Usage Values in a Multinode System for Prospective Studies in Energy

Camila Martínez Parra

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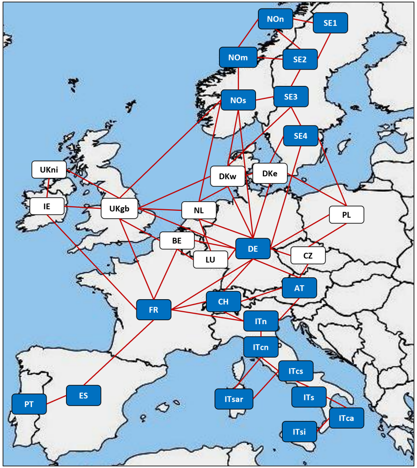
Cermics, ENPC

Supervisors:

Michel De Lara, Jean-Philippe Chancelier, Pierre Carpentier and Jean-Marc Janin

Context

Large scale prospective studies at RTE



- RTE is the French transmission system operator
- RTE conducts prospective studies at the European scale
- Increasing interest in usage value calculation for stored energy

Motivation

What do we understand by "prospective studies"? a picture of the future...

- Prospective studies are carried out by simulating the operation of an electricity system one year in the future
- In this simulation the electrical system is modelled as an oriented graph
  - the market zones are the nodes
  - the aggregated interconnections between market zones are the arcs

- Usage values are used to design policies for storage management in simulation

What are usage values?

- Usage values are the marginal value of stored energy at week s

$$\text{Usage Value} = -\frac{d}{d\ \text{storage level}}\text{cost-to-go}$$

- The cost-to-go or value of the storage, is the minimum cost that we can expect to achieve from week s to the end of the horizon, starting from a given storage level

Objective:

Cost-to-go functions and usage values that accounts for three dimensions of complexity

1. **Spatial:**
   - Multiple market zones (nodes) linked by representative interconnections (arcs)
   - The market zones are **coupled** by the **node/arcs balancing equations** (supply-demand balance in the whole system)
2. **Temporal:**
   - Two-timescale multistage optimisation problem that considers 52 weeks with 168 hours each
   - The hours and the weeks are **coupled** by the **storage dynamics equations** (storage energy balances)
3. **Stochastic:**
   - At each hour, at each node, the demand, the availability of dispatchable units and the inflows in storages are uncertain
   - **Information coupling:**
     what the DM knows at each stage of the multistage optimisation problem

Multinode multistage stochastic optimisation problem

$$\mathcal{J}_0\big(q_{(\underline{s},\underline{h})}\big) = \min_{\boldsymbol{\Phi},\boldsymbol{\Psi}} \sum_{n\in\mathcal{N}} \underbrace{\begin{array}{c}\text{expected nodal}\\ \text{intertemporal}\\ \text{production cost}\end{array}\big(q^n_{(\underline{s},\underline{h})}, \boldsymbol{\Phi}^n\big)}_{\begin{array}{l}\text{subject to}\\ \text{nodal storage dynamics}\end{array}} + \sum_{a\in\mathcal{A}} \begin{array}{c}\text{expected arc}\\ \text{intertemporal}\\ \text{transport cost}\end{array}\big(\boldsymbol{\Psi}^a\big)$$

subject to:

node/arcs balancing equations (Φ and Ψ)

information constraints over Φ and Ψ

$q_{(\underline{s},\underline{h})}$: initial storage levels
ϕ: import/export from/to nodes (n)
ψ: flow through arcs (a)

Spatial coupling
Temporal coupling
Stochastic coupling

Outline of the presentation

Part I:

Two-timescale information structures in a single-node system

temporal + stochastic

Part II:

Mixing temporal and spatial decomposition in a multi-node system

spatial + temporal + stochastic

Part I: Two-timescale information structures in a single-node system

temporal + stochastic

Article submitted:
martinezparra:hal-04681616, **martinezparra:hal-04681616**
C. Martinez Parra, M. De Lara, J.-P. Chancelier, P. Carpentier, J.-M. Janin, M. Ruiz

Multinode multistage stochastic optimisation problem

Part I

$$\mathcal{J}_0\big(q_{(\underline{s},\underline{h})}\big) = \min_{\boldsymbol{\Phi},\boldsymbol{\Psi}} \sum_{n\in\mathcal{N}} \underbrace{\text{expected nodal intertemporal production cost}\big(q^n_{(\underline{s},\underline{h})}, \boldsymbol{\Phi}^n\big)}_{\text{subject to nodal storage dynamics}} + \sum_{a\in\mathcal{A}} \text{expected arc intertemporal transport cost}\big(\boldsymbol{\Psi}^a\big)$$

subject to:

node/arcs balancing equations (Φ and Ψ)

information constraints over Φ and Ψ

$q_{(\underline{s},\underline{h})}$: initial storage levels
ϕ: import/export from/to nodes (n)
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Spatial coupling
Temporal coupling
Stochastic coupling

Outline

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Outline

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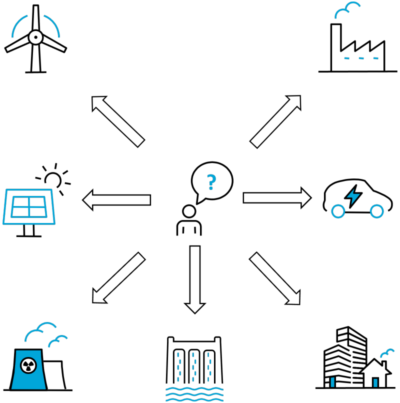
We focus on single-node system



We consider a single-node system composed of

- one storage unit (aggregated dam)
- dispatchable units

submitted to uncertainties

- demand
- inflows
- dispatchable unit's availability

Why two timescales?

- Hourly energy balance constraints
- Weekly temporal decomposition to compute cost-to-go functions
- We need a two-timescale approach to account for both

An overlook over the two-timescale timeline complexity

52 weeks $\quad \mathbb{S} \quad \underline{s} \prec \cdots \prec s^- \prec s \prec s^+ \prec \cdots \prec \overline{s}$

168 hours $\quad \mathbb{H} \quad \underline{h} \prec \cdots \prec h^- \prec h \prec h^+ \prec \cdots \prec \overline{h}$

Complete timeline $\overline{\mathbb{S} \times \mathbb{H}} = \mathbb{S} \times \mathbb{H} \cup \{(\overline{s}^+, \underline{h})\}$

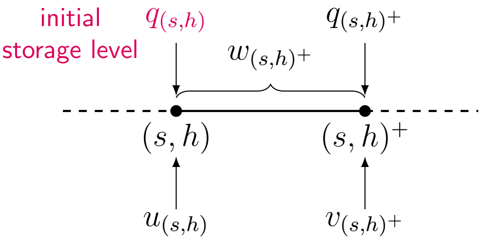
$(\underline{s}, \underline{h}) \quad (s, \underline{h}) \quad (s, \underline{h}^+) \quad (s, h)^- \quad (s, h) \quad (s, h)^+ \quad (s^+, \underline{h}) \quad (\overline{s}, \overline{h}) \quad (\overline{s}^+, \underline{h})$

Weekly vector definitions

$$\llbracket s \llbracket = \big((s, \underline{h}), (s, \underline{h})^+, \dots, (s, \overline{h})^-, (s, \overline{h})\big)$$
$$\rrbracket s \rrbracket = \big((s, \underline{h})^+, \dots, (s, \overline{h})^-, (s, \overline{h}), (s^+, \underline{h})\big)$$

Variables definition in the hourly interval

Level of stock

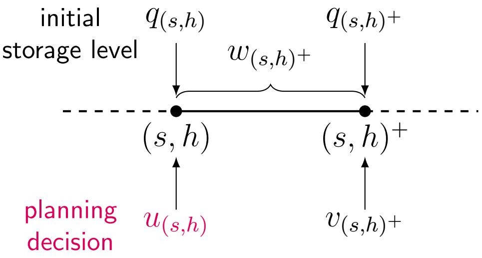


(hourly)

- *At the beginning of the hour, the planning (nonanticipative) decision* $u_{(s,h)}$ *is made*
- *The uncertainties* $w_{(s,h)^+}$ *(demand, inflows, availability) materialize during the hour*
- *Finally, the corrective (recourse) decision* $v_{(s,h)^+}$ *is made*

Variables definition in the hourly interval

Nonanticipative or planning controls



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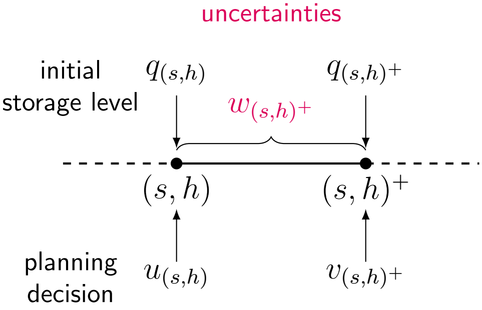
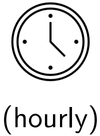
Variables definition in the hourly interval

Uncertain variables





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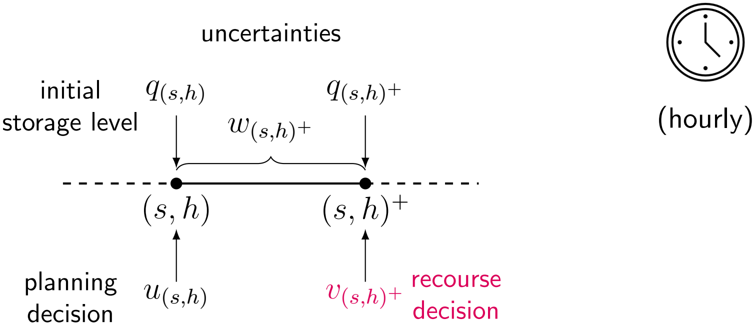
Variables definition in the hourly interval

Recourse controls



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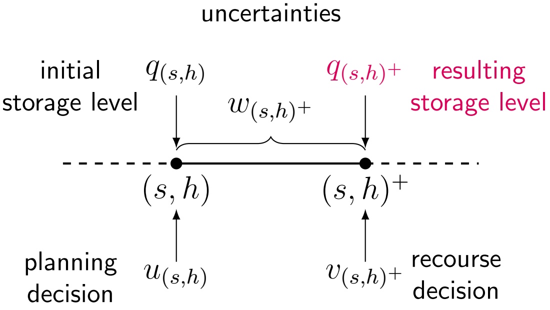
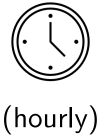
- *Finally, the corrective (recourse) decision* $v_{(s,h)^+}$ *is made*

Variables definition in the hourly interval





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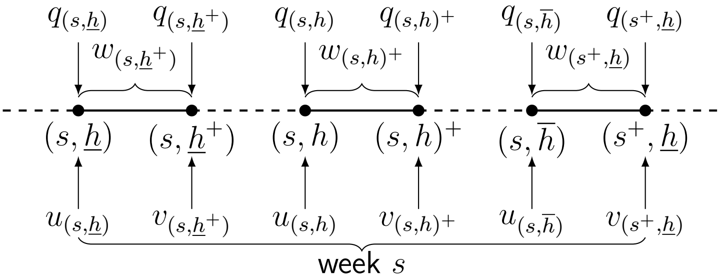
- *Finally, the corrective (recourse) decision* $v_{(s,h)^+}$ *is made*

Compact notation for weekly variables

For the week s: 1 week $=$ 168 hours

(weekly)



$$
\text{weekly}
\begin{cases}
\text{planning} & u_{\llbracket s \llbracket} = \big(u_{(s,\underline{h})}, u_{(s,\underline{h}^+)}, \dots, u_{(s,h)}, \dots, u_{(s,\overline{h})}\big) \\
\text{uncertainty} & w_{\rrbracket s \rrbracket} = \big(w_{(s,\underline{h}^+)}, \dots, w_{(s,h)^+}, \dots, w_{(s,\overline{h})}, w_{(s^+,\underline{h})}\big) \\
\text{recourse} & v_{\rrbracket s \rrbracket} = \big(v_{(s,\underline{h}^+)}, \dots, v_{(s,h)^+}, \dots, v_{(s,\overline{h})}, v_{(s^+,\underline{h})}\big)
\end{cases}
$$

Optimisation problem formulation with weekly variables

$$\min_{\mathbf{U},\mathbf{V}} \overbrace{\mathbb{E}\Big[\sum_{s\in\mathbb{S}} \underbrace{L_s(\mathbf{Q}_{(s,\underline{h})}, \mathbf{U}_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, \mathbf{V}_{]\!]s]\!]})}_{\text{weekly cost}} + \underbrace{K(\mathbf{Q}_{(\overline{s}^+,\underline{h})})}_{\text{final cost}}\Big]}^{\text{Expected intertemporal cost}}$$

subject to:

initial condition	$\mathbf{Q}_{(\underline{s},\underline{h})} = q_{(\underline{s},\underline{h})}$
storage dynamics	$\mathbf{Q}_{(s^+,\underline{h})} = f_s(\mathbf{Q}_{(s,\underline{h})}, \mathbf{U}_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, \mathbf{V}_{]\!]s]\!]})\ , \quad \forall s \in \mathbb{S}$
energy balance	$g_{[\![s[\![}(\mathbf{U}_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, \mathbf{V}_{]\!]s]\!]}) = 0\ , \quad \forall s \in \mathbb{S}$

Information constraints over planning decisions $\mathbf{U}_{[\![s[\![}$ and recourse decisions $\mathbf{V}_{]\!]s]\!]}$

From the optimisation problem to cost-to-go functions

schematic dynamic programming equations

Under suitable assumptions, cost-to-go functions can be computed recursively by temporal decomposition of the multistage stochastic optimisation problem

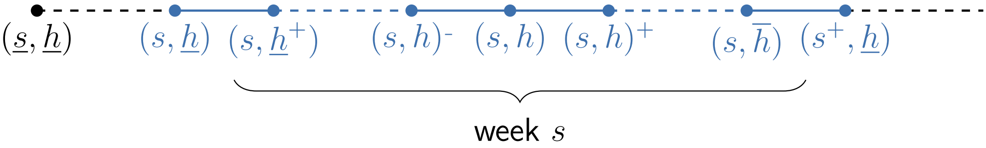
$$\text{cost-to-go}_s = \min_{\text{decisions}} \Big[\text{weekly cost}_s + \text{cost-to-go}_{s+} \Big]$$

They depend on the information constraints modelling

Outline

Information structures

Modelling what the decision maker knows
when making the decisions at week s



We are interested in
decision-hazard-decision
information structures

Decision-hazard-decision in the literature

in multistage stochastic optimisation problems

Studied in:

- **carpentier2023time**, **carpentier2023time**, **carpentier2023time**
- **street2020assessing**, **street2020assessing**, **street2020assessing**

Mentioned in:

- **Valladao-Silva-Poggi:2019**, **Valladao-Silva-Poggi:2019**, **Valladao-Silva-Poggi:2019**
- **dowson2020policy**, **dowson2020policy**, **dowson2020policy**

Most of these works are specifically tied to SDDP implementations

Contributions of this work

Exaustive analysis of two-timescale information structures:

- **Weekly hazard-decision**: current practice at RTE
- **Weekly decision-hazard-decision**: proposed alternative
- Other information structures not presented here: weekly decision-hazard and weekly decision-hazard-decision with hourly recourse

For each information structure:

- Mathematical formalization in a general framework
- Cost-to-go functions and dynamic programming equations

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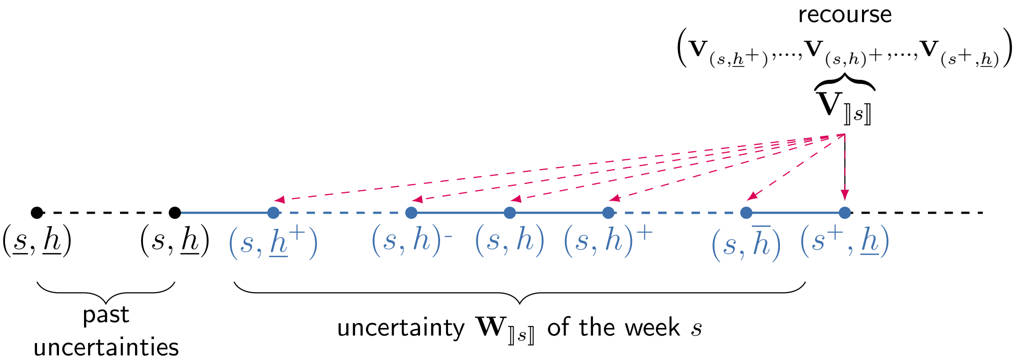
Weekly hazard-decision information structure

All the decisions are recourse decisions

The arrows from right to left represent ANTICIPATIVITY



The decision maker knows all the uncertainties for the week in advance, even the outages!

Cost-to-go functions and dynamic programming equations

for weekly hazard-decision information structure

- Dynamic programming state: $x = q_{(s,\underline{h})}$ (storage level at the begining of the week)
- Cost-to-go functions: B_s^{HD} for all $s \in \mathbb{S} \cup \{\overline{s}^+\}$

If the sequence $\big(\mathbf{W}_{\rrbracket \underline{s} \rrbracket}, \dots, \mathbf{W}_{\rrbracket s \rrbracket}, \dots, \mathbf{W}_{\rrbracket \overline{s} \rrbracket}\big)$ of uncertainties are (weekly) independent, the cost-to-go functions satisfy the following dynamic programming equations:

$$B_{\overline{s}^+}^{\mathsf{HD}}(x) = K(x)$$

$$\underbrace{B_s^{\mathsf{HD}}(x)}_{\substack{\text{cost-to-go} \\ \text{at week } s}} = \mathbb{E}\Bigg[\underbrace{\min_{v_{\rrbracket s \rrbracket}}}_{\substack{\text{anticipative} \\ \text{decisions}}}\Big\{\overbrace{L_s(x, \cancel{u_{\llbracket s \llbracket}}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket})}^{\text{weekly cost}} + \underbrace{B_{s^+}^{\mathsf{HD}}\big(\overbrace{f_s(x, \cancel{u_{\llbracket s \llbracket}}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket})}^{\text{weekly dynamics}}\big)}_{\substack{\text{cost-to-go} \\ \text{at week } s^+}}\Big\}\Bigg]$$

subject to $g_{\llbracket s \llbracket}(\cancel{u_{\llbracket s \llbracket}}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket}) = 0$

Where do we stand?

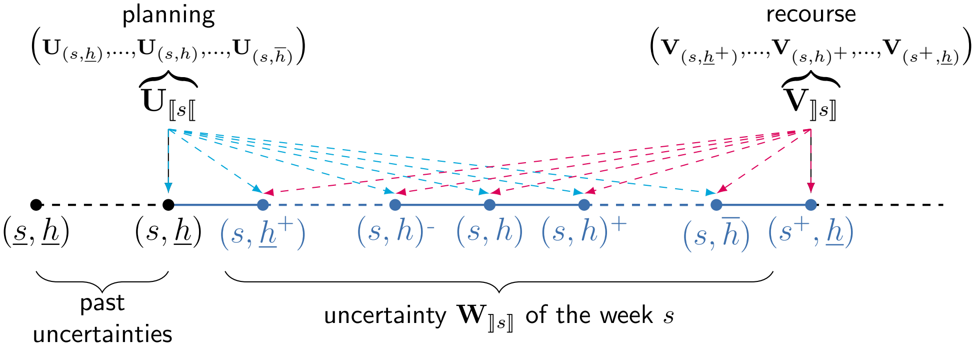
- The weekly hazard-decision structure assumes
  the weekly uncertainties are known in advance
  - Not bad when considering uncertainties with available accurate forecast
  - Delicate for the units outages
- A completely decision-hazard structure cannot guarantee feasibility
- This is why we turn to decision-hazard-decision

Outline

Weekly decision-hazard-decision

The arrows from left to right represent NONANTICIPATIVITY

The arrows from right to left represent ANTICIPATIVITY



Cost-to-go functions and dynamic programming equations

for weekly decision-hazard-decision information structure

Proposition

- *Dynamic programming state:* $x = q_{(s,\underline{h})}$ *(storage level at the begining of the week)*
- *Cost-to-go functions:* B_s^{DHD} *for all* $s \in \mathbb{S} \cup \{\overline{s}^+\}$

If the sequence $\left(\mathbf{W}_{\rrbracket \underline{s} \rrbracket}, \dots, \mathbf{W}_{\rrbracket s \rrbracket}, \dots, \mathbf{W}_{\rrbracket \overline{s} \rrbracket}\right)$ *of uncertainties are (weekly) independent, the cost-to-go functions satisfy the following dynamic programming equations:*

$$B_{\overline{s}^+}^{\mathsf{DHD}}(x) = K(x)$$

$$\underbrace{B_s^{\mathsf{DHD}}(x)}_{\substack{\text{cost-to-go} \\ \text{at week } s}} = \min_{u_{\llbracket s \llbracket}} \mathbb{E}\left[\min_{v_{\rrbracket s \rrbracket}} \left\{ \overbrace{L_s(x, u_{\llbracket s \llbracket}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket})}^{\text{weekly cost}} + \underbrace{B_{s^+}^{\mathsf{DHD}}\left(\overbrace{f_s(x, u_{\llbracket s \llbracket}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket})}^{\text{weekly dynamics}}\right)}_{\substack{\text{cost-to-go} \\ \text{at week } s^+}} \right\}\right]$$

$$\text{subject to } g_{\llbracket s \llbracket}(u_{\llbracket s \llbracket}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket}) = 0$$

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Cost-to-go functions and dynamic programming equations

for weekly decision-hazard-decision information structure

Proposition

- *Dynamic programming state:* $x = q_{(s,\underline{h})}$ *(storage level at the begining of the week)*
- *Cost-to-go functions:* B_s^{DHD} *for all* $s \in \mathbb{S} \cup \{\overline{s}^+\}$

If the sequence $\big(\mathbf{W}_{\rrbracket \underline{s} \rrbracket}, \dots, \mathbf{W}_{\rrbracket s \rrbracket}, \dots, \mathbf{W}_{\rrbracket \overline{s} \rrbracket}\big)$ *of uncertainties are (weekly) independent, the cost-to-go functions satisfy the following dynamic programming equations:*

$$B_{\overline{s}^+}^{\mathsf{DHD}}(x) = K(x)$$

$$\underbrace{B_s^{\mathsf{DHD}}(x)}_{\substack{\textit{cost-to-go} \\ \textit{at week } s}} = \underbrace{\min_{u_{\llbracket s \llbracket}}}_{\substack{\textit{nonanticipative} \\ \textit{decisions}}} \mathbb{E}\Big[\underbrace{\min_{v_{\rrbracket s \rrbracket}}}_{\substack{\textit{anticipative} \\ \textit{decisions}}} \Big\{ \overbrace{L_s(x, u_{\llbracket s \llbracket}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket})}^{\textit{weekly cost}} + \underbrace{B_{s^+}^{\mathsf{DHD}}\big(\overbrace{f_s(x, u_{\llbracket s \llbracket}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket})}^{\textit{weekly dynamics}}\big)}_{\substack{\textit{cost-to-go} \\ \textit{at week } s^+}} \Big\} \Big]$$

$$\textit{subject to } g_{\llbracket s \llbracket}(u_{\llbracket s \llbracket}, \mathbf{W}_{\rrbracket s \rrbracket}, v_{\rrbracket s \rrbracket}) = 0$$

Comparing dynamic programming equations

Hazard-Decision:

$$\underbrace{B_s^{\mathsf{HD}}(x)}_{\substack{\text{cost-to-go}\\ \text{at week } s}} = \mathbb{E}\Big[\underbrace{\min_{v_{]\!]s]\!]}}_{\substack{\text{anticipative}\\ \text{decisions}}} \Big\{ \overbrace{L_s(x, \cancel{u_{[\![s[\![}}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]})}^{\text{weekly cost}} + \underbrace{B_{s^+}^{\mathsf{HD}}\big(\overbrace{f_s(x, \cancel{u_{[\![s[\![}}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]})}^{\text{weekly dynamics}}\big)}_{\substack{\text{cost-to-go}\\ \text{at week } s^+}} \Big\}\Big]$$

$$\text{subject to } g_{[\![s[\![}(\cancel{u_{[\![s[\![}}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]}) = 0$$

Decision-Hazard-Decision:

$$\underbrace{B_s^{\mathsf{DHD}}(x)}_{\substack{\text{cost-to-go}\\ \text{at week } s}} = \underbrace{\min_{u_{[\![s[\![}}}_{\substack{\text{nonanticipative}\\ \text{decisions}}} \mathbb{E}\Big[\underbrace{\min_{v_{]\!]s]\!]}}_{\substack{\text{anticipative}\\ \text{decisions}}} \Big\{ \overbrace{L_s(x, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]})}^{\text{weekly cost}} + \underbrace{B_{s^+}^{\mathsf{DHD}}\big(\overbrace{f_s(x, u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]})}^{\text{weekly dynamics}}\big)}_{\substack{\text{cost-to-go}\\ \text{at week } s^+}} \Big\}\Big]$$

$$\text{subject to } g_{[\![s[\![}(u_{[\![s[\![}, \mathbf{W}_{]\!]s]\!]}, v_{]\!]s]\!]}) = 0$$

we know

$$B_s^{\mathsf{HD}}(x) \quad \leq \quad B_s^{\mathsf{DHD}}(x)$$

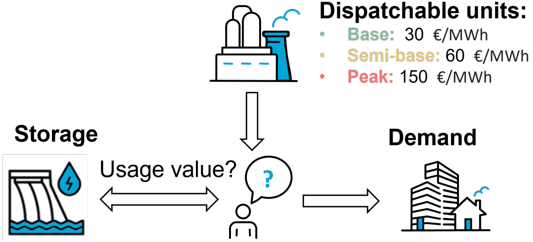
we do not know

$$\mathsf{UV}_s^{\mathsf{HD}}(x) \quad \gtreqless \quad \mathsf{UV}_s^{\mathsf{DHD}}(x)$$

Outline

Numerical results for a small case study

Computing cost-to-go functions using weekly marginal distributions of uncertainties



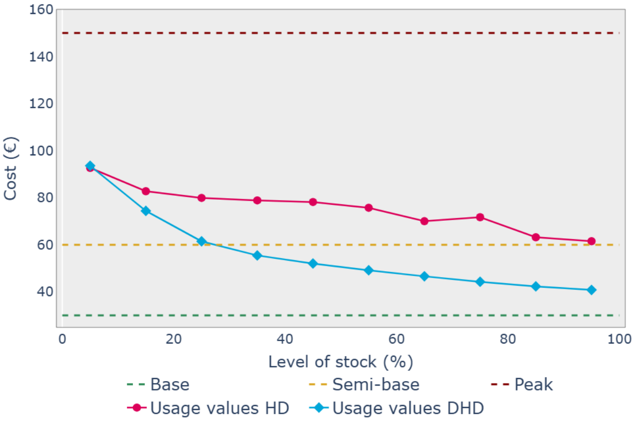
Unit	Physical decision	HD model	DHD model
base	on/off	✓	✓
	modulation	✓	✓
semi-base	on/off	✓	✓
	modulation	✓	✓
peak	on/off	✓	✓
	modulation	✓	✓
storage	charge	✓	✓
	discharge	✓	✓

✓ recourse ✓ planning

- In a HD model, all decisions are recourse decisions (✓)
- In a DHD model, decisions are split into planning (✓) and recourse decisions (✓)

Usage values comparison and merit order effects

Usage values and production costs at week 20:



Merit order comparison:

HD	**DHD**
base unit	base unit
semi-base unit	**storage**
storage	semi-base unit
peak unit	peak unit

Different merit orders, then

- different energy allocation
- different storage trajectories

Outline

Conclusions

- The current approach simplifies the information structure by being anticipative within the week
- A decision-hazard information structure cannot guarantee the balance equality
- As a compromise, we propose a decision-hazard-decision information structure:
  - we wrote dynamic programming equations
  - we performed numerical experiments on a small case study

Part II: mixing temporal and spatial decomposition

spatial + temporal + stochastic

Article submitted:
martinezparra:DADP, **martinezparra:DADP**

C. Martinez Parra, M. De Lara, J.-P. Chancelier, P. Carpentier, J.-M. Janin

Multinode multistage stochastic optimisation problem

objective: temporal decomposition to compute cost-to-go functions

$$\mathcal{J}_0\big(q_{(\underline{s},\underline{h})}\big) = \min_{\mathbf{\Phi},\mathbf{\Psi}} \sum_{n\in\mathcal{N}} \underbrace{\text{expected nodal intertemporal production cost}\,\big(q^n_{(\underline{s},\underline{h})}, \mathbf{\Phi}^n\big)}_{\substack{\text{subject to:}\\ \text{nodal storage dynamics}}} + \sum_{a\in\mathcal{A}} \text{expected arc intertemporal transport cost}\big(\mathbf{\Psi}^a\big)$$

subject to:

node/arcs balancing equations (Φ and Ψ)

information constraints over Φ and Ψ

$q_{(\underline{s},\underline{h})}$: storage levels at the initial time
ϕ: import/export from/to nodes (n)
ψ: flow through arcs (a)

Spatial coupling
Temporal coupling
Stochastic coupling

Our approach to tackle the three axes of complexity

Spatial + Temporal + Stochastic

$$\Downarrow$$

Spatial decomposition + Stochastic Dynamic Programming

Dual Approximate Dynamic Programming (DADP)[1]

[1] pacaud2021distributed

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Lower approximation of global cost-to-go functions

- The complex multivariate global cost-to-go functions are approximated by a sum of **univariate nodal cost-to-go functions** through price decomposition:

$$\sum_{n \in \mathcal{N}} \begin{array}{c}\text{univariate}\\\text{nodal}\\\text{cost-to-go}\end{array}[p]\big(x^n\big) + \begin{array}{c}\text{transport}\\\text{term}\end{array}[p] \leq \begin{array}{c}\text{multivariate}\\\text{global}\\\text{cost-to-go}\end{array}\Big(\big(x^n\big)_{n \in \mathcal{N}}\Big)$$

- The main advantage of DADP is that we avoid the curse of dimensionality
  - **nodal cost-to-go functions** are univariate functions of the storage level at the node
  - they can be independently computed as in Part I using **stochastic dynamic programming**

DADP in the literature

- **Girardeau:2010**, **Girardeau:2010** , **Girardeau:2010**
- **Barty˙RAIRO˙2010**, **Barty˙RAIRO˙2010** , **Barty˙RAIRO˙2010**
- **leclere:tel-01148466**, **leclere:tel-01148466** , **leclere:tel-01148466**
- **pacaud:tel-02134163**, **pacaud:tel-02134163** , **pacaud:tel-02134163**

The method is suitable for weakly coupled systems, as the ones considered in prospectives studies at RTE

Contributions of this work

- Extension of DADP to two-timescale multistage stochastic optimisation problems, accounting for different information structures within the nodal problems
- A Bellman-like algorithm to generate proxies of the gradients[2] for use in a gradient-based method to compute the price decomposition process p
- Practical guidelines for selecting the price process p, including price-aggregation schemes
- Numerical evaluation of the proposed DADP algorithm on multi-node systems

[2] **franc2023differentiabilityregularizationparametricconvex**

Outline

2. Mixing temporal and spatial decomposition

2.1 Spatial decomposition with prices

2.2 How to improve the price decomposition process?

2.3 Numerical results for DADP

2.4 Conclusions and perspectives

Outline

Multinode multistage stochastic optimisation problem

objective: temporal decomposition to compute cost-to-go functions

$$\mathcal{J}_0 = \min_{\phi,\psi} \left\{ \underbrace{\sum_{n\in\mathcal{N}} \left(\min_{u^n, v^n} \left\{ \begin{array}{l} \mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^n \left(q^n_{(s,\underline{h})}, u^n_{\rrbracket s \rrbracket}, w^n_{\rrbracket s \rrbracket}, v^n_{\rrbracket s \rrbracket}, \phi^n_{\rrbracket s \rrbracket} \right) \right) \right] \\ \text{s.t.: hourly nodal energy balances} \\ \quad\;\; \text{nodal storage dynamics} \\ \quad\;\; \text{information constraints over } u^n_{\llbracket s \llbracket} \text{ and } v^n_{\rrbracket s \rrbracket} \end{array} \right\} \right)}_{\text{expected nodal intertemporal production cost}} + \underbrace{\sum_{a\in\mathcal{A}} \left(\mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^a \left(\psi^a_{\rrbracket s \rrbracket} \right) \right) \right] \text{(quadratic cost)} \right)}_{\text{expected arc intertemporal transport cost}} \right\}$$

subject to, for all week $s \in \mathbb{S}$:

$$\phi_{\rrbracket s \rrbracket} - \mathbb{A}\psi_{\rrbracket s \rrbracket} = 0$$

- one balance equation for each node, hour, scenario

information constraints over $\phi_{\rrbracket s \rrbracket}$ and $\psi_{\rrbracket s \rrbracket}$

$q_{(\underline{s},\underline{h})}$: storage levels at the initial time
ϕ: import/export from/to nodes
ψ: flow through arcs

Spatial coupling
Temporal coupling
Stochastic coupling

Le réseau de transport d'électricité

École nationale des Ponts et Chaussées

Institut Polytechnique de Paris

Spatial decomposition:

lower bound of the original problem

Lower bound by Lagrangian of relaxation of the spatial coupling

$$\underline{\mathcal{J}_0}[p] = \min_{\phi,\psi} \left\{ \sum_{n\in\mathcal{N}} \left(\min_{u^n, v^n} \left\{ \begin{array}{l} \mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^n\big(q^n_{(s,\underline{h})}, u^n_{]\!]s]\!]}, w^n_{]\!]s]\!]}, v^n_{]\!]s]\!]}, \phi^n_{]\!]s]\!]}\big) \right) \right] \\ \text{s.t.: hourly nodal energy balances} \\ \quad \text{nodal storage dynamics} \\ \quad \text{information constraints over } u^n_{[\![s[\![} \text{ and } v^n_{]\!]s]\!]} \end{array} \right\} \right) + \sum_{a\in\mathcal{A}} \left(\mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^a\big(\psi^a_{]\!]s]\!]}\big) \right) \right] \right) \right\}$$

subject to:

$$\phi_{]\!]s]\!]} - \mathbb{A}\psi_{]\!]s]\!]} = 0$$

information constraints over $\phi_{]\!]s]\!]}$ and $\psi_{]\!]s]\!]}$

Spatial coupling
Temporal coupling
Stochastic coupling

Spatial decomposition:
lower bound of the original problem

Lower bound by Lagrangian of relaxation of the spatial coupling

$$\underline{\mathcal{J}_0}[p] = \min_{\phi,\psi} \left\{ \sum_{n\in\mathcal{N}} \left(\min_{u^n,v^n} \left\{ \begin{array}{l} \mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^n\big(q^n_{(s,\underline{h})}, u^n_{]\!]s]\!]}, w^n_{]\!]s]\!]}, v^n_{]\!]s]\!]}, \phi^n_{]\!]s]\!]}\big) \right) \right] \\ \text{s.t.: hourly nodal energy balances} \\ \qquad \text{nodal storage dynamics} \\ \qquad \text{information constraints over } u^n_{[\![s[\![} \text{ and } v^n_{]\!]s]\!]} \end{array} \right\} \right) + \sum_{a\in\mathcal{A}} \left(\mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^a\big(\psi^a_{]\!]s]\!]}\big) \right) \right] \right) \\ + \underbrace{\mathbb{E}\left[\sum_{s\in\mathbb{S}} \left\langle p_s, \ \phi_{]\!]s]\!]} - \mathbb{A}\psi_{]\!]s]\!]} \right\rangle \right]}_{\text{Spatial coupling relaxation}} \right\}$$

subject to:

$$\cancel{\phi_{]\!]s]\!]} - \mathbb{A}\psi_{]\!]s]\!]} = 0}$$

information constraints over $\phi_{]\!]s]\!]}$ and $\psi_{]\!]s]\!]}$

Spatial coupling
Temporal coupling
Stochastic coupling

Spatial decomposition:

lower bound of the original problem

Lower bound by Lagrangian of relaxation of the spatial coupling

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subject to:

$\cancel{\phi_{]\!]s]\!]} - \mathbb{A}\psi_{]\!]s]\!]} = 0}$

information constraints over $\phi_{]\!]s]\!]}$ and $\psi_{]\!]s]\!]}$

Spatial coupling relaxation

deterministic price decomposition process

(multiplier)

size: $52 \times 168 \times |\mathcal{N}|$

3 nodes: 26 208

Spatial coupling
Temporal coupling
Stochastic coupling

Spatial decomposition:

lower bound of the original problem

Lower bound by Lagrangian of relaxation of the spatial coupling

$$\underline{\mathcal{J}_0}[p] = \min_{\phi,\psi} \Bigg\{ \sum_{n\in\mathcal{N}} \Bigg(\min_{u^n, v^n} \Bigg\{ \mathbb{E}\Big[\sum_{s\in\mathbb{S}} \Big(L_s^n\big(q^n_{(s,\underline{h})}, u^n_{\rrbracket s \rrbracket}, w^n_{\rrbracket s \rrbracket}, v^n_{\rrbracket s \rrbracket}, \phi^n_{\rrbracket s \rrbracket}\big)\Big)\Big] \Bigg\} \Bigg) + \sum_{a\in\mathcal{A}} \Bigg(\mathbb{E}\Big[\sum_{s\in\mathbb{S}} \Big(L_s^a\big(\psi^a_{\rrbracket s \rrbracket}\big)\Big)\Big] \Bigg)$$

s.t.: hourly nodal energy balances
nodal storage dynamics
information constraints over $u^n_{\rrbracket s \rrbracket}$ and $v^n_{\rrbracket s \rrbracket}$

$$+ \underbrace{\mathbb{E}\Big[\sum_{s\in\mathbb{S}} \langle p_s, \phi_{\rrbracket s \rrbracket} - A\psi_{\rrbracket s \rrbracket} \rangle \Big]}_{\text{Spatial coupling relaxation}} \Bigg\}$$

subject to:

$\cancel{\phi_{\rrbracket s \rrbracket} - A\psi_{\rrbracket s \rrbracket} = 0}$

information constraints over $\phi_{\rrbracket s \rrbracket}$ and $\psi_{\rrbracket s \rrbracket}$

Spatial coupling relaxation
deterministic price
decomposition process
(multiplier)
size: $52 \times 168 \times |\mathcal{N}|$
3 nodes: 26 208

Spatial coupling
Temporal coupling
Stochastic coupling

Spatial decomposition + stochastic dynamic programming?

$$
\underline{\mathcal{J}_0}[p] = \left\{
\begin{array}{c}
\displaystyle\sum_{n\in\mathcal{N}} \left(\min_{u^n, v^n, \phi^n} \left\{
\begin{array}{l}
\mathbb{E}\left[\displaystyle\sum_{s\in\mathbb{S}} \left(L_s^n\big(q^n_{(s,\underline{h})}, u^n_{\rrbracket s\rrbracket}, w^n_{\rrbracket s\rrbracket}, v^n_{\rrbracket s\rrbracket}, \phi^n_{\rrbracket s\rrbracket}\big) + \langle p_s^n, \phi^n_{\rrbracket s\rrbracket} \rangle \right) \right] \\
\text{s.t.: hourly nodal energy balances} \\
\qquad \text{nodal storage dynamics} \\
\qquad \text{information constraints over } u^n_{\llbracket s\llbracket}, v^n_{\rrbracket s\rrbracket} \text{ and } \phi^n_{\rrbracket s\rrbracket}
\end{array}
\right\} \right) \\
+ \\
\displaystyle\min_{\psi} \sum_{a\in\mathcal{A}} \left(\mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^a\big(\psi^a_{\rrbracket s\rrbracket}\big) - p_s \mathbb{A}\psi_{\rrbracket s\rrbracket} \right) \right] \right) \\
\text{s.t.: information constraints over } \psi_{\rrbracket s\rrbracket}
\end{array}
\right.
$$

Spatial coupling
Temporal coupling
Stochastic coupling

Spatial decomposition + stochastic dynamic programming?

same type of problems as Part I

$$\underline{\mathcal{J}}_0[p] = \left\{ \begin{array}{c} \sum_{n\in\mathcal{N}} \left(\min_{u^n, v^n, \phi^n} \left\{ \begin{array}{l} \mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^n(q^n_{(s,\underline{h})}, u^n_{]\!]s]\!]}, w^n_{]\!]s]\!]}, v^n_{]\!]s]\!]}, \phi^n_{]\!]s]\!]}) + \langle p^n_s, \phi^n_{]\!]s]\!]} \rangle \right) \right] \\ \text{s.t.: hourly nodal energy balances} \\ \quad\quad \text{nodal storage dynamics} \\ \quad\quad \text{information constraints over } u^n_{[\![s[\![}, v^n_{]\!]s]\!]} \text{ and } \phi^n_{]\!]s]\!]} \end{array} \right\} \right) \\ + \\ \min_{\psi} \sum_{a\in\mathcal{A}} \left(\mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L^a_s(\psi^a_{]\!]s]\!]}) - p_s \mathbb{A} \psi_{]\!]s]\!]} \right) \right] \right) \\ \text{s.t.: information constraints over } \psi_{]\!]s]\!]} \end{array} \right.$$

$\Longrightarrow$

Nodal price value functions

For a given p^n, temporal decomposition using one-dimensional SDP

$\Downarrow$

univariate nodal price cost-to-go functions $B^n_s[p^n](x^n)$

Spatial coupling
Temporal coupling
Stochastic coupling

Spatial decomposition + stochastic dynamic programming?

same type of problems as Part I

$$\underline{\mathcal{J}_0}[p] = \left\{ \sum_{n\in\mathcal{N}} \left(\min_{u^n,v^n,\phi^n} \left\{ \begin{array}{l} \mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^n(q^n_{(s,\underline{h})}, u^n_{]\!]s]\!]}, w^n_{]\!]s]\!]}, v^n_{]\!]s]\!]}, \phi^n_{]\!]s]\!]}) + \langle p^n_s, \phi^n_{]\!]s]\!]} \rangle \right) \right] \\ \text{s.t.: hourly nodal energy balances} \\ \quad\quad \text{nodal storage dynamics} \\ \quad\quad \text{information constraints over } u^n_{[\![s[\![}, v^n_{]\!]s]\!]} \text{ and } \phi^n_{]\!]s]\!]} \end{array} \right\} \right) + \begin{array}{l} \min_{\psi} \sum_{a\in\mathcal{A}} \left(\mathbb{E}\left[\sum_{s\in\mathbb{S}} \left(L_s^a(\psi^a_{]\!]s]\!]}) - p_s \mathbb{A}\psi_{]\!]s]\!]} \right) \right] \right) \\ \text{s.t.: information constraints over } \psi_{]\!]s]\!]} \end{array} \right\}$$

$\Rightarrow$ Nodal price value functions

For a given p^n, temporal decomposition using one-dimensional SDP

$\Downarrow$

univariate nodal price cost-to-go functions $B_s^n[p^n](x^n)$

$\Downarrow$

Arc price value functions

Easy to solve week by week without temporal coupling

Spatial coupling
Temporal coupling
Stochastic coupling

Relation between nodal and global cost-to-go functions

Proposition

For any deterministic price decomposition process p, *if the sequence* $\left(\mathbf{W}_{\rrbracket \underline{s} \rrbracket}, \dots, \mathbf{W}_{\rrbracket s \rrbracket}, \dots, \mathbf{W}_{\rrbracket \overline{s} \rrbracket}\right)$ *of uncertainties is weekly independent, then for all* $s \in \mathbb{S}$,

$$\sum_{n \in \mathcal{N}} \underbrace{B^n_s[p^n]\big(x^n\big)}_{\substack{\text{univariate}\\ \text{nodal price}\\ \text{cost-to-go}}} + \underbrace{B^{\mathcal{A}}_s[p]}_{\substack{\text{arc price}\\ \text{cost-to-go}}} \leq \underbrace{B_s\Big(\big(x^n\big)_{n \in \mathcal{N}}\Big)}_{\substack{\text{multivariate}\\ \text{global}\\ \text{cost-to-go}}}$$

Relation between nodal and global cost-to-go functions

Proposition

For any deterministic price decomposition process p, *if the sequence* $\big(\mathbf{W}_{\rrbracket \underline{s} \rrbracket}, \dots, \mathbf{W}_{\rrbracket s \rrbracket}, \dots, \mathbf{W}_{\rrbracket \overline{s} \rrbracket}\big)$ *of uncertainties is weekly independent, then for all* $s \in \mathbb{S}$,

$$\uparrow \sum_{n \in \mathcal{N}} \underbrace{B_s^n[p^n]\big(x^n\big)}_{\substack{\text{univariate} \\ \text{nodal price} \\ \text{cost-to-go}}} + \underbrace{B_s^{\mathcal{A}}[p]}_{\substack{\text{arc price} \\ \text{cost-to-go}}} \leq \underbrace{B_s\big((x^n)_{n \in \mathcal{N}}\big)}_{\substack{\text{multivariate} \\ \text{global} \\ \text{cost-to-go}}}$$

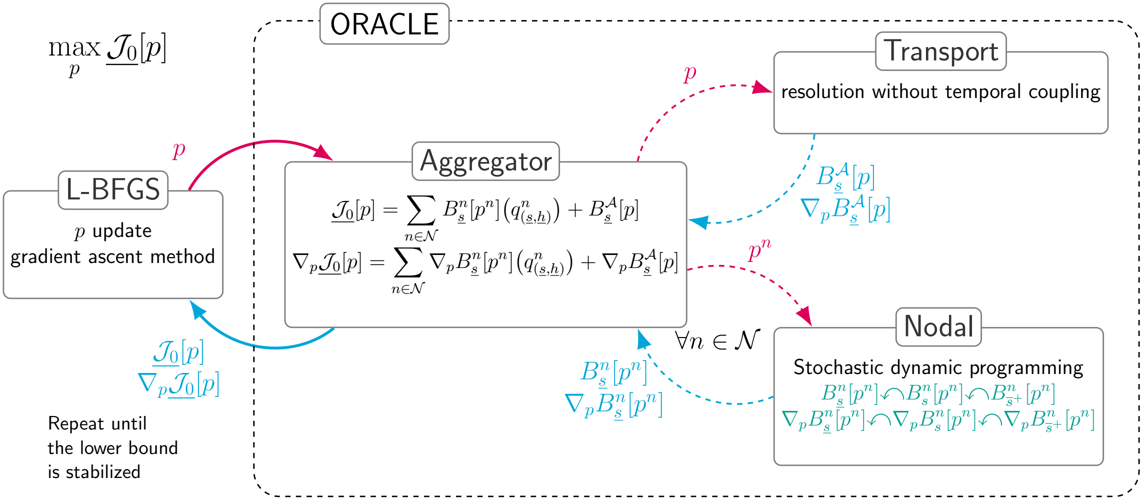
We look for a value of p to improve the global cost-to-go functions approximation

Outline

Outline

Find a value of p that improves the lower bound $\underline{\mathcal{J}_0}[p] \le \mathcal{J}_0$

dual problem for a deterministic decomposition price



Find a value of p that improves the lower bound $\underline{\mathcal{J}_0}[p] \leq \mathcal{J}_0$

dual problem for a deterministic decomposition price

$$\max_p \underline{\mathcal{J}_0}[p]$$

L-BFGS

p update
gradient ascent method

p

$\underline{\mathcal{J}_0}[p]$
$\nabla_p \underline{\mathcal{J}_0}[p]$

Repeat until
the lower bound
is stabilized

ORACLE

Aggregator

$$\underline{\mathcal{J}_0}[p] = \sum_{n \in \mathcal{N}} B^n_{\underline{s}}[p^n]\big(q^n_{(\underline{s},\underline{h})}\big) + B^{\mathcal{A}}_{\underline{s}}[p]$$

$$\nabla_p \underline{\mathcal{J}_0}[p] = \sum_{n \in \mathcal{N}} \nabla_p B^n_{\underline{s}}[p^n]\big(q^n_{(\underline{s},\underline{h})}\big) + \nabla_p B^{\mathcal{A}}_{\underline{s}}[p]$$

p

Transport

resolution without temporal coupling

$B^{\mathcal{A}}_{\underline{s}}[p]$
$\nabla_p B^{\mathcal{A}}_{\underline{s}}[p]$

p^n

$\forall n \in \mathcal{N}$

Nodal

Stochastic dynamic programming

$B^n_{\underline{s}}[p^n] \curvearrowleft B^n_s[p^n] \curvearrowleft B^n_{\overline{s}+}[p^n]$
$\nabla_p B^n_{\underline{s}}[p^n] \curvearrowleft \nabla_p B^n_s[p^n] \curvearrowleft \nabla_p B^n_{\overline{s}+}[p^n]$

$B^n_{\underline{s}}[p^n]$
$\nabla_p B^n_{\underline{s}}[p^n]$

Focus on the nodal problems

for a given p^n, using weekly marginal distributions of uncertainties

Nodal price value $B^n_{\underline{s}}[p^n]\big(q^n_{(\underline{s},\underline{h})}\big)$ by SDP as in Part I with $x^n_s = q^n_{(s,\underline{h})}$:

$$B^n_s[p^n]\big(x^n_s\big) = \min \mathbb{E}\Big[\min\Big\{L^n_s(x^n_s, u^n_{[\![s[\![}, w_{]\!]s]\!]}, v^n_{]\!]s]\!]}, \phi^n_{]\!]s]\!]}) + p^n_s \phi^n_{]\!]s]\!]} + B^n_{s+}[p^n]\big(x^n_{s+}\big)\Big\}\Big]$$

[3] franc2023differentiabilityregularizationparametricconvex

Focus on the nodal problems

for a given p^n, using weekly marginal distributions of uncertainties

Nodal price value $B^n_{\underline{s}}[p^n]\big(q^n_{(\underline{s},\underline{h})}\big)$ by SDP as in Part I with $x^n_s = q^n_{(s,\underline{h})}$:

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Nodal price gradient proxy $\nabla_p B^n_{\underline{s}}[p^n]\big(q^n_{(\underline{s},\underline{h})}\big)$ by backward recursion alongside SDP[3]:

$$\nabla_p B^n_s[p^n]\big(x^n_s\big) = \mathbb{E}\Big[\nabla_p\Big(L^n_s(x^n_s, {u^n_{[\![s[\![}}^*, w_{]\!]s]\!]}, {v^n_{]\!]s]\!]}}^*, {\phi^n_{]\!]s]\!]}}^*) + p^n_s {\phi^n_{]\!]s]\!]}}^*\Big) + \nabla_p B^n_{s+}[p^n]\big({x^n_{s+}}^*\big)\Big]$$

with ${u^n_{[\![s[\![}}^*$, ${v^n_{]\!]s]\!]}}^*$, ${\phi^n_{]\!]s]\!]}}^*$ and ${x^n_{s+}}^*$ $\in \arg\min$ of the decomposed problem at week s

[3] **franc2023differentiabilityregularizationparametricconvex**

Outline

Simplifying the deterministic price decomposition process p

The size of p is a challenge

We propose three possibilities to model the deterministic price decomposition process p:

- **Hourly**: p of size 8736 at each node
  - one coupling constraint each hour
- **In 8-hour blocks**: p of size 1092 at each node
  - one aggregated spatial coupling constraint each 8 hours
- **In weekly blocks**: p of size 52 at each node
  - one aggregated spatial coupling constraint each week

Outline

We consider three different case studies

Computing cost-to-go functions

Case study	Cost-to-go functions using				
	DADP			SDP	SDDP
	hourly prices	8-hour prices	weekly prices		
3 nodes HD	✓	✓	✓	✓	✓
3 nodes DHD	✓	✓	✓	✓	
30 nodes HD	✓	✓	✓		✓

We consider three different case studies

Computing cost-to-go functions

Case study	Cost-to-go functions using				
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- In all cases, we consider continuous decision variables and linear costs

We consider three different case studies

Computing cost-to-go functions

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30 nodes HD	✓	✓	✓		✓

- In all cases, we consider continuous decision variables and linear costs
- Weekly marginal probability distributions of uncertainties

$$\widehat{\mathbb{P}} = \bigotimes_{s \in \mathbb{S}} \frac{1}{|\mathcal{C}_\beta|} \sum_{c \in \mathcal{C}_\beta} \delta_{(w_{\rrbracket s \rrbracket})^c} \qquad \widehat{\mathbb{E}}[\zeta(\mathbf{W}_{\rrbracket s \rrbracket})] = \frac{1}{|\mathcal{C}_\beta|} \sum_{c \in \mathcal{C}_\beta} \zeta\big((w_{\rrbracket s \rrbracket})^c\big) \qquad |\mathcal{C}_\beta| = 10$$

Acronyms

- SDP: Stochastic Dynamic Programming
- SDDP: Stochastic Dual Dynamic Programming
- DADP: Dual Approximate Dynamic Programming

Outline

Comparing algorithms with HD information structure

- Algorithms comparison
  (Cost-to-go computation phase with the weekly marginal distribution $\widehat{\mathbb{P}}$)
  - ✓ Computing times
  - ✓ Lower bounds
  - ✓ Statistical upper bounds (by simulation in scenarios sampled with $\widehat{\mathbb{P}}$)
  - ✓ Relative statistical gap

Algorithms comparison: cost-to-go computation

3-node system, HD information structure

Method (HD)	Time (h)	Lower Bound (B€)	Statistical upper bound (B€)	Relative statistical gap
SDP	42.74	20.863	20.847 ± 0.042	-0.08%
SDDP	4.57	19.813	20.876 ± 0.041	5.37%
DADP hourly prices	1.92	19.987	20.861 ± 0.042	4.37%
DADP 8-hour prices	2.03	19.959	20.867 ± 0.042	4.55%
DADP weekly prices	3.42	19.906	21.060 ± 0.053	5.80%

Table: Computing times, lower bounds and statistical upper bounds

- The tight difference between SDP bounds suggests it is a good benchmark
- DADP is faster than SDP and SDDP
- DADP is comparable to SDDP in terms of statistical gap

Algorithms comparison: cost-to-go computation

3-node system, HD information structure

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- DADP is comparable to SDDP in terms of statistical gap

Comparing algorithms with DHD information structure

- Algorithms comparison
  (Cost-to-go computation phase with the weekly marginal distribution $\widehat{\mathbb{P}}$)
  ✓ Computing times
  ✓ Lower bounds
  ✓ Statistical upper bounds (by simulation in scenarios sampled with $\widehat{\mathbb{P}}$)
  ✓ Relative statistical gap

Algorithms comparison: cost-to-go computation

3-node system, DHD information structure

Method (DHD)	Time (h)	Lower Bound (B€)	Statistical upper bound (B€)	Relative statistical gap
SDP	118.5	21.680	21.665 ± 0.042	-0.07%
SDDP	–	–	–	–
DADP hourly prices	1.86	20.978	21.802 ± 0.043	3.93%
DADP 8-hour prices	1.95	20.919	21.805 ± 0.043	4.24%
DADP weekly prices	2.44	20.988	21.866 ± 0.044	4.18%

Table: Computing times, lower bounds and statistical upper bounds

- The tight difference on SDP bounds suggests it is a good benchmark
- DADP obtains satisfactory lower bounds and statistical gap in considerably shorter computing times than SDP

Algorithms comparison: cost-to-go computation

3-node system, DHD information structure

Method (DHD)	Time (h)	Lower Bound (B€)	Statistical upper bound (B€)	Relative statistical gap
SDP	118.5	21.680	21.665 ± 0.042	-0.07%
SDDP	–	–	–	–
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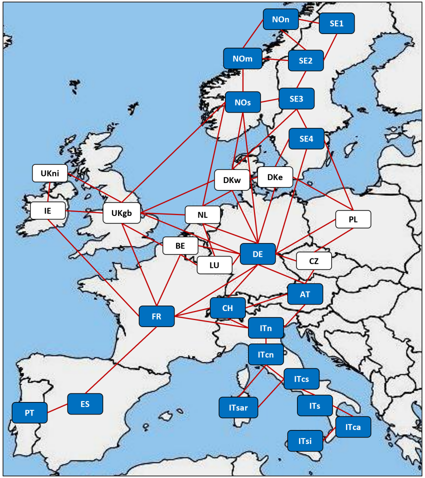
Summing up

3-node system with HD and DHD information structures

- DADP gives results comparable to SDP and SDDP
- DADP has significant computational potential when scaling to larger systems
- We test DADP on a 30-node system

Outline

30-node system



- We consider a 30-node system with 58 arcs representing its interconnections
- Each node has at most one storage unit and multiple dispatchable units
- Weekly hazard-decision information structure
- Linear costs and continuous decision variables

Comparing algorithms

- Algorithms comparison
  (Cost-to-go computation phase with the weekly marginal distribution $\widehat{\mathbb{P}}$)
  ✓ Computing times
  ✓ Lower bounds
  ✓ Statistical upper bounds (by simulation in scenarios sampled with $\widehat{\mathbb{P}}$)
  ✓ Relative statistical gap

- Algorithms comparison in simulation with RTE reference chronicles
  (without independence assumption)
  ✓ Operational costs
  ✓ Energy not supplied (ENS, highly penalized)
  ✓ Storage trajectories

Algorithms comparison: cost-to-go computation

30-node system, HD information structure

Method (HD)	Time (h)	Lower Bound (B€)	Statistical upper bound (B€)	Relative statistical gap
SDP	–	–	–	–
SDDP[1]	24.0	45.175	45.919 ± 0.057	1.65%
DADP hourly prices[1]	24.0	44.696	45.891 ± 0.087	2.67%
DADP 8-hour prices	11.7	44.924	45.477 ± 0.060	1.23%
DADP weekly prices	15.0	44.810	45.567 ± 0.062	1.69%

Table: Computing times, lower bounds and statistical upper bounds

- [1]SDDP and DADP hourly prices were both terminated due to the time limit
- SDDP gets the highest lower bound
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  ✓ Storage trajectories

Algorithms comparison with RTE reference chronicles

30-node system

Method (HD)	Operational cost (B€)	ENS (GWh) (% of total demand)
SDP	–	–
SDDP[1]	45.628	99.59 (0.005%)
DADP hourly prices[1]	46.530	489.54 (0.026%)
DADP 8-hour prices	45.489	124.93 (0.007%)
DADP weekly prices	45.645	160.44 (0.008%)

Table: Mean operational costs and mean ENS

- DADP with 8-hour and weekly prices have mean operational costs close to SDDP
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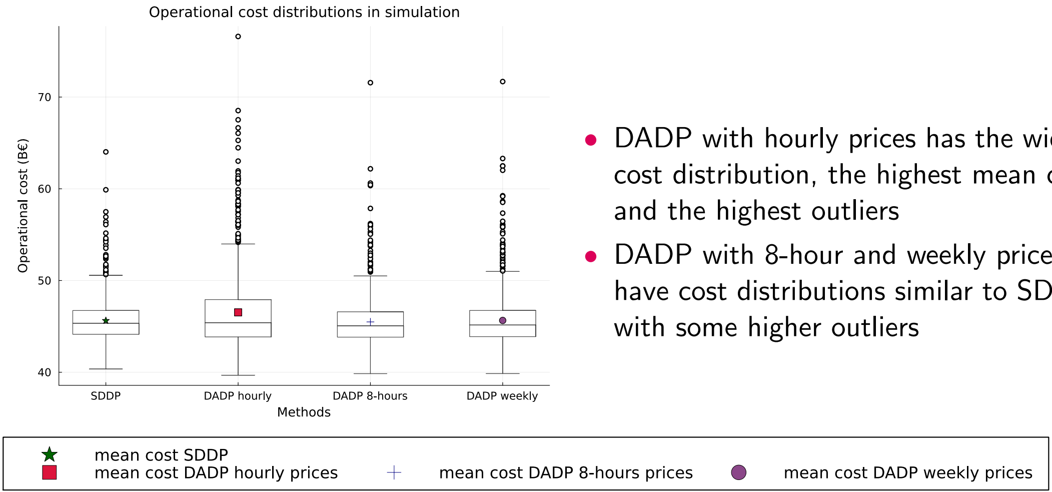
Algorithms comparison with RTE reference chronicles

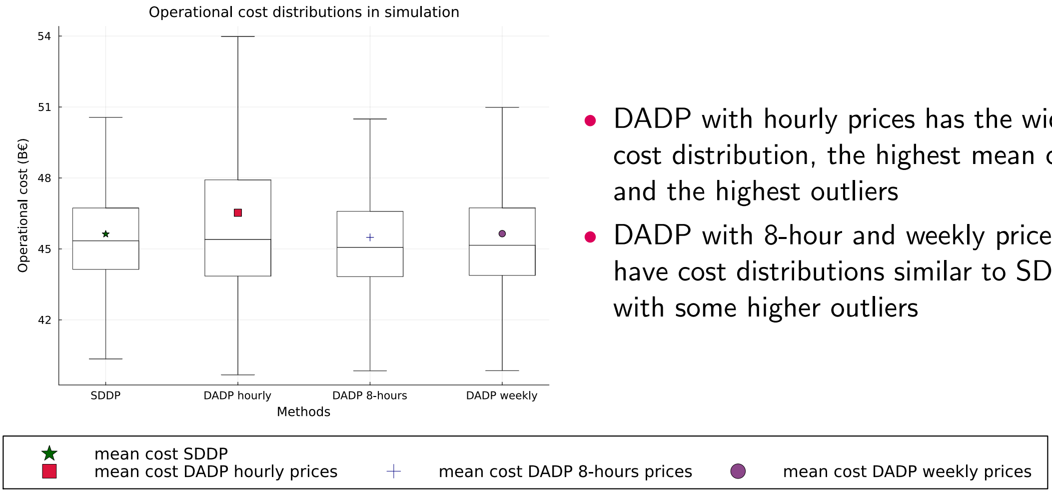
30-node system: operational cost distributions



- DADP with hourly prices has the widest cost distribution, the highest mean cost and the highest outliers
- DADP with 8-hour and weekly prices have cost distributions similar to SDDP, with some higher outliers

Algorithms comparison with RTE reference chronicles

30-node system: operational cost distributions



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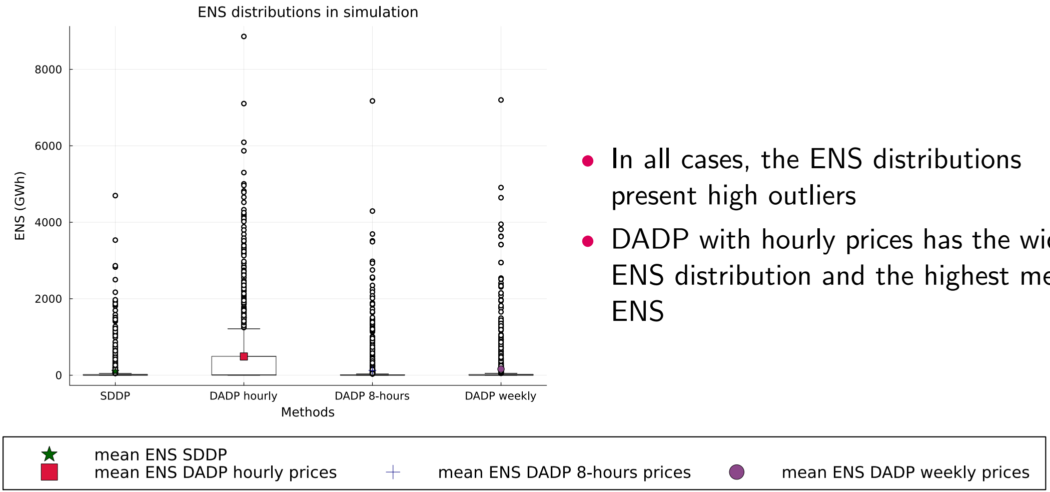
Algorithms comparison with RTE reference chronicles

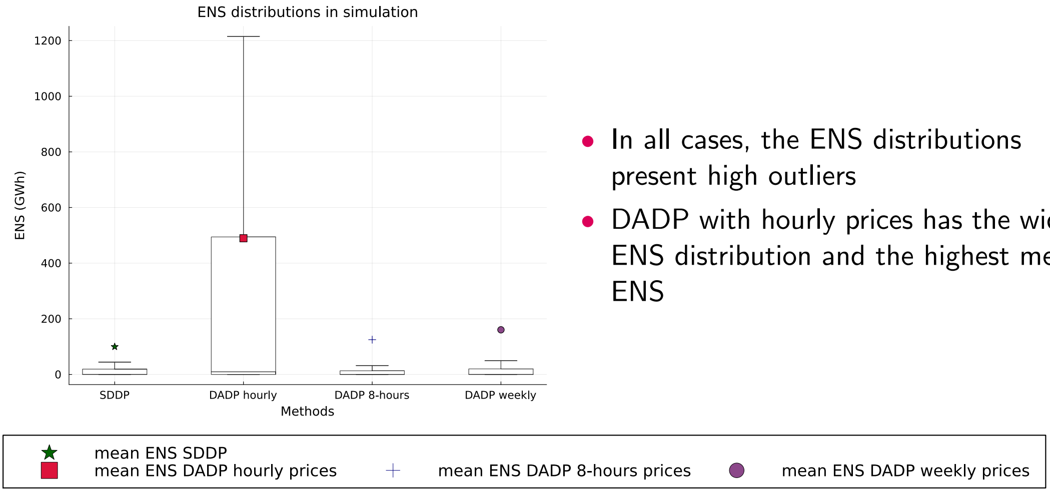
30-node system: ENS distributions



- In all cases, the ENS distributions present high outliers
- DADP with hourly prices has the widest ENS distribution and the highest mean ENS

Algorithms comparison with RTE reference chronicles

30-node system: ENS distributions



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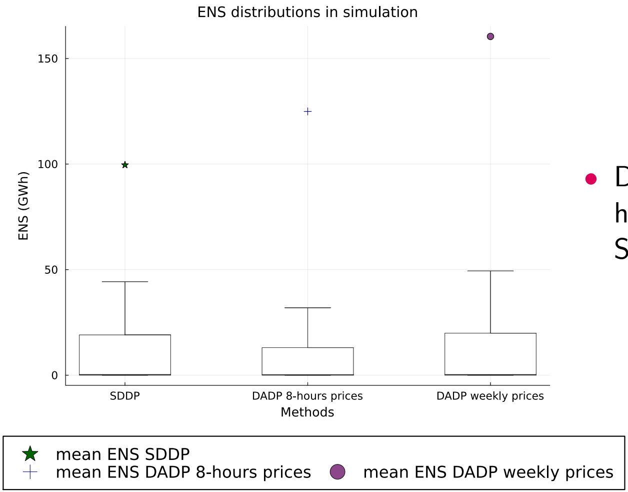
Algorithms comparison with RTE reference chronicles

30-node system: ENS distributions

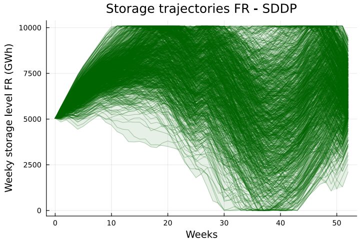


- DADP using 8-hour and weekly prices have ENS distributions similar to SDDP, but with higher mean ENS

Algorithms comparison with RTE reference chronicles

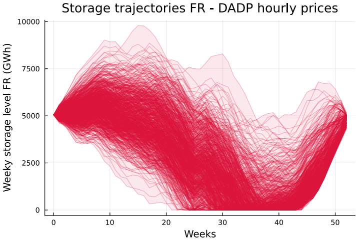
30-node system: trajectories for France

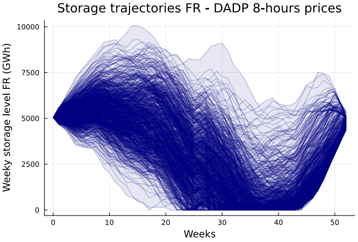
SDDP
DADP hourly prices
DADP 8-hour prices
DADP weekly prices

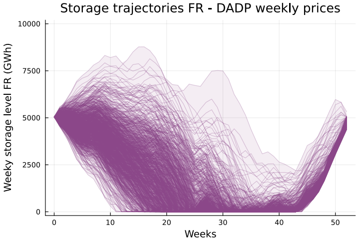


Penalization cost at the end of the year

$$K^n(x^n) = \begin{cases} 150 \times \left(\frac{X^n_{\max}}{2} - x^n \right) & \text{if } x^n < \frac{X^n_{\max}}{2} \,, \\ 0 & \text{if } x^n \geq \frac{X^n_{\max}}{2} \,. \end{cases}$$







Le réseau de transport d'électricité
ÉCOLE NATIONALE DES PONTS ET CHAUSSÉES
INSTITUT POLYTECHNIQUE DE PARIS

Algorithm comparison with RTE reference chronicles

Summing up

While having similar operational costs

- SDDP uses less the energy in storage than DADP methods (higher storage levels)
  - higher thermal costs
  - lower ENS costs
- DADP methods tend to use more the energy in storage than SDDP
  - lower thermal costs
  - higher ENS costs
- Trade-off between storage usage and ENS costs

Outline

We tackle the three complexity axes in a real scale problem

- Two-timescale multistage stochastic optimisation problem treated by spatial decomposition
- We studied different information structures
- DADP obtains results comparable to SDDP, with DADP being faster
- Univariate cost-to-go functions from DADP are easier to handle in simulation

Numerical challenges

- The update of the price decomposition process p is challenging
  - when is p good enough?
  - the size of the price decomposition process
  - we are using a gradient method to update p in a nonsmooth context
- Finding a compromise between gradient methods adaptability to nonsmooth problems and the scalability when the price decomposition process p is large

Perspectives

- Carry out a systematic numerical study on the 30-node system with DHD information structure
- DADP with nonlinear cost functions and binary/integer decisions (e.g. unit commitment)
- Multiple storage facilities per node

Outline

1. **Two-timescale information structures in a single-node system**
2. **Mixing temporal and spatial decomposition**
3. **General conclusions**

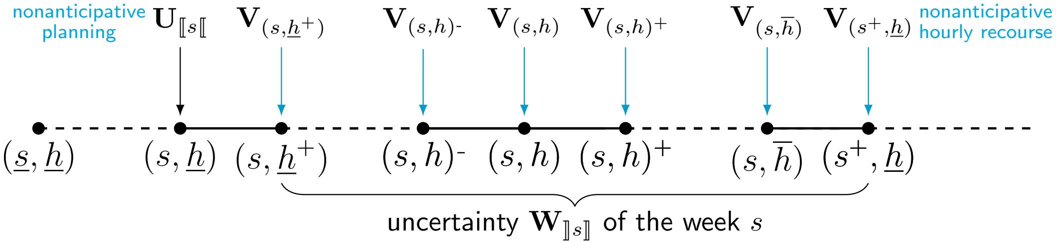
General conclusions

- Thesis motivated by RTE's need to compute usage values: this work connects academic methods with industrial practice
- Comprehensive study of two-timescale information structures and temporal decomposition methods for usage values calculation
- Application of our improved DADP to a large real-scale European case, showing good performance and outlining remaining challenges

Thank you for your attention

Ideal information structure

weekly decision-hazard-decision information structure with hourly recourse



$$B_s^{\mathsf{DHD^h}}(x) = \min_{u_{\llbracket s \llbracket}} \mathbb{E}\Bigg[\min_{v_{(s,\underline{h}^+)}} \mathbb{E}\Big[\ldots \min_{v_{(s,\overline{h})}} \mathbb{E}\Big[\min_{v_{(s^+,\underline{h})}} \Big(\overbrace{L_s}^{\substack{L_s(x,u_{\llbracket s \llbracket},w_{\rrbracket s \rrbracket},v_{\rrbracket s \rrbracket}) \\ \uparrow}} + B_{s^+}^{\mathsf{DHD^h}}\big(\overbrace{x_{s^+}}^{\substack{f_s(x,u_{\llbracket s \llbracket},w_{\rrbracket s \rrbracket},v_{\rrbracket s \rrbracket}) \\ \uparrow}}\big)\Big) \;\Big|\; w_{(s,\underline{h}^+)}, w_{(s,\underline{h}^+)^+}, \ldots, w_{(s,\overline{h})} \Big] \cdots \;\Big|\; w_{(s,\underline{h}^+)} \Big] \Bigg]$$

Algorithms comparison: cost-to-go computation

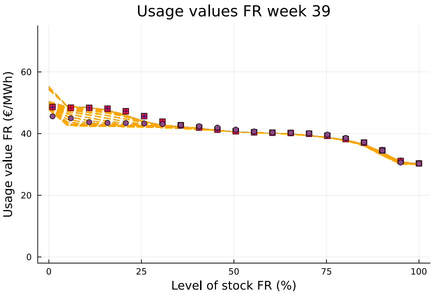
3-node system, HD information structure

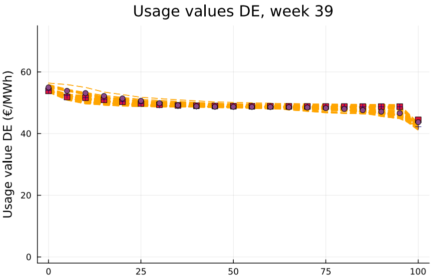
Method (HD)	Iterations	Lin. search	Time (h)	Lower Bound (B€)
SDP	–	–	42.74	20.863
SDDP	504	–	4.57	19.813
DADP hourly prices	27	71	1.92	19.987
DADP 8-hour prices	41	81	2.03	19.959
DADP weekly prices	60	109	3.42	19.906

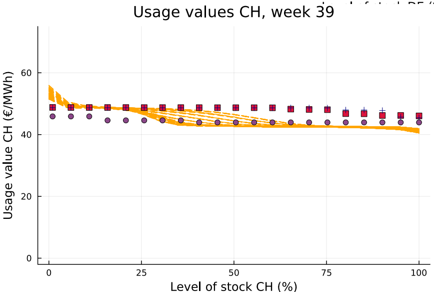
Table: Number of iterations, number of linear searches, computing times and lower bound for different resolution methods in the weekly hazard-decision case

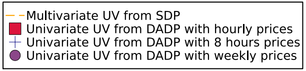
Algorithms comparison: cost-to-go computation

3-node system, HD information structure: usage values



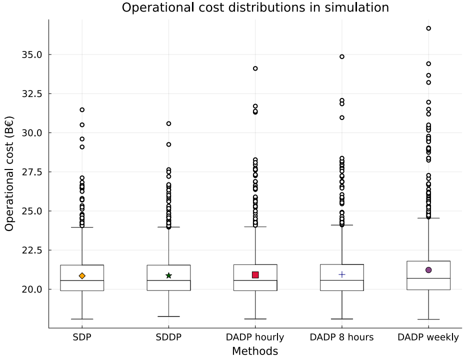




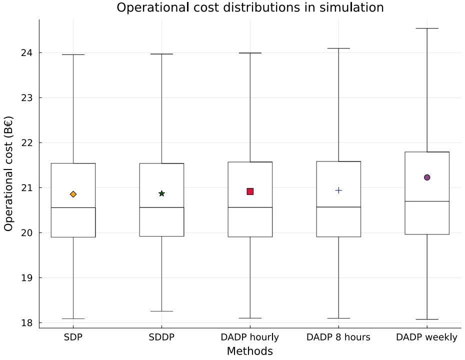


Algorithms comparison with RTE reference chronicles

3-node system: operational costs distributions



(a) Op. cost distributions with outliers

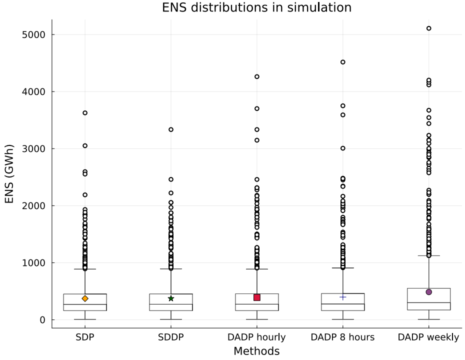


(b) Op. cost distributions without outliers

mean cost SDP
mean cost SDDP
mean cost DADP hourly prices
mean cost DADP 8-hours prices
mean cost DADP weekly prices

Algorithms comparison with RTE reference chronicles

3-node system: ENS distributions



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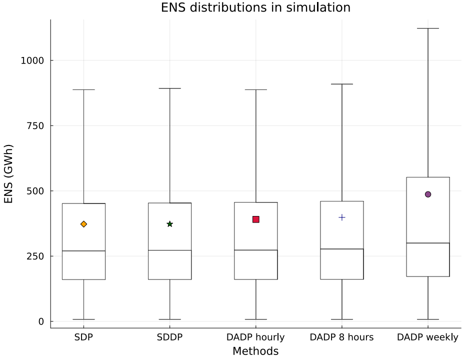

(b) ENS distributions without outliers

mean ENS SDP
mean ENS SDDP
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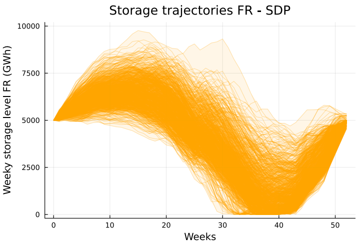
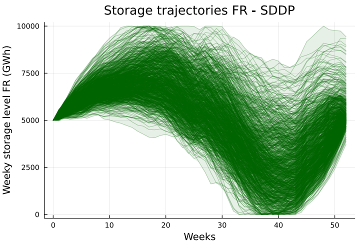
Algorithms comparison with RTE reference chronicles

3-node system: storage trajectories for France





Penalization cost at the end of the year:

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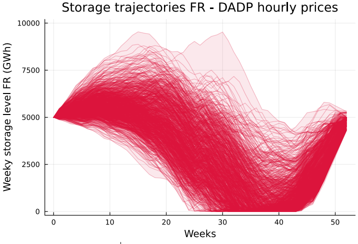
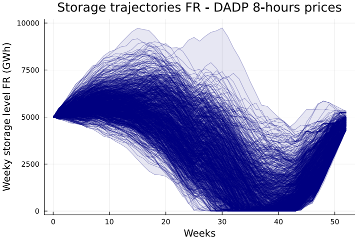
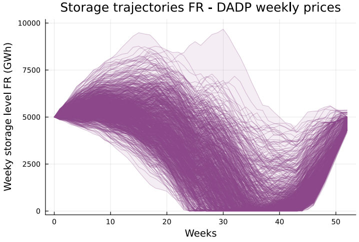
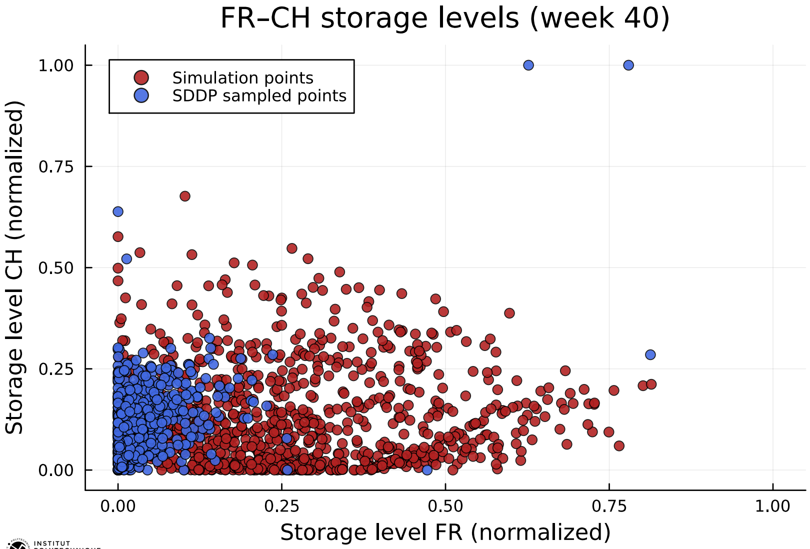






SDDP space exploration

3-node system, HD information structure



Algorithms comparison: cost-to-go computation

30-node system, HD information structure

Method (HD)	Iterations	Lin. search	Time (h)	Lower Bound (B€)
SDP	N/A	N/A	N/A	N/A
SDDP[1]	250	–	24.0	45.175
DADP hourly prices[1]	72	122	24.0	44.696
DADP 8-hour prices	128	141	11.7	44.924
DADP weekly prices	112	278	15.0	44.810

Table: Number of iterations, number of linear searches, computation times and lower bound for different solution methods in the 30-node system under the weekly hazard-decision information structure.

[1] Terminated due to the time limit