(t_0, x_0)$, and the trade-offs analysis is generally made visually.

In an economic perspective, this is as if an implicit (and visual) preference relation between aggregated statistics existed.

To the best, this relation may be materialized by indifference curves in the case of two statistics.

Stochastic viability

In the stochastic viability approach trade-offs are made explicit **before** aggregating with respect to scenarios which gives the viability probability. This latter is a common currency which allows to rank MPs.

$$\begin{aligned}\mathbb{P}\left[\Omega_{\mathfrak{u},t_0,x_0}\right] &= \mathbb{P}\big[\forall t = t_0, \ldots, T, \forall k = 1, \ldots, K, \\ &\quad \mathcal{I}_k\big(x(t), u(t), w(t)\big) \geq \iota_k\big] \\ &= \mathbb{E}\left[\prod_{t=t_0}^{T} \prod_{k=1}^{K} \mathbf{1}_{[\iota_k,+\infty[}\Big(\mathcal{I}_k\big(x(t), u(t), w(t)\big)\Big)\right]\end{aligned}$$

The multiplicative utility function $\prod_{t=t_0}^{T} \prod_{k=1}^{K} \mathbf{1}_{[\iota_k,+\infty[}(\mathcal{I}_k)$ takes the values 0 and 1, so that trade-offs between indicators are inexistent except at the thresholds where they are infinite.

Expected utility

$$\mathbb{E}\left[L\Big(\mathcal{I}_k\big(x(t),u(t),w(t)\big),\, t=t_0,\ldots,T,\, k=1,\ldots,K\Big)\right]$$

1. First, the utility function expresses trade-offs between
   - different indicators,
   - different time periods.
2. Second, mathematical expectation is taken, expressing aggregation with respect to scenarios.

CONCLUSION

A formal framework with methods to deal with

- trade-offs between different time periods (such as generations in the definition of sustainable development)
- trade-offs between conflicting issues (economic/ecological/social materialized by different indicators)
- uncertainties.

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Geromont, H. F., Oliveira, J. A. A. D., Johnston, S. J., and Cunningham, C. L. (1999). Development and application of management procedures for fisheries in Southern Africa. *ICES Jounal of Marine Science*, 56:952–966.

Oliveira, J. A. A. D. and Butterworth, D. S. (2004). Developing and refining a joint management proceduree for the multispecies South African pelagic fisheries. *ICES Jounal of Marine Science*, 61:1432–1442.

Sainsbury, K. J., Punt, A. E., and Smith, A. D. M. (2000). Design of operational management strategies for achieving fishery

Main result

Proposition

Assume that

- *the* **dynamics** G *is* **increasing with the state** x*, and* **decreasing with the control** u*;*
  [population dynamics with harvesting]
- *the* **indicator** $\mathcal{C}_1$ *is* **increasing with the state** x *and continuous in the control* u*, and is* **independent of the uncertainty** w*;*
  [economic indicator]
- *the* **indicators** $\mathcal{C}_2, \ldots, \mathcal{C}_K$ *are* **increasing with the state** x*, and* **decreasing with the control** u*.*
  [biological indicators]

Proposition

For those (t, x) *such that there exists* $u \in [u^\flat, u^\sharp]$ *satisfying* $\mathcal{C}_1(t, x, u) \geq 0$, *define the feedback* $\mathfrak{u}^\star$ *by*

$$\mathfrak{u}^\star(t, x) := \inf\{u \in [u^\flat, u^\sharp] \mid \mathcal{C}_1(t, x, u) \geq 0\} .$$

Then, $\mathfrak{u}^\star$ *is an optimal viable feedback maximizing* $\mathbb{P}\left[\Omega_{\mathfrak{u}, t_0, x_0}\right]$.