Robust and Stochastic Viable Control

Extended from Chapter 7 of
Sustainable Management of Natural Resources. Mathematical Models and Methods
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Outline of the presentation

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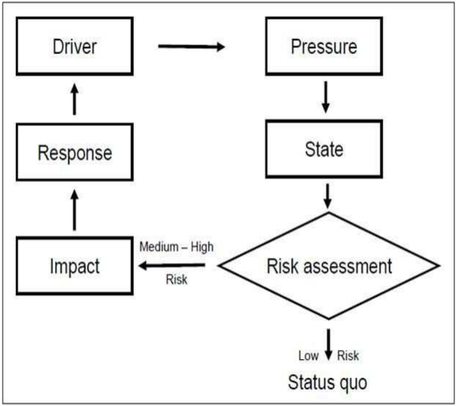
A battery of assessment frameworks have been concocted to gauge policies with respect to risk and ecological impact

- Integrated Ecosystem Assessment (IEA)
  (National Oceanic and Atmospheric Administration)
- Ecological Risk Assessment
- Ecosystem-based Management (EBM)
- Ecosystem Approach to Management
- Driver Pressure State Impact Response (DPSIR) Approach
- Management strategy evaluation (MSE)

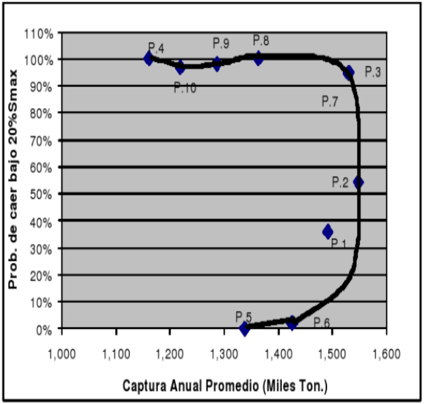
The Driver-Pressure-State-Impact-Response framework

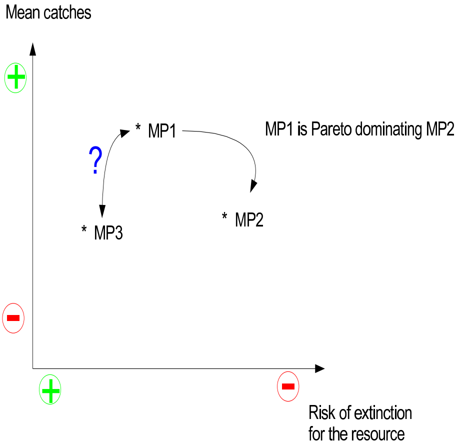


- stressor is an agent of change in the environment
- receptor
- exposure
- "effect" means the response of the receptor when it is actually exposed to the stressor
- assessment endpoint: a specific management outcome that is desired of the ecosystem

The Management Strategy Evaluation framework

"Mieux vaut être riche et bien portant que pauvre et malade"





Outline of the presentation

A control system connects input and output variables

Input variables

Control	wood logs
Uncertainty	wood humidity
	metal conductivity

Output variables

soup quality
water vapor
temperature (internal state)

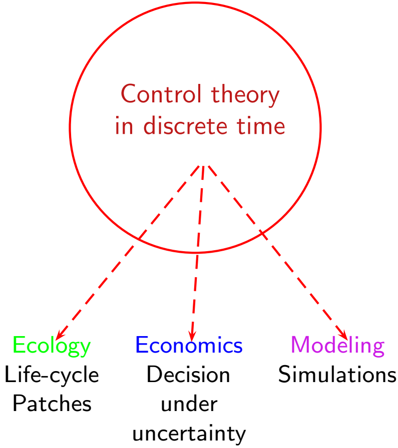
Uncertainty variables are new input variables in a discrete-time nonlinear state-control system

A specific output is distinguished, and is labeled "state" (more on this later), when the system may be written

$$x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t), w(t)\big) , \quad t \in \mathbb{T} = \{t_0, t_0+1, \dots, T-1\}$$

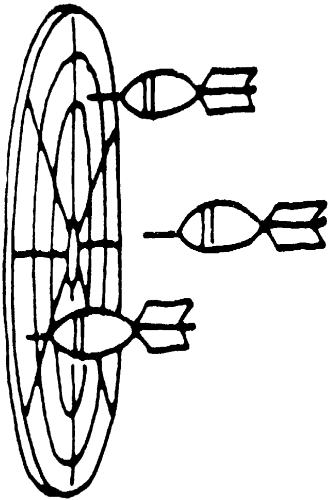
- time $t \in \overline{\mathbb{T}} = \{t_0, t_0+1, \dots, T-1, T\} \subset \mathbb{N}$
  (the time period $[t, t+1[$ may be a year, a month, etc.)
- state $x(t) \in \mathbb{X} := \mathbb{R}^n$ (biomasses, abundances, etc.)
- control $u(t) \in \mathbb{U} := \mathbb{R}^p$ (catches or harvesting effort)
- uncertainty $w(t) \in \mathbb{W} := \mathbb{R}^q$
  (recruitment or mortality uncertainties, climate fluctuations or trends, etc.)
- dynamics $\mathtt{Dyn}$ maps $\mathbb{T} \times \mathbb{X} \times \mathbb{U} \times \mathbb{W}$ into $\mathbb{X}$
  (biomass model, age-class model, economic model)

We dress natural resources management issues in the formal clothes of control theory in discrete time



- Problems are framed as
  - find controls/decisions driving a dynamical system
  - to achieve various goals
- Three main ingredients are
  - controlled dynamics
  - constraints
  - criterion to optimize

We mathematically express the objectives pursued as control and state constraints



- For a state-control system,
  we cloth objectives as constraints
- and we distinguish
  control constraints (rather easy)
  state constraints (rather difficult)
- Viability theory deals with state constraints

Constraints may be explicit on the control variable

and are rather easily handled by reducing the decision set

Examples of control constraints

- Irreversibility constraints, physical bounds
  $0 \leq a(t) \leq 1 \ , \quad 0 \leq h(t) \leq B(t)$
- Tolerable costs $c\big(a(t), Q(t)\big) \leq c^{\sharp}$

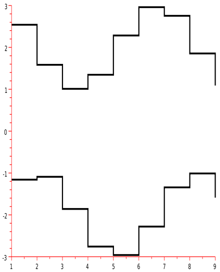
Control constraints / admissible decisions

$$\underbrace{u(t)}_{\text{control}} \in \underbrace{\mathbb{B}\big(t, x(t)\big)}_{\text{admissible set}} \ , \quad t = t_0, \ldots, T-1$$

Easy because control variables $u(t)$ are precisely those variables whose values the decision-maker can fix at any time within given bounds

Meeting constraints bearing on the state variable is delicate

due to the dynamics pipeline between controls and state



State constraints / admissible states

$$\underbrace{x(t)}_{\text{state}} \in \underbrace{\mathbb{A}(t)}_{\text{admissible set}} \quad , \quad t = t_0, \ldots, T$$

Examples ("tipping points")

- $\mathrm{CO_2}$ concentration $M(t) \leq M^\sharp$
- biomass $B^\flat \leq B(t) \leq B^\sharp$

State constraints are mathematically difficult because of "inertia"

$$x(t) = \underbrace{\text{function}}_{\text{iterated dynamics}} \big(\underbrace{u(t-1), \ldots, u(t_0)}_{\text{past controls}}, x(t_0) \big)$$

Target and asymptotic state constraints are special cases

- Final state achieves some target

$$\underbrace{x(T)}_{\text{final state}} \in \underbrace{\mathbb{A}(T)}_{\text{target set}}$$

Example: CO_2 concentration

- State converges toward a target

$$\underbrace{\lim_{t \to +\infty} x(t)}_{\text{asymptotic state}} \in \underbrace{\mathbb{A}(\infty)}_{\text{target set}}$$

Example: convergence towards an endemic state in epidemiology

Outline of the presentation

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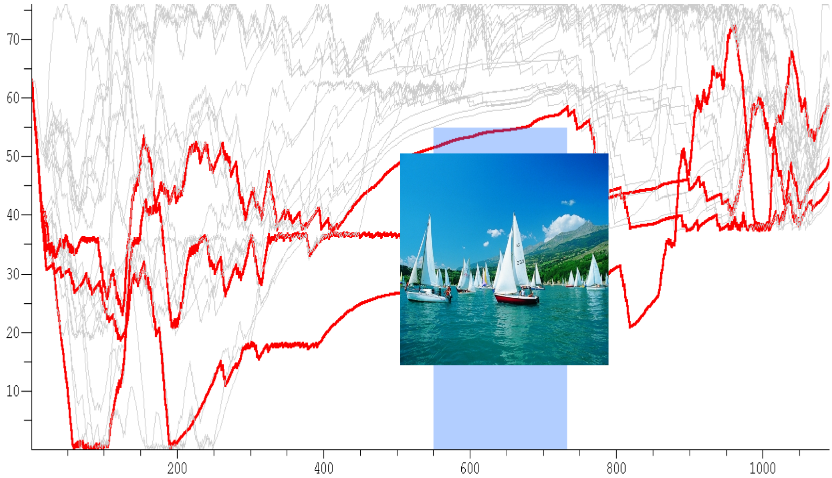
Tourism issues impose constraints upon traditional economic management of a hydro-electric dam

- Maximizing the revenue from turbinated water
- under a tourism constraint of having enough water in July and August

The red stock trajectories fail to meet the tourism constraint in July and August



We consider a single dam nonlinear dynamical model in the decision-hazard setting

We can model the dynamics of the water volume in a dam by

$$\underbrace{S(t+1)}_{\text{future volume}} = \min\{S^\sharp, \underbrace{S(t)}_{\text{volume}} - \underbrace{q(t)}_{\text{turbined}} + \underbrace{a(t)}_{\text{inflow volume}} \}$$

- $S(t)$ volume (stock) of water at the beginning of period $[t, t+1[$
- $a(t)$ inflow water volume (rain, etc.) during $[t, t+1[$
- $q(t)$ turbined outflow volume during $[t, t+1[$
    - decided at the beginning of period $[t, t+1[$
    - chosen such that $0 \leq q(t) \leq \min\{S(t), q^\sharp\}$
    - supposed to depend on the stock $S(t)$ but not on the inflow water $a(t)$
- the setting is called decision-hazard: $a(t)$ is not available at the beginning of period $[t, t+1[$

In the risk-neutral economic approach, an optimal management maximizes the expected payoff

- Suppose that
  - at the horizon, the final volume $S(T)$ has a value $\mathtt{K}(S(T))$, the "final value of water"
  - turbined water $q(t)$ is sold at price $p(t)$, related to the price at which energy can be sold at time t
  - a probability $\mathbb{P}$ is given on the set $\Omega = \mathbb{R}^{T-t_0} \times \mathbb{R}^{T-t_0}$ of water inflows scenarios $\big(a(t_0), \dots, a(T-1)\big)$ and prices scenarios $\big(p(t_0), \dots, p(T-1)\big)$
- The traditional (risk-neutral) economic problem is to maximize the intertemporal payoff (without discounting if the horizon is short)

$$\max \mathbb{E}\left[\sum_{t=t_0}^{T-1}\Big(\overbrace{p(t)}^{\text{price}}\ \overbrace{q(t)}^{\text{turbined}}\ \underbrace{-\epsilon q(t)^2}_{\text{turbined costs}}\Big) + \overbrace{\mathtt{K}(S(T))}^{\text{final volume utility}}\right]$$

We now have a stochastic optimization problem, where the tourism constraint still needs to be dressed in formal clothes

- Traditional cost minimization/payoff maximization

$$\max \mathbb{E}\left[\sum_{t=t_0}^{T-1} \overbrace{p(t)q(t) - \epsilon q(t)^2}^{\text{turbined water payoff}} + \overbrace{\mathtt{K}(S(T))}^{\text{final volume utility}}\right]$$

- Tourism constraint

$$\text{volume } S(t) \geq S^\flat , \quad \forall t \in \{ \text{July, August} \}$$

- In what sense should we consider this inequality which involves the random variables $S(t)$ for $t \in \{ \text{July, August} \}$?

Robust / almost sure / probability constraint

- Robust constraints: for all the scenarios in a subset $\overline{\Omega} \subset \Omega$

$$S(t) \geq S^{\flat} \;, \quad \forall t \in \{ \text{July, August} \}$$

- Almost sure constraints

$$\text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \text{ are above the} \\ \text{threshold } S^{\flat} \text{ for periods } t \text{ in summer} \end{array} \right\} = 1$$

- Probability constraints, with "confidence" level $p \in [0, 1]$

$$\text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \text{ are above the} \\ \text{threshold } S^{\flat} \text{ for periods } t \text{ in summer} \end{array} \right\} \geq p$$

- and also by penalization, or in the mean, etc.

Our problem may be clothed as a stochastic optimization problem under a probability constraint

- The traditional economic problem is $\max \mathbb{E}\left[P(T)\right]$ where the payoff/utility criterion is

$$P(T) = \sum_{t=t_0}^{T-1} \overbrace{p(t)q(t) - \epsilon q(t)^2}^{\text{turbined water payoff}} + \overbrace{K(S(T))}^{\text{final volume utility}}$$

- and a failure tolerance is accepted

$$\text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \geq S^\flat \\ \text{for periods } t \text{ in July and August} \end{array} \right\} \geq 90\%$$

- Details concerning the theoretical and numerical resolution are available on demand ;-)

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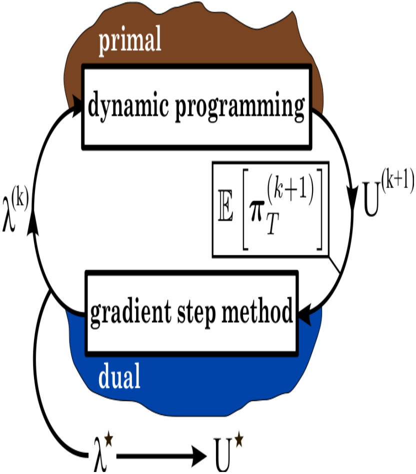
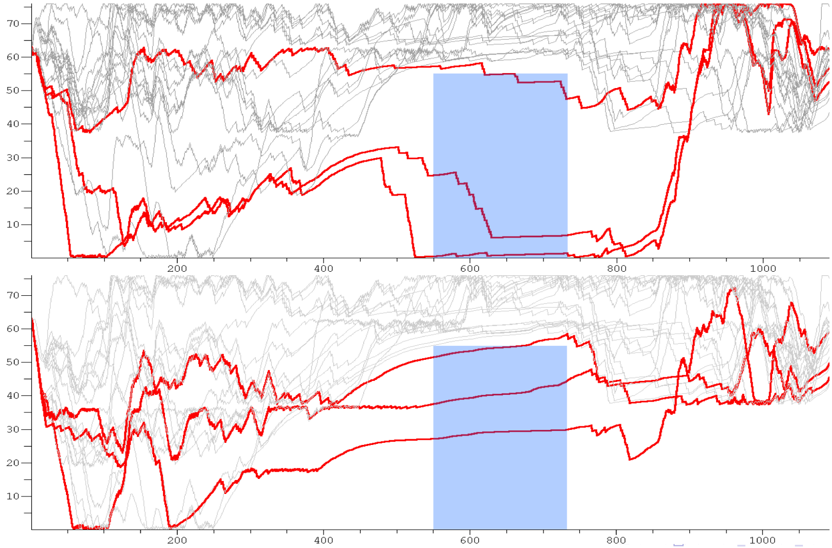
- $\pi_0 = 1$ and $\pi_{t+1} = \begin{cases} \mathbf{1}_{\{x_{t+1} \geq x_{\text{ref}}\}} \times \pi_t & \text{if } t \in \mathcal{T} \\ \pi_t & \text{else} \end{cases}$
- $\mathbb{P}\left[x_\tau \geq x_{\text{ref}}\ , \quad \forall \tau \in \mathcal{T}\right]$
  $= \mathbb{E}\left[\mathbf{1}_{\{x_\tau \geq x_{\text{ref}}\ , \quad \forall \tau \in \mathcal{T}\}}\right]$
  $= \mathbb{E}\left[\prod_{\tau \in \mathcal{T}} \mathbf{1}_{\{x_\tau \geq x_{\text{ref}}\}}\right]$
  $= \mathbb{E}\left[\pi_T\right]$



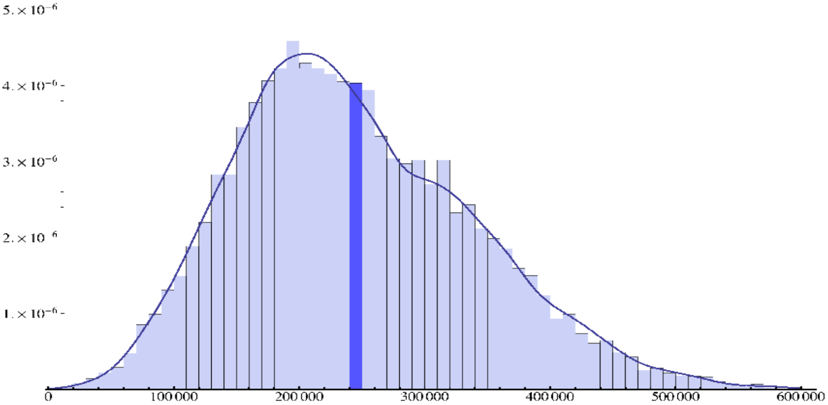
90% of the stock trajectories meet the tourism constraint in July and August



Our resolution approach brings a sensible improvement compared to standard procedures

OPTIMAL POLICIES	OPTIMIZATION		SIMULATION		
	Iterations	Time	Gain	Respect	Well behaviour
Standard	15	10 mn	**ref**	**0,9**	**no**
Convenient	**10**	**160 mn**	**-3.20%**	**0,9**	**yes**
Heuristic	10	160 mn	**-3.25%**	**0,9**	**yes**

However, though the expected payoff is optimal, the payoff effectively realized can be far from it



Outline of the presentation

Hard constraints in deterministic optimization problems

- The intertemporal deterministic optimization problem

$$\max_{x(\cdot),u(\cdot)} \left(\sum_{t=t_0}^{T-1} \mathtt{L}\big(t, x(t), u(t)\big) \right)$$

under the viability constraints

$$x(t) \in \mathbb{A}(t)\,, \quad t = t_0, \ldots, T-1$$

- is equivalent to

$$\max_{x(\cdot),u(\cdot)} \left(\sum_{t=t_0}^{T-1} \mathtt{L}\big(t, x(t), u(t)\big) - \sum_{t=t_0}^{T-1} \chi_{\mathbb{A}(t)}\big(x(t)\big) \right)$$

$$\text{where } \chi_{\mathbb{A}(t)}(x) = \begin{cases} +\infty & \text{if} \quad x \notin \mathbb{A}(t) \\ 0 & \text{if} \quad x \in \mathbb{A}(t) \end{cases}$$

Hard constraints in stochastic optimization problems

- The intertemporal stochastic optimization problem

$$\max_{x(\cdot),u(\cdot)} \mathbb{E}\left(\sum_{t=t_0}^{T-1} \mathtt{L}\big(t, x(t), u(t), w(t)\big) - \sum_{t=t_0}^{T-1} \chi_{\mathbb{A}(t)}\big(x(t)\big) \right)$$

where $\chi_{\mathbb{A}(t)}(x) = \begin{cases} +\infty & \text{if} \quad x \notin \mathbb{A}(t) \\ 0 & \text{if} \quad x \in \mathbb{A}(t) \end{cases}$

- is equivalent to

$$\max_{x(\cdot),u(\cdot)} \mathbb{E}\left(\sum_{t=t_0}^{T-1} \mathtt{L}\big(t, x(t), u(t), w(t)\big) \right)$$

under the almost sure viability constraints

$$\mathbb{P}\{x(t) \in \mathbb{A}(t) , \quad t = t_0, \ldots, T-1\} = 1$$

Outline of the presentation

A scenario is said to be viable for a given policy if the state and control trajectories satisfy the constraints

Viable scenario under given policy

A scenario $w(\cdot) \in \Omega$ is said to be viable under policy $\texttt{Pol} : \mathbb{T} \times \mathbb{X} \to \mathbb{U}$ if the trajectories $x(\cdot)$ and $u(\cdot)$ generated by the dynamics

$$x(t+1) = \texttt{Dyn}\big(t, x(t), u(t), w(t)\big) , \quad t = t_0, \ldots, T-1$$

with the policy

$$u(t) = \texttt{Pol}\big(t, x(t)\big)$$

satisfy the state and control constraints

$$\underbrace{u(t) \in \mathbb{B}\big(t, x(t)\big)}_{\text{control constraints}} \text{ and } \underbrace{x(t) \in \mathbb{A}(t)}_{\text{state constraints}} , \quad \forall t = t_0, \ldots, T$$

The set of viable scenarios is denoted by $\Omega_{\texttt{Pol}, t_0, x_0}$

We look after policies that make the corresponding set of viable scenarios "large"

Set of viable scenarios

$$\Omega_{\texttt{Pol},t_0,x_0} := \{ w(\cdot) \in \Omega \mid \quad \text{the state constraints}$$
$$X_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)](t) \in \mathbb{A}(t)$$
$$\text{and the control constraints}$$
$$U_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)] \in \mathbb{B}\big(t, x(t)\big)$$
$$\text{are satisfied for all times } t = t_0, \ldots, T \}$$

- The larger set $\Omega_{\texttt{Pol},t_0,x_0}$ of viable scenarios, the better, because the policy Pol is able to maintain the system within constraints for a large "number" of scenarios
- But "large" in what sense? Robust? Probabilistic?

Outline of the presentation

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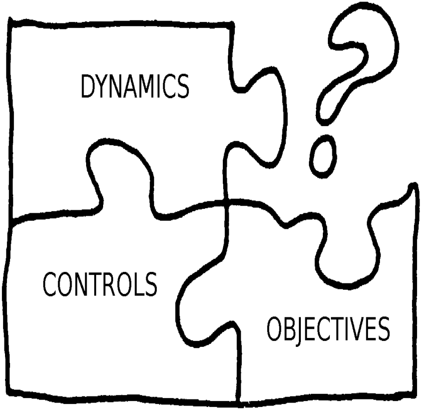
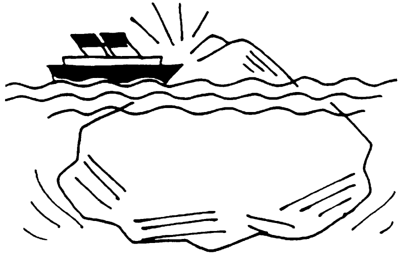
1. Multi-objectives dynamic management under uncertainty
   - Assessment frameworks are designed to deal with multiple goals
   - Recalls on uncertain dynamical systems under constraints
2. Criterion and constraints in the uncertain case
   - A dam management example
   - Constraints penalization
   - Viable scenarios
3. The robust viability problem
   - The deterministic viability approach
   - Robust viable controls and states
   - Robust viability analysis of anchovy–hake Peruvian fisheries
4. The stochastic viability problem
   - Maximal viability probability and dynamic programming equation
   - Stochastic viability kernels
   - Dam stochastic viable management
   - Nephrops-hake fishery viable management
5. Summary

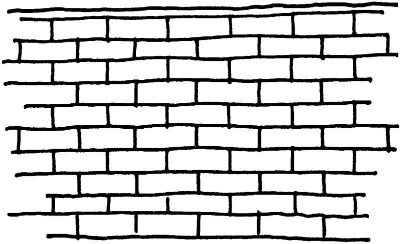
Can we solve the compatibility puzzle between dynamics and objectives by means of appropriate controls?



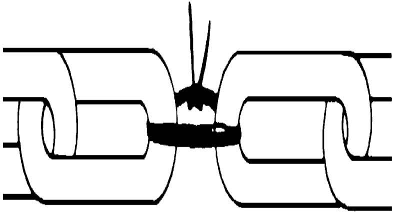
- Given a dynamics that mathematically embodies the causal impact of controls on the state
- Imposing objectives bearing on output variables (states, controls)
- Is it possible to find a control path that achieves the objectives for all times?

Crisis occurs when constraints are trespassed at least once





- An initial state is not viable if, whatever the sequence of controls, a crisis occurs
- There exists a time when one of the state or control constraints is violated



Outline of the presentation

Robust viability dissects how to channel the system inside constraints *whatever the scenarios*

Let $\overline{\Omega} \subset \Omega$ be a subset of the set Ω of scenarios

The robust viability problem

Identify the initial states $x_0 \in \mathbb{X}$ for which there exists at least one viable robust policies $\mathtt{Pol} : \mathbb{T} \times \mathbb{X} \to \mathbb{U}$ such that

- the state trajectories given by the state solution map $x(t) = X_{\mathtt{Dyn}}[t_0, x_0, \mathtt{Pol}, w(\cdot)](t)$ satisfy the following state constraints
$$x(t) \in \mathbb{A}(t) \text{ for } t = t_0, \ldots, T$$
- and the control constraints $u(t) = \mathtt{Pol}\big(t, x(t)\big) \in \mathbb{B}\big(t, x(t)\big)$ are satisfied for $t = t_0, \ldots, T-1$

for all scenarios $w(\cdot) \in \overline{\Omega}$

The robust viability kernel is the set of initial states for which the robust viability problem can be solved

Robust viability kernel

$$\mathbb{V}\mathrm{iab}_1(t_0) := \left\{ x_0 \in \mathbb{X} \;\middle|\; \begin{array}{l} \text{there exists a policy } \mathtt{Pol} \in \mathfrak{U} \\ \text{ such that for all scenario } w(\cdot) \in \overline{\Omega} \\ \text{the state constraints } x(t) \in \mathbb{A}(t) \\ \text{and the control constraints} \\ u(t) = \mathtt{Pol}\big(t, x(t)\big) \in \mathbb{B}\big(t, x(t)\big) \\ \text{are satisfied for all times } t = t_0, \ldots, T \end{array} \right\}$$

where the state $x(t) = X_{\mathtt{Dyn}}[t_0, x_0, \mathtt{Pol}, w(\cdot)](t)$ is given by the state solution map

The robust viability kernel and viable scenarios are related

$$x_0 \in \underbrace{\mathbb{V}\mathrm{iab}_1(t_0)}_{\text{robust viability kernel}} \iff \begin{cases} \text{there exists a policy } \mathtt{Pol} \in \mathfrak{U}, \\ \overline{\Omega} \subset \underbrace{\Omega_{\mathtt{Pol},t_0,x_0}}_{\text{viable scenarios}} \end{cases}$$

Robust viability kernels and robust viable policies can be defined for all times

Robust viability kernel at time t

The robust viability kernel at time t is the subset of states

$$\mathbb{V}\mathrm{iab}_1(t) := \left\{ x \in \mathbb{X} \;\middle|\; \begin{array}{l} \text{there exists } \texttt{Pol} \in \mathfrak{U}^{ad} \text{ such that} \\ \text{for all scenario } w(\cdot) \in \overline{\Omega} \\ x(s) \in \mathbb{A}(s) \text{ for } s = t, \ldots, T \end{array} \right\}$$

where $x(s) = X_{\mathtt{Dyn}}[t, x, \texttt{Pol}, w(\cdot)](s)$ is given by the state solution map

The final viability kernel is the whole target set: $\mathbb{V}\mathrm{iab}_1(T) = \mathbb{A}(T)$

Viable robust policies

$$\mathfrak{U}_1^{\mathrm{viab}}(t, x) := \left\{ \texttt{Pol} \in \mathfrak{U}^{ad} \;\middle|\; \begin{array}{l} \text{for all scenario } w(\cdot) \in \overline{\Omega} \\ X_{\mathtt{Dyn}}[t, x, \texttt{Pol}, w(\cdot)](s) \in \mathbb{A}(s) \\ \text{for } s = t, \ldots, T \end{array} \right\}$$

The viability program aims at turning state constraints into control constraints

- A priori constraints, with state constraints

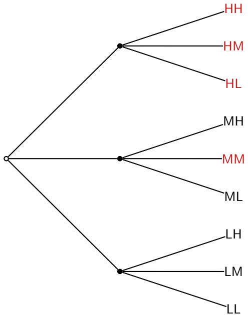
$$\begin{cases} x(t_0) \in \mathbb{X} \\ x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t), w(t)\big) \\ u(t) \in \mathbb{B}\big(t, x(t)\big) & \text{control constraints} \\ x(t) \in \mathbb{A}(t) & \text{state constraints} \end{cases}$$

- are turned into a posteriori constraints, without state constraints except for the initial state

$$\begin{cases} x(t_0) \in \mathbb{V}\mathrm{iab}(t_0) & \text{initial state constraint} \\ x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t), w(t)\big) \\ u(t) \in \mathbb{B}^{\mathrm{viab}}\big(t, x(t)\big) \subset \mathbb{B}\big(t, x(t)\big) & \text{control constraints} \end{cases}$$

- ex ante state constraints $\to$ ex post control constraints

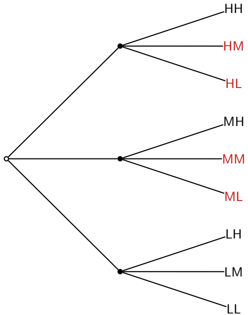
Product scenarios subsets embody time independence



There is no time independence because the range of values of $w(t+1)$ depends on the value of $w(t)$:

$w(t) = H \Rightarrow w(t+1) \in \{M, L\}$

$w(t) = M \Rightarrow w(t+1) \in \{M\}$



There is time independence because

$\overline{\Omega} = \{H, M\} \times \{M, L\} \subset \Omega$

is a product set

A priori information on the scenarios may be set membership

The product case

- Uncertain variables may be restricted to subsets, period by period

$$w(t) \in \mathbb{S}(t)$$

so that some scenarios are selected and the rest are excluded

$$w(\cdot) \in \mathbb{S}(t_0) \times \cdots \times \mathbb{S}(T) \subset \Omega = \mathbb{W}^{T+1-t_0}$$

Bounded water inflows in a dam

If only an upper bound on water inflows is known,
we represent off-line information by

$$0 \leq a(t) \leq a^\sharp$$

The robust dynamic programming equation is a backward equation relating the robust viability kernels

Let $\overline{\Omega} \subset \Omega$ be a subset of the set Ω of scenarios

Robust dynamic programming equation

If the scenarios vary within a rectangle $\overline{\Omega} = \mathbb{S}(t_0) \times \cdots \times \mathbb{S}(T)$ (corresponding to independence in the stochastic setting), the robust viability kernels satisfy the following backward induction, where t runs from $T-1$ down to t_0

$$
\begin{aligned}
\mathbb{V}\mathrm{iab}_1(T) &= \mathbb{A}(T) \\
\mathbb{V}\mathrm{iab}_1(t) &= \left\{ x \in \mathbb{A}(t) \;\middle|\; \begin{array}{l} \text{there exists an admissible control } u \in \mathbb{B}(t,x) \\ \text{such that for all scenarios } w \in \mathbb{S}(t) \\ \text{one has that } \mathtt{Dyn}(t,x,u,w) \in \mathbb{V}\mathrm{iab}_1(t+1) \end{array} \right\}
\end{aligned}
$$

The robust dynamic programming equation yields the robust viable controls

Robust viable controls

For any time t and state x, robust viable controls are

$$\mathbb{B}_1^{\text{viab}}(t,x) := \{u \in \mathbb{B}(t,x) \mid \forall w \in \mathbb{S}(t)\ ,\ \texttt{Dyn}(t,x,u,w) \in \mathbb{V}\mathrm{iab}_1(t+1)\}$$

Proposition

Viable robust policies are those $\texttt{Pol} \in \mathfrak{U}$ *such that*

$$\texttt{Pol}(t,x) \in \mathbb{B}_1^{viab}(t,x)\ , \quad \forall t \in \mathbb{T}\ , \forall x \in \mathbb{V}\mathrm{iab}_1(t)$$

The viability program is achieved

- Robust viable controls exist at time t if and only if the state x belongs to the robust viability kernel at time t:

$$\mathbb{B}_1^{\text{viab}}(t,x) \neq \emptyset \iff x \in \mathbb{V}\text{iab}_1(t)$$

- A solution to the viability problem is
  - an initial state x_0
  - and a policy `Pol`

  such that

$$\begin{aligned} x_0 &\in \mathbb{V}\text{iab}_1(t_0) \\ \texttt{Pol}(t,x) &\in \mathbb{B}_1^{\text{viab}}(t,x) \, , \quad \forall t \in \mathbb{T} \, , \forall x \in \mathbb{V}\text{iab}_1(t) \end{aligned}$$

Outline of the presentation

We consider two species targeted by two fleets in a biomass ecosystem dynamics *with uncertainties*

We embody uncertainties, stocks and fishing interactions in a two-dimensional dynamical model

$$\overbrace{A(t+1)}^{\text{future biomass}} = A(t)\overbrace{\mathcal{R}_A\big(A(t), H(t), \underbrace{w_A(t)}_{\text{uncer tainty}}\big)}^{\text{growth factor}}\big(1 - \underbrace{E_A(t)}_{\text{effort control}}\big)$$

$$H(t+1) = H(t)\mathcal{R}_H\big(A(t), H(t), \overbrace{w_H(t)}\big)\big(1 - \overbrace{E_H(t)}\big)$$

- Uncertainties $w_A(t)$ and $w_H(t)$ are discrepancies
- State vector $(A(t), H(t))$ represents biomasses
- Control vector $(E_A(t), E_H(t))$ is fishing effort of each species
- Catches are $E_A(t)\mathcal{R}_A\big(A(t), H(t), w_A(t)\big)A(t)$ and $E_H(t)\mathcal{R}_H\big(A(t), H(t), w_H(t)\big)H(t)$

Our objectives are twofold: conservation and production

The robust viability kernel is the set of initial species biomasses $\big(A(t_0), H(t_0)\big)$
from which at least one appropriate policy
produces biomasses and effort trajectories
such that the following goals are satisfied
for all the scenarios $\big(w_A(t), w_H(t)\big)$, $t = t_0, t_0 + 1, \ldots, T$

- preservation (minimal biomass thresholds)

$$A \ \underline{\text{stocks}}: \qquad A(t) \geq S_A^\flat$$

$$H \ \underline{\text{stocks}}: \qquad H(t) \geq S_H^\flat$$

- economic/social requirements (minimal catch thresholds)

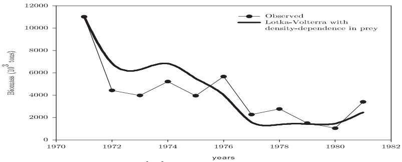
$$A \ \underline{\text{catches}}: \qquad E_A(t)\mathcal{R}_A\big(A(t), H(t), w_A(t)\big)A(t) \geq C_A^\flat$$

$$H \ \underline{\text{catches}}: \qquad E_H(t)\mathcal{R}_H\big(A(t), H(t), w_H(t)\big)H(t) \geq C_H^\flat$$

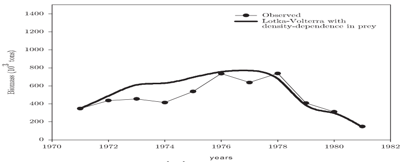
We taylor a Lotka-Volterra *decision model* to hake-anchovy Peruvian fisheries scarce data, and qualify the discrepancies as uncertainties

Hake-anchovy Peruvian fisheries data between 1971 and 1981, in thousands of tonnes (10^3 tons)

- anchoveta_stocks= [11019 4432 3982 5220 3954 5667 2272 2770 1506 1044 3407]
- merluza_stocks= [347 437 455 414 538 735 636 738 408 312 148]
- anchoveta_captures= [9184 3493 1313 3053 2673 3211 626 464 1000 223]
- merluza_captures= [26 13 133 109 85 93 107 303 93 159 69]



(a) Anchovy



(b) Hake

Figure : Comparison of observed and simulated biomasses of anchovy and hake using a Lotka-Volterra model with density-dependence in the prey. Model parameters are $R = 2.25$, $L = 0.945$, $\kappa = 67\ 113 \times 10^3$ t ($K = 37\ 285 \times 10^3$ t), $\alpha = 1.22 \times 10^{-6}\ t^{-1}$, $\beta = 4.845 \times 10^{-8}\ t^{-1}$.

Here is the Lotka-Volterra *decision model* with uncertainty

- A is the prey biomass (anchovy)
- H is the predator biomass (hake)
- The discrete-time Lotka-Volterra system with uncertainty is

$$
\begin{aligned}
A(t+1) &= A(t) \overbrace{\Big(w_A(t) + R - \frac{R}{\kappa} A(t) - \alpha H(t)\Big)}^{\mathcal{R}_A\big(A(t),H(t),w_A(t)\big)} \big(1 - E_A(t)\big) \\
H(t+1) &= H(t) \underbrace{\big(w_H(t) + L + \beta A(t)\big)}_{\mathcal{R}_H\big(A(t),H(t),w_H(t)\big)} \big(1 - E_H(t)\big),
\end{aligned}
$$

We make a heroic assumption about the set of scenarios

- An uncertainty scenario is a time sequence of uncertainty couples

$$\big(w_A(\cdot), w_H(\cdot)\big) = \Big(\big(w_A(t_0), w_H(t_0)\big), \dots, \big(w_A(T-1), w_H(T-1)\big)\Big)$$

- We assume that, at each time t,
  the uncertainties $\big(w_A(t), w_H(t)\big)$ can take any value in a two-dimensional set

$$\big(w_A(t), w_H(t)\big) \in \mathbb{S}(t) \subset \mathbb{R}^2$$

- Therefore, from one time t to the next $t+1$,
  uncertainties can be drastically different,
  since $\big(w_A(t), w_H(t)\big)$ is not related to $\big(w_A(t+1), w_H(t+1)\big)$
- Such an independence assumption is materialized by the property that
  a scenario can take any value in a product set

$$\big(w_A(\cdot), w_H(\cdot)\big) \in \prod_{t=t_0}^{T-1} \mathbb{S}(t)$$

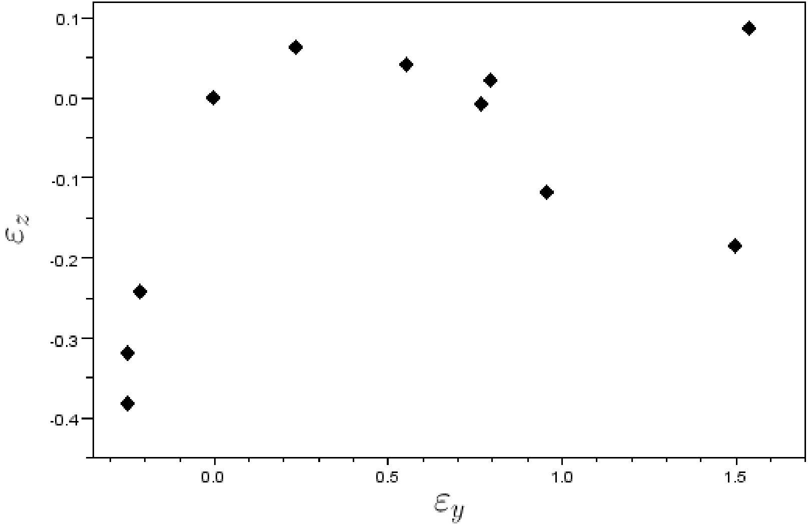
In practice, we consider stationary uncertainty sets forged from empirical data

- In practice, we consider stationary uncertainty sets $\mathbb{S}(t) = \mathbb{S}$
- We define $\overline{w}_A(t)$ and $\overline{w}_H(t)$ such that

$$\begin{cases} \overline{A}(t+1) &= \overline{A}(t)\big(\overline{w}_A(t) + R - \frac{R}{\kappa}\overline{A}(t) - \alpha\overline{H}(t)\big)\big(1 - \overline{v}_A(t)\big) \\ \overline{H}(t+1) &= \overline{H}(t)\big(\overline{w}_H(t) + L + \beta\overline{A}(t)\big)\big(1 - \overline{v}_H(t)\big) \end{cases}$$

  where $(\overline{A}(t), \overline{H}(t))_{t=t_0,\ldots,T}$ and $(\overline{v}_A(t), \overline{v}_H(t))_{t=t_0,\ldots,T-1}$ denote the empirical biomass and effort trajectories
- Therefore, our tough assumption on the set of scenarios is:
  any of the possible uncertainty of any year
  can materialize any *other* year

Empirical distribution of the uncertainties $(\overline{w}_A(t), \overline{w}_H(t))_{t=t_0,\ldots,T-1}$



We first consider the empirical uncertainty set and a refinement

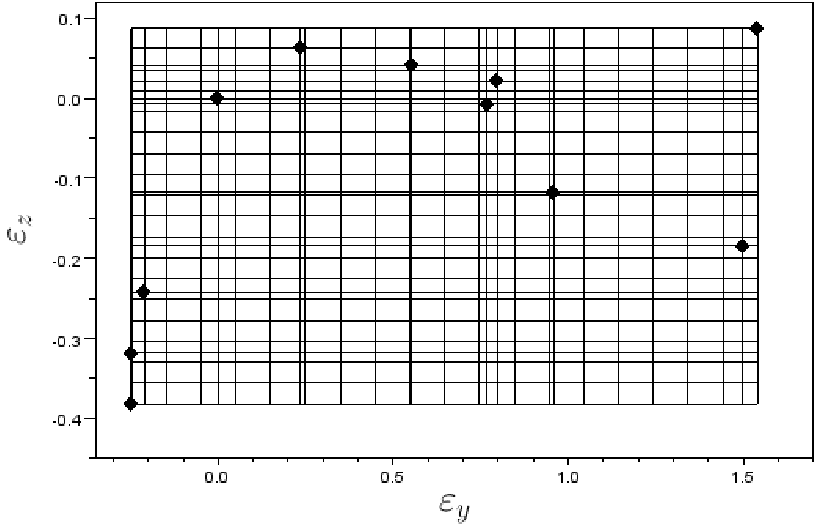
- The empirical uncertainties set is

$$\mathbb{S}^E = \underbrace{\{(\overline{w}_A(t), \overline{w}_H(t)) | t = t_0, \ldots, T-1\}}_{\text{empirical discrepancies}} \cup \underbrace{\{(0,0)\}}_{\text{deterministic case}}$$

- The refined empirical uncertainties set $\mathbb{S}^{ER}$ is made of 900 uncertainty couples delineated by a 30×30 grid over the surface $[\overline{w}_A^{min}, \overline{w}_A^{max}] \times [\overline{w}_H^{min}, \overline{w}_H^{max}]$, including all the uncertainty couples in $\mathbb{S}^E$
- Since $\{(0,0)\} \subset \mathbb{S}^E \subset \mathbb{S}^{ER}$, the corresponding robust and deterministic viability kernels satisfy

$$\mathbb{V}\text{iab}_1^{ER}(t_0) \subset \mathbb{V}\text{iab}_1^{E}(t_0) \subset \mathbb{V}\text{iab}(t_0)$$

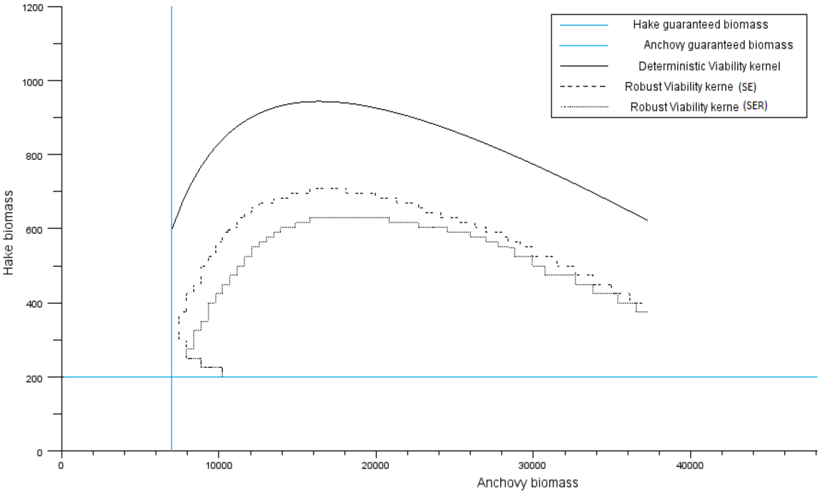
Figure : Uncertainty sets $\mathbb{S}^{E}$ (diamonds) and $\mathbb{S}^{ER}$ (grid)



Numerical resolution of the dynamic programming equation

- We discretize biomass, harvesting effort and uncertainty values
- A top loop for time steps embraces two nested loops for state variables A and H, respectively
- Next, loops over uncertainties nested in loops over harvesting efforts allow us to obtain the set of images associated with a biomass couple
- Images for target constraints that are not satisfied are set equal to zero
- We then project these images on the value function grid of the previous period, through linear interpolation
- At given efforts, we retain the minimum value obtained over all uncertainties
- Then, we retain the highest value produced by an effort couple among all
- It is this value that is multiplied with the value function of the current time period, at the location of the biomass couple at stake
- The robust viability kernel is defined by the set of grid points where the value function is equal to 1
- This implies that biomass couples for which all images do not fall between four 1 in the interpolation are excluded from the robust viability kernel (in the sense that we provide robustness with respect to grid approximation)

The robust viability kernels are noticeably smaller than the deterministic one



Now, we focus on worst-case uncertainties

- Numerical simulations led us to consider the three following uncertainty sets
  - $$\mathbb{S}^L = \{(\frac{\overline{w}_A^{min}}{2}, \frac{\overline{w}_H^{min}}{2}), (\frac{\overline{w}_A^{min}}{2}, \frac{\overline{w}_H^{max}}{2})\}$$
  - $$\mathbb{S}^M = \{(\overline{w}_A^{min}, \overline{w}_H^{min}), (\overline{w}_A^{min}, \overline{w}_H^{max})\}$$
  - $$\mathbb{S}^H = 1.1 * \mathbb{S}^M$$
- Since $\{(0,0)\} \subset \mathbb{S}^L \subset \mathbb{S}^M \subset \mathbb{S}^H$, the corresponding robust and deterministic viability kernels satisfy

$$\mathbb{V}\mathrm{iab}_1^H(t_0) \subset \mathbb{V}\mathrm{iab}_1^M(t_0) \subset \mathbb{V}\mathrm{iab}_1^L(t_0) \subset \mathbb{V}\mathrm{iab}(t_0)$$

Figure : Uncertainty sets $\mathbb{S}^L$ (crosses), $\mathbb{S}^M$ (diamonds) and $\mathbb{S}^H$ (triangles)

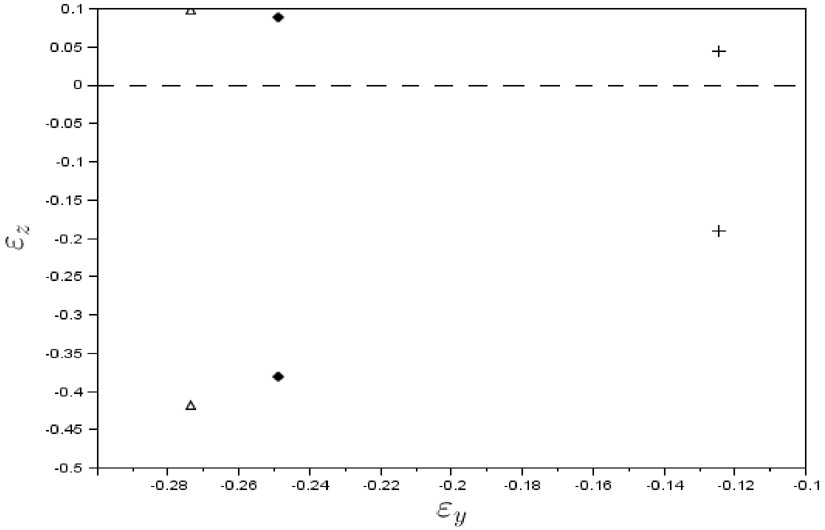
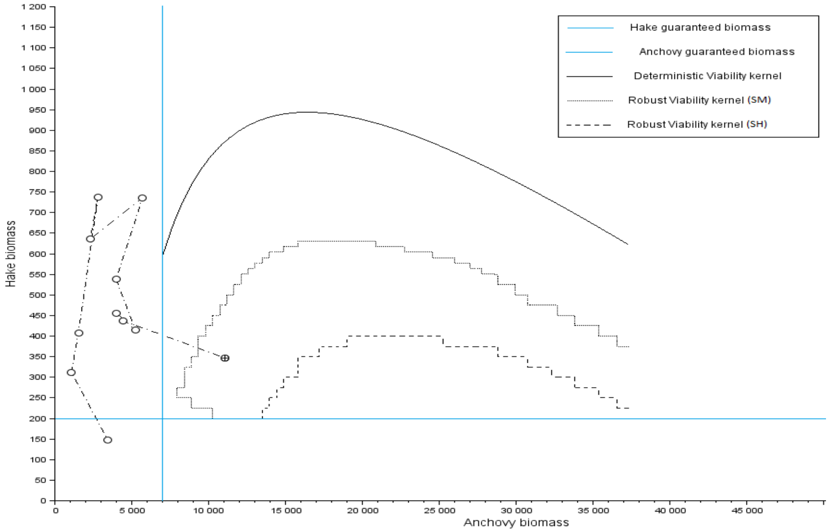


Figure : Robust viability kernels $\mathbb{V}\mathrm{iab}_1^L(t_0)$, $\mathbb{V}\mathrm{iab}_1^M(t_0)$ and $\mathbb{V}\mathrm{iab}_1^H(t_0)$ and the deterministic viability kernel



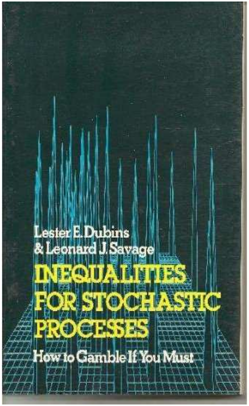
Summary

- We introduce uncertainties in the growth rates of interacting populations
- When populations start from a robust viable state, the fisheries can be managed so that both preservation and conservation objectives are met, *whatever the scenarios of uncertainties*
- To compute robust viable states, we make the strong assumption that, from one year t to the next $t+1$, uncertainties can be drastically different (independence)
- With this assumption, we compute the robust viability kernel by dynamic programming, for different sets of uncertainties
- We observe that the robust viability kernels are noticeably smaller than the deterministic ones
- We also identify uncertainties and scenarios that really matter for a precautionary approach: low growth for both species alternating with low growth of anchovy/high growth of hake

Outline of the presentation

Outline of the presentation

Maximizing the probability of success may be an objective



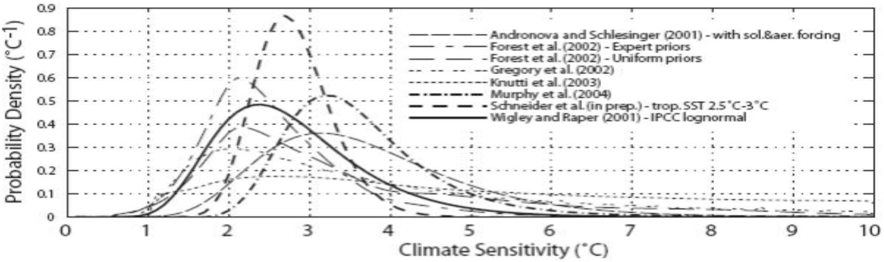
How to gamble if you must, L.E. Dubbins and L.J. Savage, 1965

Imagine yourself at a casino with $1,000. For some reason, you desperately need $10,000 by morning; anything less is worth nothing for your purpose.

The only thing possible is to gamble away your last cent, if need be, in an attempt to reach the target sum of $10,000.

- The question is how to play, not whether. What ought you do? How should you play?
  - Diversify, by playing 1 $ at a time?
  - Play boldly and concentrate, by playing 10,000 $ only one time?
- What is your decision criterion?

The set Ω of scenarios can be equipped with a probability $\mathbb{P}$ (though this is a delicate issue!)



In practice, one often assumes that the components $\big(w(t_0), \ldots, w(T-1)\big)$ form an independent and identically distributed sequence of random variables, or form a Markov chain, or a time series

The viability probability is the probability of satisfying constraints under a policy

Viability probability

The viability probability associated with
the initial time t_0, the initial state x_0 and the policy $\texttt{Pol}$
is the probability $\mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right]$ of the set $\Omega_{\texttt{Pol},t_0,x_0}$ of viable scenarios

$$\mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right] = \text{Proba} \left\{ \begin{array}{l} \text{scenarios along which} \\ \text{the state } x(\cdot) \text{ and control } u(\cdot) \text{ trajectories} \\ \text{generated by dynamics } \texttt{Dyn} \text{ and policy } \texttt{Pol} \\ \text{starting from initial state } x_0 \text{ at initial time } t_0 \\ \text{satisfy the constraints} \\ \text{from initial time } t_0 \text{ to horizon } T \end{array} \right\}$$

The maximal viability probability is the upper bound for the probability of satisfying constraints

Maximal viability probability and optimal viable policy

The maximal viability probability is

$$\max_{\mathtt{Pol}} \mathbb{P}\left[\Omega_{\mathtt{Pol},t_0,x_0}\right]$$

An optimal viable policy $\mathtt{Pol}^{\star}$ satisfies

$$\mathbb{P}\left[\Omega_{\mathtt{Pol}^{\star},t_0,x_0}\right] \geq \mathbb{P}\left[\Omega_{\mathtt{Pol},t_0,x_0}\right]$$

In a sense, any optimal viable policy makes the set of viable scenarios the “largest” possible

Let us introduce the stochastic viability Bellman function

Suppose that the primitive random variables
$\big(w(t_0), w(t_0+1), \ldots, w(T-2), w(T-1)\big)$
are independent under the probability $\mathbb{P}$

Bellman function / stochastic viability value function

Define the probability-to-go as

$$V(t,x) :=$$

$$\max_{\texttt{Pol}} \mathbb{P}\Big(w(\cdot) \in \Omega \mid \overbrace{\texttt{Pol}\big(s, x(s)\big) \in \mathbb{B}\big(s, x(s)\big)}^{\text{control constraints}} \text{ and } \overbrace{x(s) \in \mathbb{A}(s)}^{\text{state constraints}} \text{ for } s \geq t \Big)$$

where $x(s+1) = \texttt{Dyn}\big(s, x(s), \texttt{Pol}\big(s, x(s)\big), w(s)\big)$ and $x(t) = x$

- The function $V(t,x)$ is called stochastic viability value function or Bellman function
- The original problem is $V(t_0, x_0)$

The dynamic programming equation is a backward equation satisfied by the stochastic viability value function

Proposition

If the primitive random variables $\big(w(t_0), w(t_0+1), \dots, w(T-2), w(T-1)\big)$ *are independent under the probability* $\mathbb{P}$*, the stochastic viability value function* $V(t,x)$ *satisfies the following backward induction, where* t *runs from* $T-1$ *down to* t_0

$$
\begin{aligned}
V(T,x) &= \mathbf{1}_{\mathbb{A}(T)}(x) \\
V(t,x) &= \mathbf{1}_{\mathbb{A}(t)}(x) \max_{u \in \mathbb{B}(t,x)} \mathbb{E}_{w(t)}\Big[V\Big(t+1, \mathtt{Dyn}\big(t,x,u,w(t)\big)\Big)\Big]
\end{aligned}
$$

The stochastic viable dynamic programming equation yields stochastic viable policies

For any time t and state x, let us assume that the set

$$\mathbb{B}^{\text{viab}}(t,x) := \operatorname*{argmax}_{u \in \mathbb{B}(t,x)} \Big(\mathbf{1}_{\mathbb{A}(t)}(x) \mathbb{E}_{w(t)} \Big[V\Big(t+1, \mathtt{Dyn}(t,x,u,w(t))\Big) \Big] \Big)$$

of viable controls is not empty

Proposition

Then, any (measurable) policy $\mathtt{Pol}$ *such that* $\mathtt{Pol}^{\star}(t,x) \in \mathbb{B}^{viab}(t,x)$ *is an optimal viable policy which achieves the maximal viability probability*

$$V(t_0, x_0) = \max_{\mathtt{Pol}} \mathbb{P}\left[\Omega_{\mathtt{Pol}, t_0, x_0}\right]$$

Outline of the presentation

In the dam management multi-objective problem, the "tourism" constraint must be met with probability 90% at least

Dam management under "tourism" probability constraint

- Traditional cost minimization/payoff maximization

$$\max \mathbb{E}\left[\sum_{t=t_0}^{T-1} \overbrace{p(t)q(t)}^{\text{turbined water payoff}} + \overbrace{\mathtt{K}(S(T))}^{\text{final volume utility}}\right]$$

- For "tourism" reasons, the following probability constraint is added

$$\text{Probability}\left\{\begin{array}{l}\text{water inflow scenarios along which}\\ \text{the volumes } S(t) \text{ are above the}\\ \text{threshold } S^{\flat} \text{ for periods } t \text{ in summer}\end{array}\right\} \geq 90\%$$

Stochastic viability kernels

In stochastic viability, state constraints are to be met along time with a given confidence level

$$\mathbb{P}\Big(w(\cdot) \in \Omega \mid x(t) \in \mathbb{A}(t) \text{ for } t = t_0, \ldots, T \Big) \geq \beta$$

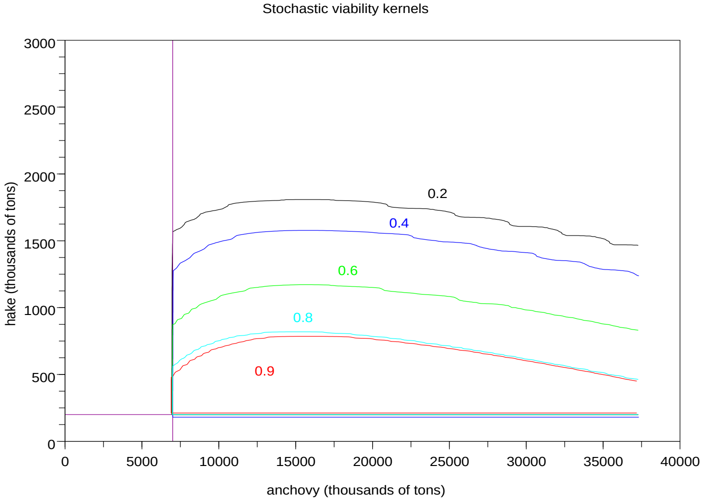
Stochastic viability kernels

The stochastic viability kernel at confidence level $\beta \in [0,1]$ is

$$\mathbb{V}\mathrm{iab}_\beta(t_0) := \left\{ x_0 \in \mathbb{X} \;\middle|\; \begin{array}{l} \text{there exists a policy } \mathtt{Pol} \in \mathfrak{U}^{ad} \text{ such that} \\ \mathbb{P}\Big(w(\cdot) \in \Omega \mid x(t) \in \mathbb{A}(t) \text{ for } t = t_0, \ldots, T \Big) \geq \beta \end{array} \right\}$$

where the state $x(t) = X_{\mathtt{Dyn}}[t_0, x_0, \mathtt{Pol}, w(\cdot)](t)$ is the outcome of the state solution map

Stochastic viability kernels $\mathbb{V}\mathrm{iab}_\beta(t_0)$ for a hake-anchovy fisheries model



Stochastic viable policies

Stochastic viable policies

Stochastic viable policies are

$$\mathcal{U}_{\beta}^{\text{viab}}(t_0, x_0) := \left\{ \texttt{Pol} \in \mathcal{U}^{ad} \;\middle|\; \mathbb{P}\Big(w(\cdot) \in \Omega \mid x(t) \in \mathbb{A}(t) \;\text{ for }\; t = t_0, \ldots, T \Big) \geq \beta \right\}$$

where the state $x(t) = X_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)](t)$ corresponds to the state solution map

The dynamic programming equation yields the viability kernels, as well as stochastic viable policies

- The viability kernel at confidence level β is the section of level β of the stochastic value function:

$$V(t_0, x_0) \geq \beta \iff x_0 \in \mathbb{V}\mathrm{iab}_\beta(t_0)$$

- If, for any time t and state x, the set $\mathbb{B}^{\text{viab}}(t,x) :=$

$$\underset{u \in \mathbb{B}(t,x)}{\operatorname{argmax}} \Big(\mathbf{1}_{\mathbb{A}(t)}(x) \mathbb{E}_{w(t)} \Big[V\Big(t+1, \mathtt{Dyn}(t,x,u,w(t)) \Big) \Big] \Big)$$

is not empty, then any (measurable) $\mathtt{Pol}^\star \in \mathfrak{U}$ such that $\mathtt{Pol}^\star(t,x) \in \mathbb{B}^{\text{viab}}(t,x)$ belongs to $\mathfrak{U}^{\text{viab}}_\beta(t_0,x_0)$ for $x_0 \in \mathbb{V}\mathrm{iab}_\beta(t_0)$

Outline of the presentation

1. Multi-objectives dynamic management under uncertainty
   - Assessment frameworks are designed to deal with multiple goals
   - Recalls on uncertain dynamical systems under constraints
2. Criterion and constraints in the uncertain case
   - A dam management example
   - Constraints penalization
   - Viable scenarios
3. The robust viability problem
   - The deterministic viability approach
   - Robust viable controls and states
   - Robust viability analysis of anchovy–hake Peruvian fis
4. The stochastic viability problem
   - Maximal viability probability and dynamic programmin
   - Stochastic viability kernels
   - Dam stochastic viable management
   - Nephrops-hake fishery viable management
5. Summary

We propose a stochastic viability formulation to treat symmetrically and to guarantee both environmental and economic objectives

- Given two thresholds to be guaranteed
  - a volume $S^\flat$ (measured in cubic hectometers hm^3)
  - a payoff $P^\flat$ (measured in numeraire \$)
- we look after policies achieving the maximal viability probability

$$\Pi(S^\flat, P^\flat) = \max \text{Proba} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \geq S^\flat \\ \text{for all time } \ t \in \{ \text{July, August} \} \\ \text{and the final payoff } \ P(T) \geq P^\flat \end{array} \right\}$$

- $\Pi(S^\flat, P^\flat)$ is the maximal probability to guarantee to be above the thresholds $S^\flat$ and $P^\flat$

The stochastic viability formulation requires to redefine state and dynamics

- The state is the couple $x(t) = \big(S(t), P(t)\big)$ volume/payoff
- The control $u(t) = q(t)$ is the turbined water
- The dynamics is

$$
\begin{aligned}
\underbrace{S(t+1)}_{\text{future volume}} &= \min\{S^\sharp, \underbrace{S(t)}_{\text{volume}} - \underbrace{q(t)}_{\text{turbined}} + \underbrace{a(t)}_{\text{inflow volume}} \}, \\
& \quad t = t_0, \ldots, T-1 \\
\underbrace{P(t+1)}_{\text{future payoff}} &= \underbrace{P(t)}_{\text{payoff}} + \underbrace{p(t)q(t) - \epsilon q(t)^2}_{\text{turbined water payoff}}, \quad t = t_0, \ldots, T-2 \\
P(T) &= P(T-1) + \underbrace{\mathtt{K}\big(S(T)\big)}_{\text{final volume utility}}
\end{aligned}
$$

In the stochastic viability formulation, objectives are dressed as state constraints

- The control constraints are

$$u(t) \in \mathbb{B}\big(t, x(t)\big) \iff 0 \leq q(t) \leq S(t)$$

- The state constraints are

$$x(t) \in \mathbb{A}(t) \iff \begin{cases} S(t) \geq S^{\flat} & , \quad \forall t \in \{ \text{July, August} \} \\ P(T) \geq P^{\flat} & \end{cases}$$

For each couple of thresholds on payoff and stock, we write a dynamic programming equation

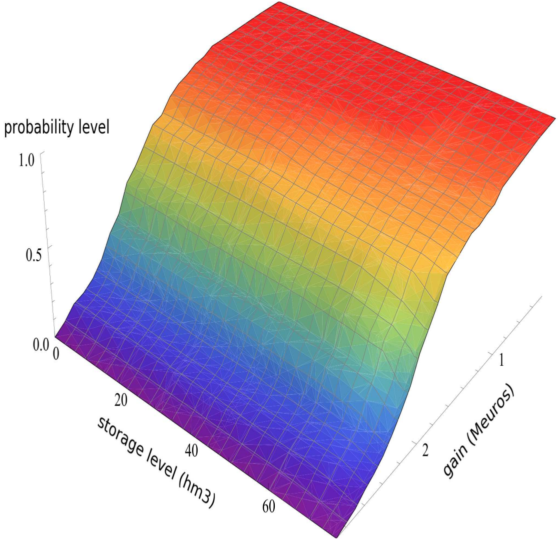
- Abstract version

$$
\begin{aligned}
V(T,x) &= \mathbf{1}_{\mathbb{A}(T)}(x) \\
V(t,x) &= \mathbf{1}_{\mathbb{A}(t)}(x) \max_{u \in \mathbb{B}(t,x)} \mathbb{E}_{w(t)}\Big[V\Big(t+1, \mathtt{Dyn}\big(t,x,u,w(t)\big)\Big)\Big]
\end{aligned}
$$

- Specific version

$$
\begin{aligned}
V(T,S,P) &= \mathbf{1}_{\{P \geq P^\flat\}} \\
V(T-1,S,P) &= \max_{0 \leq q \leq S} \mathbb{E}_{a(t)}\Big[V\Big(t+1, S-q+a(t), P+\mathtt{K}(S)\Big)\Big] \\
V(t,S,P) &= \max_{0 \leq q \leq S} \mathbb{E}_{a(t)}\Big[V\Big(t+1, S-q+a(t), P+pq-\epsilon q^2\Big)\Big] \;, \; t \notin \{\,\text{July, August}\,\} \\
V(t,S,P) &= \mathbf{1}_{\{S \geq S^\flat\}} \max_{0 \leq q \leq S} \mathbb{E}_{a(t)}\Big[V\Big(t+1, S-q+a(t), P+pq-\epsilon q^2\Big)\Big] \;, \\
& \quad t \in \{\,\text{July, August}\,\}
\end{aligned}
$$

We plot the maximal viability probability $\Pi(S^\flat, P^\flat)$ as a function of guaranteed thresholds $S^\flat$ and $P^\flat$

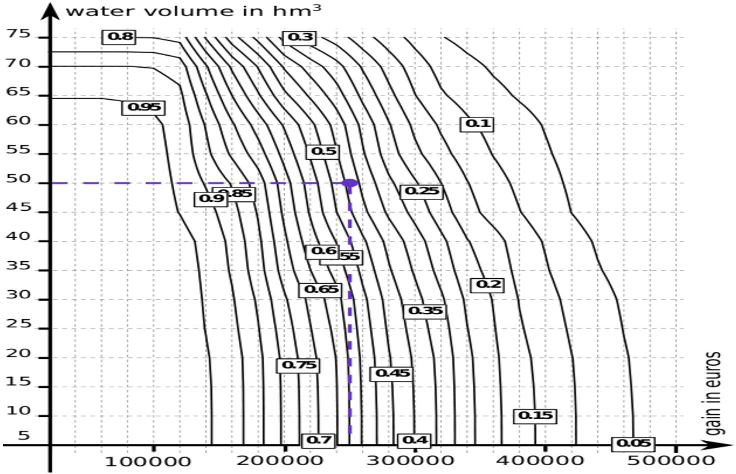


For example, the probability to guarantee

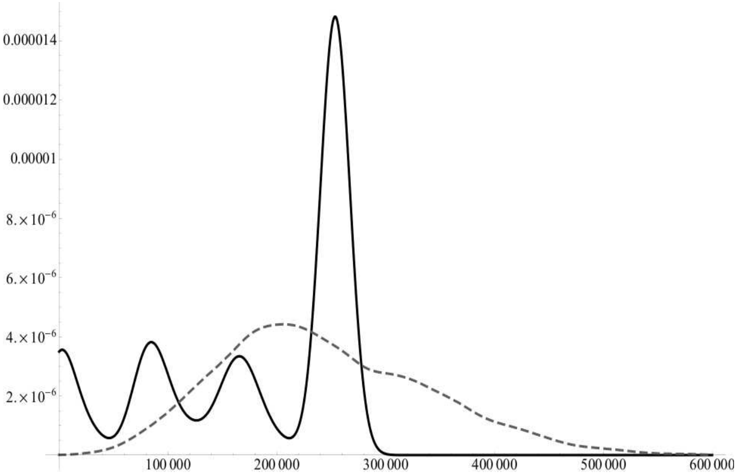
- a final payoff above $P^\flat = 1$ Meuros
- and a volume above $S^\flat = 40$ hm^3 in July and August

is about 90%

We plot iso-values for the maximal viability probability as a function of guaranteed thresholds $S^\flat$ and $P^\flat$



The probability distribution of the random gain reflects the viability objectives



Outline of the presentation

We tackle the issue of bycatch in nets

M. De Lara and V. Martinet.
Multi-criteria dynamic decision under uncertainty: a stochastic viability analysis and an application to sustainable fishery management.
In Mathematical Biosciences, Volume 217, Issue 2, February 2009, Pages 118-124

Hake and nephrops in technical interaction

$$N_1^h(t+1) = w^h(t) \text{ uncertain hake recruitment}$$
$$N_1^n(t+1) = w^n(t) \text{ uncertain nephrops recruitment}$$

$$N_a^h(t+1) = N_{a-1}^h(t) \left(1 - M_{a-1}^h - \overbrace{u(t) F_{a-1}^{nh}}^{\text{hake bycatch}} - F_{a-1}^{hh} \right)$$

$$N_a^n(t+1) = N_{a-1}^n(t) \left(1 - M_{a-1}^n - \overbrace{u(t) F_{a-1}^{nn}}^{\text{nephrops fishing mortality}} \right)$$

$$N_A^h(t+1) = N_{A-1}^h(t) \left(1 - M_{A-1}^h - u(t) F_{A-1}^{nh} - F_{A-1}^{hh} \right) + N_A^h(t) \left(1 - M_A^h - u(t) F_A^{nh} - F_A^{hh} \right)$$

$$N_A^n(t+1) = N_{A-1}^n(t) \left(1 - M_{A-1}^n - u(t) F_{A-1}^{nn} \right) + N_A^n(t) \left(1 - M_A^n - u(t) F_A^{nn} \right)$$

The relative effort of the nephrops fleet has to be controlled to ensure both nephrops fleet profitability and hake preservation

- Economic objective: nephrops fishery is economically viable if the gross return is greater than a threshold

$$\underbrace{P\big(N^n(t), u(t)\big)}_{\text{payoff}} \geq P^\flat$$

- Ecological objective: fishery is ecologically viable if its impact by bycatch on the hake biology is compatible with sufficient recruitment of mature hakes

$$\underbrace{N_4^h(t)}_{\text{fourth age-class}} \qquad \geq (N_4^h)^\flat$$

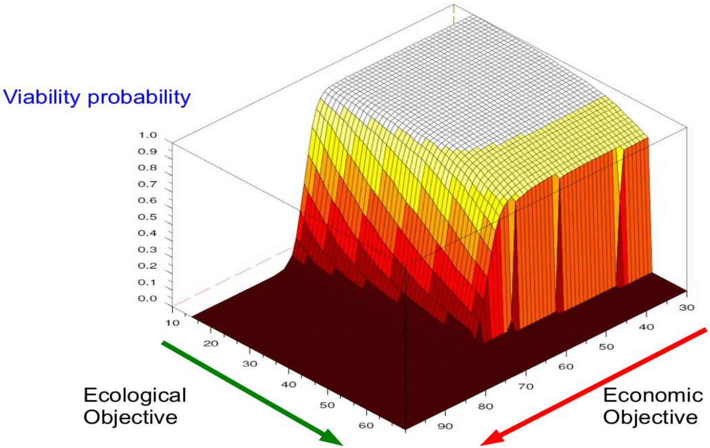
An optimal viable policy can be calculated thanks to monotonicity properties

- Due to monotonicity properties
  - of the dynamics, increasing in the state variable and decreasing in the control
  - of the constraints, increasing in the state variable and decreasing in the control
- we can prove that

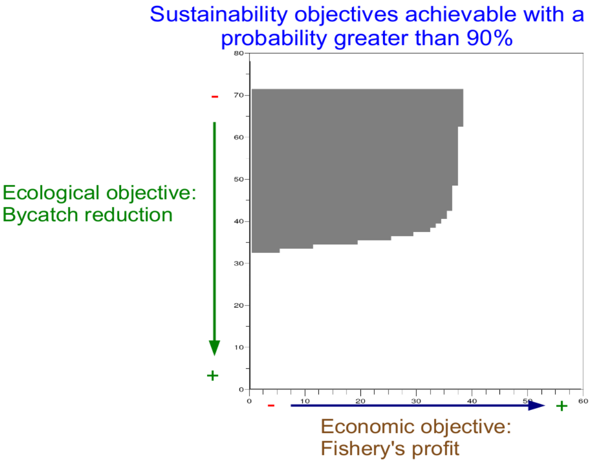
$$\mathtt{Pol}^{\star}(t, N) = \inf\{u \in [0, u^{\sharp}] \mid P(N^n, u) \geq P^{\flat}\}$$

is an optimal viable policy

Maximal viability probability function of $P^\flat$ and $(N_4^h)^\flat$



Iso-values for the maximal viability probability as a function of guaranteed thresholds $P^\flat$ and $(N_4^h)^\flat$



Outline of the presentation

Summary

- When uncertainties affect the dynamics at each time step, the state outcome is no longer unique
- The state trajectory is now contingent on policy *and* scenario
- Therefore, state constraints are met or not depending on the scenario
- In the robust setting, state constraints have to be met for all the scenarios in a subset of scenarios
- In the probabilistic setting, state constraints have to be met with a given confidence level, possibly the highest