IMCA Instituto de Matemática y Ciencias Afines

Smart Energy and Stochastic Optimization

◇

WORKSHOP

# Mixing Dynamic Programming and Spatial Decomposition Methods

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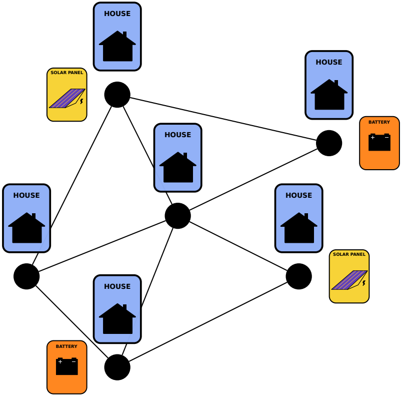
# Motivation

We consider a *peer-to-peer* microgrid where houses share equipments and exchange energy, and we formulate it as a large-scale stochastic optimization problem




**How to manage it in an (sub)optimal manner?**

# Motivation

We will see that, for a large district microgid, e.g.

- 48 buildings
- 16 batteries
- 71 edges network

methods mixing temporal decomposition (dynamic programming) and spatial decomposition (price or resource allocation) give better results than the standard SDDP algorithm (implemented using approximations)

- in terms of CPU time: $\times 3$ faster

| | |
|---|---|
| SDDP CPU time: 453' | Decomp CPU time: 128' |

- in terms of cost gap: 1.5% better

| | |
|---|---|
| SDDP policy cost: 3550 | Decomp policy cost: 3490 |

# Lecture outline

**Tools for mixing spatial and temporal decomposition methods**
- Upper and lower bounds using spatial decomposition
- Temporal decomposition using dynamic programming
- The case of deterministic coordination processes

**Application to the management of urban microgrids**
- Nodal decomposition of a network optimization problem
- Numerical results on urban microgrids of increasing size

**Another point of view: decentralized information structure**
- Centralized versus decentralized information structure
- Bounds for the decentralized information structure
- Analysis of the upper bound

Tools for mixing spatial and temporal decomposition methods

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Temporal decomposition using dynamic programming

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# An abstract optimization problem

We consider the following optimization problem

$$V_0^* = \min_{u^1 \in \mathcal{U}^1_{\mathrm{ad}}, \cdots, u^N \in \mathcal{U}^N_{\mathrm{ad}}} \sum_{i=1}^{N} J^i(u^i)$$

$$\text{s.t.} \quad \underbrace{\big(\Theta^1(u^1), \cdots, \Theta^N(u^N)\big) \in S}_{\text{coupling constraint}}$$

with

- $u^i \in \mathcal{U}^i$ be a local decision variable
- $J^i : \mathcal{U}^i \to \mathbb{R}, \; i \in [\![1, N]\!]$ be a local objective
- $\mathcal{U}^i_{\mathrm{ad}}$ be a subset of $\mathcal{U}^i$
- $\Theta^i : \mathcal{U}^i \to \mathcal{C}^i$ be a local constraint mapping
- $S$ be a subset of $\mathcal{C} = \mathcal{C}^1 \times \cdots \times \mathcal{C}^N$

We denote by $S^o$ the polar cone of $S$

$$S^o = \Big\{ (p^1, \ldots, p^N) \in \mathcal{C}^\star \;\text{ s.t. }\; \sum_{i=1}^{N} \big\langle p^i, \, r^i \big\rangle \leq 0 \quad \forall (r^1, \ldots, r^N) \in S \Big\}$$

# Price and resource value functions

For each $i \in [\![1, N]\!]$,

- for any price $p^i \in (\mathcal{C}^i)^\star$, we define the local price value

$$\underline{V}_0^i[p^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} \; J^i(u^i) + \langle p^i , \Theta^i(u^i) \rangle$$

- for any resource $r^i \in \mathcal{C}^i$, we define the local resource value

$$\overline{V}_0^i[r^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} \; J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

## Theorem 1 (Upper and lower bounds for optimal value)

*For any*

- *admissible price* $p = (p^1, \cdots, p^N) \in S^o$
- *admissible resource* $r = (r^1, \cdots, r^N) \in S$

$$\sum_{i=1}^N \underline{V}_0^i[p^i] \;\leq\; V_0^* \;\leq\; \sum_{i=1}^N \overline{V}_0^i[r^i]$$

# The case of multistage stochastic optimization I

Assume that the local price value

$$\underline{V}_0^i[p^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) + \left\langle p^i , \Theta^i(u^i) \right\rangle ,$$

corresponds to a stochastic optimal control problem

$$\underline{V}_0^i[\mathbf{P}^i](x_0^i) = \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E}\left[ \sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \left\langle \mathbf{P}_t^i , \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) \right\rangle + K^i(\mathbf{X}_T^i) \right]$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) , \quad \mathbf{X}_0^i = x_0^i$$

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0, \cdots, \mathbf{W}_t)$$

This local control problem can be effectively solved at optimality by Dynamic Programming (DP) under restrictive assumptions:

- the dimension of the state variable $x^i$ is small
- the noise process $\mathbf{W}$ is a white noise process
- the price process $\mathbf{P}^i$ follows a dynamics in small dimension

DP leads to a collection $\left\{ \underline{V}_t^i[\mathbf{P}^i] \right\}_{t \in [\![0,T]\!]}$ of local price value functions

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# The case of multistage stochastic optimization II

Similar considerations hold true for the local resource value

$$\overline{V}_0^i[r^i] = \min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

considered as a stochastic optimal control problem

$$\overline{V}_0^i[\mathbf{R}^i](x_0^i) = \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E}\left[\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i)\right]$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) \ , \ \ \mathbf{X}_0^i = x_0^i$$

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0, \cdots, \mathbf{W}_t)$$

$$\Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = \mathbf{R}_t^i$$

This local control problem can be solved by Dynamic Programming, hence a collection $\left\{\overline{V}_t^i[\mathbf{R}^i]\right\}_{t \in [\![0, T]\!]}$ of local resource value functions

# Mix of spatial and temporal decompositions

For any admissible price process $\mathbf{P} \in S^o$ and any admissible resource process $\mathbf{R} \in S$, we have bounds of the optimal value $V_0^*$

$$\sum_{i=1}^{N} \underline{V}_0^i[\mathbf{P}^i](x_0^i) \;\leq\; V_0^* \;\leq\; \sum_{i=1}^{N} \overline{V}_0^i[\mathbf{R}^i](x_0^i)$$

1. To obtain the bounds, we perform spatial decompositions giving
   - a collection $\left\{\underline{V}_0^i[\mathbf{P}^i](x_0^i)\right\}_{i\in[\![1,N]\!]}$ of price local values
   - a collection $\left\{\overline{V}_0^i[\mathbf{R}^i](x_0^i)\right\}_{i\in[\![1,N]\!]}$ of resource local values

   *The computation of these local values can be performed in parallel*

2. To compute each local value, we perform temporal decomposition based on Dynamic Programming. For each $i$, we obtain
   - a sequence $\left\{\underline{V}_t^i[\mathbf{P}^i]\right\}_{t\in[\![0,T]\!]}$ of price local value functions
   - a sequence $\left\{\overline{V}_t^i[\mathbf{R}^i]\right\}_{t\in[\![0,T]\!]}$ of resource local value functions

   *The computation of these local values functions is done sequentially*

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# Mix of spatial and temporal decompositions II

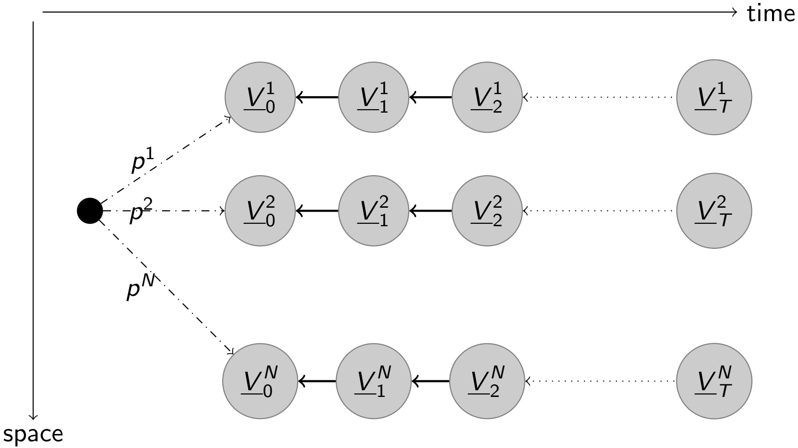



Figure: The case of price decomposition

# The case of deterministic price and resource processes I

We assume that $\mathbf{W}$ is a white noise process, and we restrict ourselves to **deterministic** admissible processes $p \in S^o$ and $r \in S$

- The local value functions $\underline{V}^i_t[p^i]$ and $\overline{V}^i_t[r^i]$ are easy to compute because they only depend on the local state variable $x^i$
- It is easy to obtain tighter bounds by maximizing the lower bound w.r.t. prices and minimizing the upper bound w.r.t. resources

$$\sup_{p \in S^o} \sum_{i=1}^{N} \underline{V}^i_0[p^i](x^i_0) \leq V^*_0 \leq \inf_{r \in S} \sum_{i=1}^{N} \overline{V}^i_0[r^i](x^i_0)$$

But limiting ourselves to deterministic processes could prove restrictive...

# The case of deterministic price and resource processes II

The local value functions $\underline{V}_t^i[p^i]$ and $\overline{V}_t^i[r^i]$ allow the computation of global policies by solving static optimization problems

- In the case of local price value functions, the policy is obtained as

$$\underline{\gamma}_t(x_t^1, \cdots, x_t^N) \in \underset{u_t^1, \cdots, u_t^N}{\arg\min} \ \mathbb{E}\left[\sum_{i=1}^N L_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) + \sum_{i=1}^N \underline{V}_{t+1}^i[p^i](\mathbf{X}_{t+1}^i)\right]$$

$$\text{s.t.} \ \mathbf{X}_{t+1}^i = g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) \ , \ \forall i \in [\![1, N]\!]$$

$$\big(\Theta_t(x_t^1, u_t^1), \cdots, \Theta_t(x_t^N, u_t^N)\big) \in S_t$$

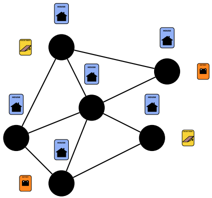
- Another policy based on local resource value functions is available

Estimating the expected cost of these two policies by Monte Carlo simulation leads to statistical upper bounds of the optimal cost of the problem since the two policies are admissible

# Progress status

- First, we have highlighted lower and upper bounds for a global optimization problem with coupling constraints thanks to two spatial decomposition schemes
  - price decomposition
  - resource decomposition
- Second, we have computed the lower and upper bounds by dynamic programming (temporal decomposition)
- Third, we have devised two online policies for the global problem based on the price and resource Bellman value functions

- Now, we apply these decomposition schemes to large-scale microgrids



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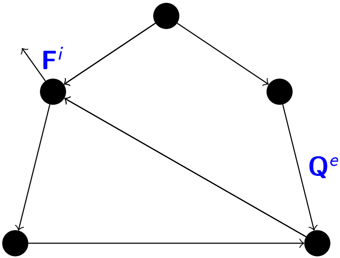
# Network and flows

Directed graph $G = (\mathcal{V}, \mathcal{E})$




- $\mathbf{Q}_t^e$ flow through edge $e$,
- $\mathbf{F}_t^i$ flow imported at node $i$

Let $A$ be the *node-edge* incidence matrix

Each node corresponds to a building with its own devices (battery, hot water tank, solar panel...)

Each edge allows energy exchanges between two nodes

At each time $t \in [\![0, T-1]\!]$, the Kirchhoff current law couples node and edge flows

$$A\mathbf{Q}_t + \mathbf{F}_t = 0 \quad \text{at time } t$$

or equivalently

$$\mathbb{A}\mathbf{Q} + \mathbf{F} = 0$$

# Optimization problem at a given node

At each **node** $i \in \mathcal{V}$, given a node flow process $\mathbf{F}^i$, we minimize the house cost

$$J^i_{\mathcal{V}}(\mathbf{F}^i) = \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E}\left[\sum_{t=0}^{T-1} L^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{W}^i_{t+1}) + K^i(\mathbf{X}^i_T)\right]$$

subject to, for all $t \in [\![0, T-1]\!]$

i) nodal dynamics constraints (battery, hot water tank)

$$\mathbf{X}^i_{t+1} = g^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{W}^i_{t+1})$$

ii) non-anticipativity constraints (future remains unknown)

$$\sigma(\mathbf{U}^i_t) \subset \sigma(\mathbf{W}_0, \cdots, \mathbf{W}_{t+1})$$

iii) nodal load balance equations (demand - production = import)

$$\Delta^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{W}^i_{t+1}) = \mathbf{F}^i_t$$

*Remarks*

- Local noise $\mathbf{W}^i_t$ in the formulation of problem at node $i$
- Global noise $\mathbf{W}_t = (\mathbf{W}^1_{t+1}, \ldots, \mathbf{W}^N_{t+1})$ in the non-anticipativity constraint

# Transportation cost and global optimization problem

We define the network cost as the sum over time and **edges** of the costs of flows $\mathbf{Q}_t^e$ through the edges of the network

$$J_{\mathcal{E}}(\mathbf{Q}) = \mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{e \in \mathcal{E}} l_t^e(\mathbf{Q}_t^e)\right]$$

This transportation cost is additive in space, in time and in uncertainty!

The global optimization problem is obtained by gathering all elements

$$V_0^* = \min_{\mathbf{F},\mathbf{Q}} \sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(\mathbf{F}^i) + J_{\mathcal{E}}(\mathbf{Q})$$

$$\text{s.t. } \mathbb{A}\mathbf{Q} + \mathbf{F} = 0$$

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# Price and resource decompositions

The formalism developed previously leads to the following

- **Price** problem:

$$\underline{V}_0[p] = \min_{\mathbf{F},\mathbf{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\mathbf{F}^i) + J_{\mathcal{E}}(\mathbf{Q}) + \big\langle p, \mathbb{A}\mathbf{Q} + \mathbf{F} \big\rangle$$

$$= \sum_{i\in\mathcal{V}} \underbrace{\Big(\min_{\mathbf{F}_i} \; J^i_{\mathcal{V}}(\mathbf{F}^i) + \big\langle p^i, \mathbf{F}^i \big\rangle\Big)}_{\text{Node } i\text{'s subproblem}} + \underbrace{\Big(\min_{\mathbf{Q}} \; J_{\mathcal{E}}(\mathbf{Q}) + \big\langle \mathbb{A}^\top p, \mathbf{Q} \big\rangle\Big)}_{\text{Network subproblem}}$$

- **Resource** problem:

$$\overline{V}_0[r] = \min_{\mathbf{F},\mathbf{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\mathbf{F}^i) + J_{\mathcal{E}}(\mathbf{Q}) \quad \text{s.t.} \quad \mathbb{A}r + \mathbf{F} = 0 \;, \;\; \mathbf{Q} = r$$

$$= \sum_{i\in\mathcal{V}} \Big(\min_{\mathbf{F}_i} \; J^i_{\mathcal{V}}(\mathbf{F}^i) \;\text{ s.t. }\; \mathbf{F}^i = -(\mathbb{A}r)^i\Big) + \Big(\min_{\mathbf{Q}} \; J_{\mathcal{E}}(\mathbf{Q}) \;\text{ s.t. }\; \mathbf{Q} = r\Big)$$

**Objective**

Find **deterministic** processes $\widehat{p}$ and $\widehat{r}$ with a gap as small as possible

$$\sup_p \underline{V}_0[p] \;\leq\; V_0^\sharp \;\leq\; \inf_r \overline{V}_0[r]$$

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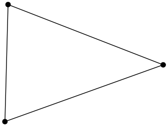
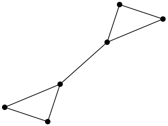
# Different urban configurations

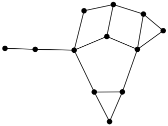
**3-Nodes**

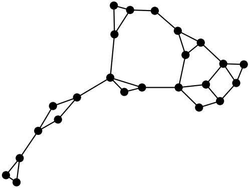


**6-Nodes**

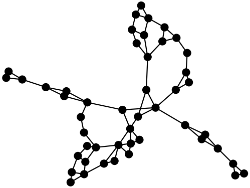


**12-Nodes**





**24-Nodes**



**48-Nodes**

with battery and solar panels at some nodes

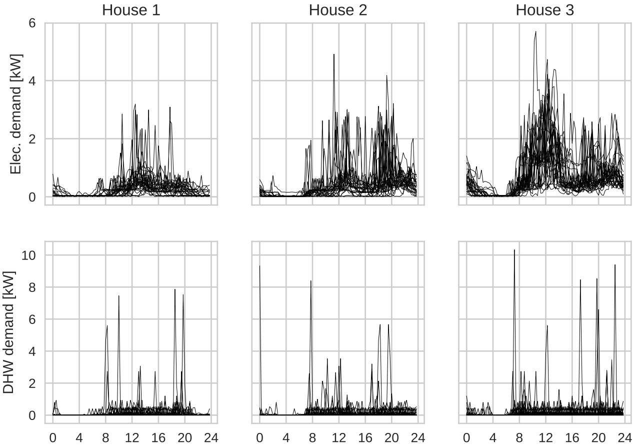
# Problem settings

Thanks to the periodicity of demands and electricity tariffs, the microgrid management problem can be solved day by day

- One day horizon with a 15mn time step: $T = 96$
- Weather corresponds to a sunny day in Paris *(June 28, 2015)*
- We mix three kinds of buildings
  1. battery + electrical hot water tank
  2. solar panel + electrical hot water tank
  3. electrical hot water tank

  and suppose that all consumers are commoners sharing their devices

# Electrical and thermal demands uncertainty




# Algorithms implemented on the problem

## SDDP

We use the SDDP algorithm to solve the problem globally. . .

- but noises $\mathbf{W}_t^1, \cdots, \mathbf{W}_t^N$ are independent node by node, so that the support size of the noise may be huge ($|\mathrm{supp}(\mathbf{W}_t^i)|^N$). We must resample the noise to be able to compute the cuts

## Price decomposition

Spatial decomposition and maximization w.r.t. a deterministic price $p$

- Each nodal subproblem solved by a DP-like method
- Maximisation w.r.t. $p$ by Quasi-Newton (BFGS) method

$$p^{(k+1)} = p^{(k)} + \rho^{(k)} H^{(k)} \nabla \underline{V}_0[p^{(k)}]$$

- Oracle $\nabla \underline{V}_0[p]$ estimated by Monte Carlo ($N^{scen} = 1,000$)

## Resource decomposition

Spatial decomposition and minimization w.r.t. a deterministic resource process $r$ with a similar implementation to the price decomposition

# Exact upper and lower bounds on the global problem

| | Network | **3-Nodes** | **6-Nodes** | **12-Nodes** | **24-Nodes** | **48-Nodes** |
|---|---|---|---|---|---|---|
| State dim. | $\vert\mathbb{X}\vert$ | 4 | 8 | 16 | 32 | 64 |
| SDDP | time | 1' | 3' | 10' | 79' | 453' |
| SDDP | LB | 225.2 | 455.9 | 889.7 | 1752.8 | 3310.3 |
| Price | time | 6' | 14' | 29' | 41' | 128' |
| Price | LB | 213.7 | 447.3 | 896.7 | 1787.0 | 3396.4 |
| Resource | time | 3' | 7' | 22' | 49' | 91' |
| Resource | UB | 253.9 | 527.3 | 1053.7 | 2105.4 | 4016.6 |

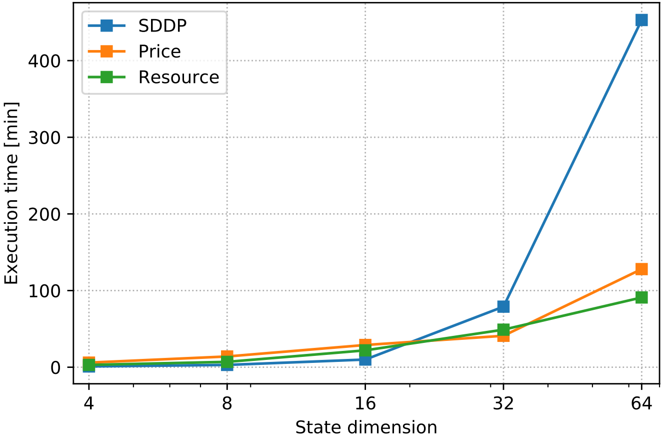
For the **48-Nodes** microgrid,

- price decomposition gives a (slightly) better exact lower bound than SDDP

$$\underbrace{3310.3}_{\underline{V}_0[sddp]} \leq \underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^* \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

- **price decomposition is more than 3 times faster than SDDP**

# Time evolution



# Policy evaluation by Monte Carlo (1,000 scenarios)

| | 3-Nodes | 6-Nodes | 12-Nodes | 24-Nodes | 48-Nodes |
|---|---|---|---|---|---|
| SDDP policy | $226 \pm 0.6$ | $471 \pm 0.8$ | $936 \pm 1.1$ | $1859 \pm 1.6$ | $3550 \pm 2.3$ |
| Price policy | $228 \pm 0.6$ | $464 \pm 0.8$ | $923 \pm 1.2$ | $1839 \pm 1.6$ | $3490 \pm 2.3$ |
| Gap | +0.9 % | -1.5% | -1.4% | -1.1% | -1.7% |
| Resource policy | $229 \pm 0.6$ | $471 \pm 0.8$ | $931 \pm 1.1$ | $1856 \pm 1.6$ | $3503 \pm 2.2$ |
| Gap | +1.3 % | 0.0% | -0.5% | -0.2% | -1.2% |

All the cost values above are statistical upper bounds of $V_0^*$

For the **48-Nodes** microgrid,

- price policy beats SDDP policy and resource policy

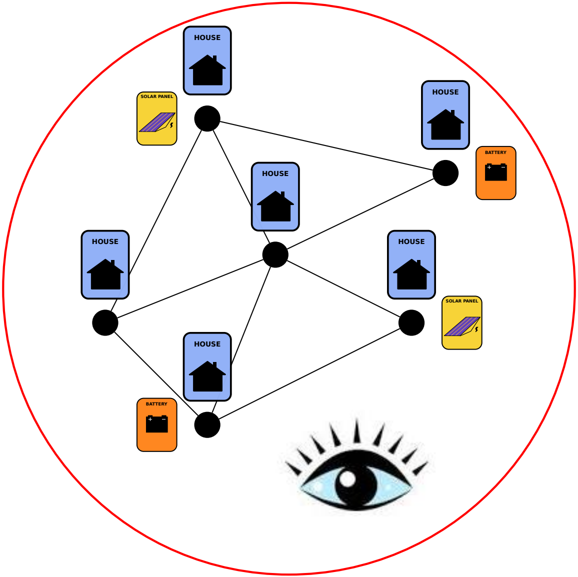
$$V_0^* \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{3550}_{C[sddp]}$$

- the exact upper bound given by resource decomposition is not so tight

$$\underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^* \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

gap <3% ≈ 3% >18%

# Motivation for decentralized information



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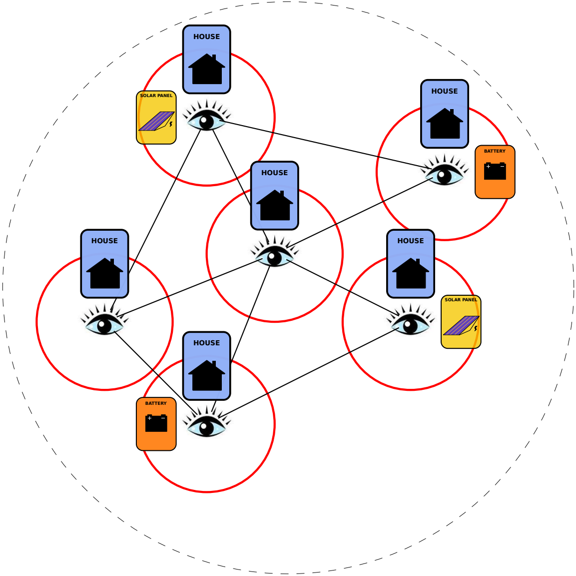

# Centralized information structure

Up to now, we have studied the following problem

$$V_0^{\mathrm{C}} = \min_{\mathbf{F},\mathbf{Q}} \Bigg( \sum_{i \in \mathcal{V}} \underbrace{\min_{\mathbf{X}^i,\mathbf{U}^i} \mathbb{E}\Big[ \sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) + K^i(\mathbf{X}_T^i) \Big]}_{J_{\mathcal{V}}^i(\mathbf{F}^i)} + \underbrace{\mathbb{E}\Big[ \sum_{t=0}^{T-1} \sum_{e \in \mathcal{E}} l_t^e(\mathbf{Q}_t^e) \Big]}_{J_{\mathcal{E}}(\mathbf{Q})} \Bigg)$$

subject to, for all $t \in [\![0, T-1]\!]$ and for all $i \in \mathcal{V}$

$$A\mathbf{Q}_t + \mathbf{F}_t = 0 \qquad \text{(network constraints)}$$

$$\mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) \qquad \text{(nodal dynamic constraints)}$$

$$\Delta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) = \mathbf{F}_t^i \qquad \text{(nodal balance equation)}$$

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0, \cdots, \mathbf{W}_{t+1}) \qquad \text{(information constraints)}$$

with $\mathbf{W}_t = (\mathbf{W}_t^1, \ldots, \mathbf{W}_t^N)$: global noise process

# Decentralized information structure

Consider now the following problem

$$V_0^{\mathrm{D}} = \min_{\mathbf{F},\mathbf{Q}} \Bigg( \sum_{i \in \mathcal{V}} \underbrace{\min_{\mathbf{X}^i,\mathbf{U}^i} \mathbb{E}\Big[ \sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) + K^i(\mathbf{X}_T^i) \Big]}_{J_{\mathcal{V}}^i(\mathbf{F}^i)} + \underbrace{\mathbb{E}\Big[ \sum_{t=0}^{T-1} \sum_{e \in \mathcal{E}} l_t^e(\mathbf{Q}_t^e) \Big]}_{J_{\mathcal{E}}(\mathbf{Q})} \Bigg)$$

subject to, for all $t \in [\![0, T-1]\!]$ and for all $i \in \mathcal{V}$

$$A\mathbf{Q}_t + \mathbf{F}_t = 0 \qquad \text{(network constraints)}$$

$$\mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) \qquad \text{(nodal dynamic constraints)}$$

$$\Delta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) = \mathbf{F}_t^i \qquad \text{(nodal balance equation)}$$

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0^i, \cdots, \mathbf{W}_{t+1}^i) \qquad \text{(information constraints)}$$

that is, the local control $\mathbf{U}_t^i$ is a feedback w.r.t. local noises $(\mathbf{W}_0^i, \ldots, \mathbf{W}_{t+1}^i)$

For such a problem, there is no Dynamic Programming Principle...

# Bounds in the decentralized case I

Consider the lower bound obtained with a deterministic price process $p$

$$\underline{V}_0[p] = \sum_{i \in \mathcal{V}} V_0^i[p^i] + V_0^{\mathcal{E}}[p] \ , \qquad \text{with}$$

$$V_0^i[p^i] = \min_{\mathbf{X}^i, \mathbf{U}^i, \mathbf{F}^i} \mathbb{E}\left[\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) + \langle p_t^i, \mathbf{F}_t^i \rangle + K^i(\mathbf{X}_T^i)\right]$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) \ , \ \ \mathbf{X}_0^i = x_0^i$$

$$\Delta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) = \mathbf{F}_t^i$$

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_1, \ldots, \mathbf{W}_{t+1})$$

# Bounds in the decentralized case

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$$\Delta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) = \mathbf{F}_t^i$$

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_1^i, \ldots, \mathbf{W}_{t+1}^i)$$

Replacing the global $\sigma$-field $\sigma(\mathbf{W}_1, \ldots, \mathbf{W}_{t+1})$ by the local $\sigma$-field $\sigma(\mathbf{W}_1^i, \ldots, \mathbf{W}_{t+1}^i)$ does not make any change in this local subproblem

**The lower bound $\underline{V}_0[p]$ is the same for both information structures**

*A similar conclusion holds true for the upper bound $\overline{V}_0[r]$*

# Bounds in the decentralized case II

Since $\mathbf{W}_t = (\mathbf{W}_t^1, \ldots, \mathbf{W}_t^N)$, for all $i$, we have the inclusion of $\sigma$-fields

$$\sigma(\mathbf{W}_0^i, \ldots, \mathbf{W}_t^i) \subset \sigma(\mathbf{W}_0, \ldots, \mathbf{W}_t)$$

We deduce that the admissible control set in case of a decentralized information structure is smaller that the one in case of a centralized information structure, and hence

$$V_0^{\mathrm{C}} \leq V_0^{\mathrm{D}}$$

Finally, we obtain the following sequence of inequalities

$$\sup_{p} \underline{V}_0[p] \leq V_0^{\mathrm{C}} \leq V_0^{\mathrm{D}} \leq \inf_{r} \overline{V}_0[r]$$

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# Bounds in the decentralized case III

$$\sup_p \underline{V}_0[p] \leq V_0^{\mathrm{C}} \leq V_0^{\mathrm{D}} \leq \inf_r \overline{V}_0[r]$$

- We have seen on the numerical experiments that the lower bound was close from the optimal value $V_0^{\mathrm{C}}$ in the centralized case

$$\underbrace{\sup_p \underline{V}_0[p] \leq V_0^{\mathrm{C}}}_{\approx 3\%}$$

- What can we say about the upper bound and the optimal value $V_0^{\mathrm{D}}$ in the decentralized case?

$$\underbrace{V_0^{\mathrm{C}} \leq \inf_r \overline{V}_0[r]}_{\approx 18\%} \quad , \quad \underbrace{V_0^{\mathrm{D}} \leq \inf_r \overline{V}_0[r]}_{\textbf{Value of the gap?}}$$

# Analysis of the decentralized case

Consider the decentralized information structure

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0^i, \ldots, \mathbf{W}_{t+1}^i)$$

and the constraints that have to be met at node $i$

$$\mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) \qquad \text{(nodal dynamic constraints)}$$

$$\Delta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) = \mathbf{F}_t^i \qquad \text{(nodal balance equation)}$$

- Thanks to the nodal dynamic constraints, the state $\mathbf{X}_t^i$ is measurable w.r.t. the $\sigma$-field $\sigma(\mathbf{W}_0^i, \ldots, \mathbf{W}_t^i)$
- Thanks to the nodal balance equation, the node flow $\mathbf{F}_t^i$ is measurable w.r.t. the $\sigma$-field $\sigma(\mathbf{W}_0^i, \ldots, \mathbf{W}_{t+1}^i)$

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# Analysis of the decentralized case II

Suppose that $(\mathbf{W}^1, \cdots, \mathbf{W}^N)$ are independent random processes
Otherwise stated, we add an **independence assumption in space**

At time $t$, consider now the global coupling constraints $A\mathbf{Q}_t + \mathbf{F}_t = 0$.
Summing these constraints leads to the aggregate coupling constraint

$$\sum_{i \in \mathcal{V}} \mathbf{F}_t^i = 0$$

Since $\mathbf{F}_t^i$ is measurable w.r.t. the $\sigma$-field $\sigma(\mathbf{W}_0^i, \ldots, \mathbf{W}_{t+1}^i)$ and from the independence assumption in space, we deduce that the random variables $\mathbf{F}_t$ (and hence $\mathbf{Q}_t$) are in fact **deterministic variables**

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# Analysis of the decentralized case III

According to this conclusion, under the space independence assumption, in case of a decentralized information structure, the global minimisation problem depends on deterministic node flows $f$ and edge flows $q$ variables

$$
\begin{aligned}
V_0^{\mathrm{D}} &= \min_{f,q} \Big( \sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(f^i) + J_{\mathcal{E}}(q) \Big) \quad \text{s.t.} \quad \mathbb{A}q + f = 0 \\
&= \inf_{r} \bigg( \sum_{i \in \mathcal{V}} \Big( \min_{f_i} \; J_{\mathcal{V}}^i(f^i) \;\text{ s.t. }\; f^i = -(\mathbb{A}r)^i \Big) + \Big( \min_{q} \; J_{\mathcal{E}}(q) \;\text{ s.t. }\; q = r \Big) \bigg) \\
&= \inf_{r} \overline{V}_0[r]
\end{aligned}
$$

The upper bound $\inf_r \overline{V}_0[r]$ and the optimal value $V_0^{\mathrm{D}}$ are **the same**

# Information gap

Recall the sequence of inequalities relating optimal values and bounds

$$\sup_p \underline{V}_0[p] \;\leq\; V_0^{\mathrm{C}} \;\leq\; V_0^{\mathrm{D}} \;\leq\; \inf_r \overline{V}_0[r]$$

Gathering all the theoretical and numerical results obtained, we have

$$\underbrace{\sup \underline{V}_0[p] \;\leq\; V_0^{\mathrm{C}}}_{\approx 3\%} \;,\; \underbrace{V_0^{\mathrm{C}} \;\leq\; V_0^{\mathrm{D}}}_{\approx 18\%} \;,\; V_0^{\mathrm{D}} \;=\; \inf \overline{V}_0[r]$$

that provides a way to **quantify the information gap** in our application.

# Conclusions

- We have two algorithms that decompose spatially and temporally a large-scale optimization problem under coupling constraints.
- In our case study, price decomposition beats SDDP for large instances ($\geq$ 24 nodes)
  - in computing time (more than twice faster)
  - in precision (more than 1% better)
- Price decomposition gives (in a surprising way) a tight lower bound, whereas the upper bound given by resource decomposition is weak (which is understandable on the case study)
- We have studied the case of a decentralized information structure to explain this weakness

# Future works

- Obtaining tighter bounds (*mainly for resource decomposition*)

  If we select properly price **P** and resource **R** processes among the class of Markovian processes (instead of deterministic ones) we can obtain “better” nodal value functions (with an extended local state)

- Solving large problems with a large number of time steps

  Prospective investment studies in the energy field at the European scale involve both a large spatial dimension (dozens of countries) and an optimization horizon of several years that must be finely discretized (tens of thousands of time steps). Now, the goal is to mix spatial decomposition and time-block decomposition

# Further details in

F. Pacaud
*Decentralized Optimization Methods*
*for Efficient Energy Management under Stochasticity*
**PhD Thesis, Université Paris Est**, 2018

P. Carpentier, J.-P. Chancelier, M. De Lara and F. Pacaud
*Mixed Spatial and Temporal Decompositions*
*for Large-Scale Multistage Stochastic Optimization Problem*
**Journal of Optimization Theory and Applications**, 186, 985–1005, 2020

F. Pacaud, M. De Lara, J.-P. Chancelier and P. Carpentier
*Distributed Multistage Optimization*
*of Large-Scale Microgrids under Stochasticity*
**IEEE Transactions on Power Systems**, 37(1), 204–211, 2022

THANK YOU FOR YOUR ATTENTION