IMCA Instituto de Matemática y Ciencias Afines

Smart Energy and Stochastic Optimization

◇

WORKSHOP

Mixing Time Blocks and Price/Resource Decompositions Methods

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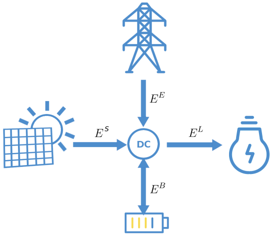
A battery management problem over a long time horizon



We present a battery management problem over several years

- optimize long-term investment decisions
  — here the renewal of a battery in an energy system
- but the optimal long-term decisions highly depend on short-term operating decisions
  — here the way the battery is operated in real-time.

Battery management involves two time scales

- When to renew a battery (long term decision – day)?
- How to control the battery (short time decision – $\frac{1}{2}$ hour)?
- Impact of the battery control on aging?

Large number of stages: $350,400 = \underbrace{7300}_{\text{days}} \times \underbrace{48}_{\frac{1}{2}\text{hours}}$

Fortunately the problem displays a **two-time-scale** structure...

We decompose the two time scales

Fast time scale: $\frac{1}{2}$ hour (battery charge/discharge)
Slow time scale: day (battery renewal)

- Using Dynamic Programming, we compute Bellman value functions V_d at the slow time scale, each computation involving a stochastic multistage optimization problem at the fast time scale
- We propose numerical schemes providing upper and lower bounds of the family of daily Bellman value functions V_d, based on resource and price decomposition/coordination techniques

We introduce notations for two time scales

Time is described by to indices $(d, m) \in \mathbb{T}$

$$\mathbb{T} = \{0, \dots, D\} \times \{0, \dots, M\} \cup \{(D+1, 0)\}$$

1. Battery charge, decision every half hour $m \in \{0, \dots, M\}$ of every day $d \in \{0, \dots, D\}$
   $\rightarrow$ half hours in day d are $(d, 0)$, $(d, 1)$,..., (d, M)
2. Renewal of the battery, decision every day $d \in \{0, \dots, D+1\}$
   $\rightarrow$ Start of days are $(0, 0)$,..., $(d, 0)$,..., $(D+1, 0)$
3. Compatibility between days:[1] $(d, M+1) = (d+1, 0)$

Equipped with the *lexicographical order*, $\mathbb{T}$ is a totally ordered set

$$(d, m) < (d', m') \iff (d < d') \vee \big(d = d' \wedge m < m'\big)$$

[1] It could be practical to add a time interval between $(d, M+1)$ and $(d+1, 0)$: we will not detail this point

Lecture outline

Two-time-scale battery management problem

Resource and price decomposition methods

- Time blocks and resource decomposition
- Time blocks and price decomposition
- Producing fast time-scale policies

Managing a battery over 20 years

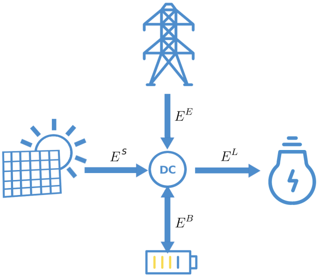
Outline of the presentation

Two-time-scale battery management problem

Resource and price decomposition methods

Managing a battery over 20 years

Physical model: a home with load, solar panel and storage



- Two-time-scale uncertainties
  - $\mathbf{D}_{d,m}$: Net demand $(= \mathbf{E}^L_{d,m} - \mathbf{E}^S_{d,m})$
  - $\mathbf{P}^b_d$: Uncertain battery price
- Two-time-scale controls
  - $\mathbf{E}^B_{d,m}$: Battery charge/discharge
  - $\mathbf{E}^E_{d,m}$: National grid import
  - $\mathbf{R}_d$: Battery renewal
- Two-time-scale states
  - $\mathbf{S}_{d,m}$: Battery state of charge
  - $\mathbf{H}_{d,m}$: Battery health
  - $\mathbf{C}_d$: Battery capacity

Fast time scale: system operation

- The national grid import ensures energy balance

$$\mathbf{E}^{E}_{d,m} = \mathbf{D}_{d,m} + (\mathbf{E}^{B}_{d,m})^{+} - (\mathbf{E}^{B}_{d,m})^{-}$$

and induces an operating cost

$$\pi^{e}_{d,m} \times \big(\mathbf{D}_{d,m} + (\mathbf{E}^{B}_{d,m})^{+} - (\mathbf{E}^{B}_{d,m})^{-}\big)$$

- The battery state of charge and health evolve at the fast time scale

$$\mathbf{S}_{d,m+1} = \mathbf{S}_{d,m} + \rho^{\mathrm{c}}(\mathbf{E}^{B}_{d,m})^{+} - \rho^{\mathrm{d}}(\mathbf{E}^{B}_{d,m})^{-}$$
$$\mathbf{H}_{d,m+1} = \mathbf{H}_{d,m} - (\mathbf{E}^{B}_{d,m})^{+} - (\mathbf{E}^{B}_{d,m})^{-}$$

whereas the battery capacity remains unchanged at this scale

$$\mathbf{C}_{d,m+1} = \mathbf{C}_{d}$$

$$\longrightarrow \quad (\mathbf{S}_{d,m+1}, \mathbf{H}_{d,m+1}, \mathbf{C}_{d,m+1}) = \varphi(\mathbf{S}_{d,m}, \mathbf{H}_{d,m}, \mathbf{C}_{d}, \mathbf{E}^{B}_{d,m})$$

Slow time scale: renewal model

- At the end of every day d, we can buy a new battery at cost $\mathbf{P}_d^b \times \mathbf{R}_d$

$$\text{Storage capacity: } \mathbf{C}_{d+1} = \begin{cases} \mathbf{R}_d \ , & \text{if } \mathbf{R}_d > 0 \\ \mathbf{C}_d \ , & \text{otherwise} \end{cases}$$

- A new battery can make a maximum number of cycles $N_c(\mathbf{R}_d)$:

$$\text{Storage health: } \mathbf{H}_{d+1,0} = \begin{cases} 2 \times N_c(\mathbf{R}_d) \times \mathbf{R}_d \ , & \text{if } \mathbf{R}_d > 0 \\ \mathbf{H}_{d,M} \ , & \text{otherwise} \end{cases}$$

- A new battery is empty

$$\text{Storage state of charge: } \mathbf{S}_{d+1,0} = \begin{cases} 0 \ , & \text{if } \mathbf{R}_d > 0 \\ \mathbf{S}_{d,M} \ , & \text{otherwise} \end{cases}$$

$$\longrightarrow \quad (\mathbf{S}_{d+1,0}, \mathbf{H}_{d+1,0}, \mathbf{C}_{d+1}) = \psi(\mathbf{S}_{d,M}, \mathbf{H}_{d,M}, \mathbf{C}_d, \mathbf{R}_d)$$

We build an optimization problem at the daily scale

- Uncertainties

$$\mathbf{W}_d = \left(\mathbf{D}_{d,0}, \dots, \mathbf{D}_{d,m}, \dots, \mathbf{D}_{d,M-1}, \begin{pmatrix} \mathbf{D}_{d,M} \\ \mathbf{P}^b_d \end{pmatrix}\right)$$

- Controls

$$\mathbf{U}_d = \left(\mathbf{E}^B_{d,0}, \dots, \mathbf{E}^B_{d,m}, \dots, \mathbf{E}^B_{d,M-1}, \begin{pmatrix} \mathbf{E}^B_{d,M} \\ \mathbf{R}_d \end{pmatrix}\right)$$

- States and dynamics

$$\mathbf{X}_d = \left(\mathbf{S}_{d,0}, \mathbf{H}_{d,0}, \mathbf{C}_d\right) \quad \text{and} \quad \mathbf{X}_{d+1} = \overbrace{f_d\left(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d\right)}^{\text{composition of } \varphi \text{ and } \psi}$$

- Objective to be minimized

$$\mathbb{E}\left(\sum_{d=0}^{D} \underbrace{\left(\mathbf{P}^b_d \times \mathbf{R}_d + \sum_{m=0}^{M} \pi^e_{d,m} \times \left(\mathbf{D}_{d,m} + (\mathbf{E}^B_{d,m})^+ - (\mathbf{E}^B_{d,m})^-\right)\right)}_{L_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d)} + K(\mathbf{X}_{D+1})\right)$$

Two-time-scale non standard stochastic control problem

We now write the associated stochastic multistage optimization problem, whose optimal value is a function V^{e} of the initial state x_O

$$\mathcal{P}^{\mathrm{e}}: \quad V^{\mathrm{e}}(x_0) = \min_{(\mathbf{X}_{0:D+1}, \mathbf{U}_{0:D})} \mathbb{E}\left(\sum_{d=0}^{D} L_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d) + K(\mathbf{X}_{D+1}) \right)$$

$$\begin{aligned} \text{s.t} \quad & \mathbf{X}_0 = x_0 \ , \ \ \mathbf{X}_{d+1} = f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d) \\ & \mathbf{U}_d = (\mathbf{U}_{d,0}, \ldots, \mathbf{U}_{d,m}, \ldots, \mathbf{U}_{d,M}) \\ & \mathbf{W}_d = (\mathbf{W}_{d,0}, \ldots, \mathbf{W}_{d,m}, \ldots, \mathbf{W}_{d,M}) \\ & \sigma(\mathbf{U}_{d,m}) \subset \sigma\big(\mathbf{W}_{d',m'}, \ (d', m') \leq (d, m)\big) \end{aligned}$$

$\mathcal{P}^{\mathrm{e}}$ is a non standard SOC problem: the nonanticipativity constraint is **written every half hour** whereas dynamics is written **every day**!

Bellman equation with daily time blocks

The Bellman value functions V_d^{e} associated to Problem $\mathcal{P}^{\mathrm{e}}$ are obtained by setting $V_{D+1}^{\mathrm{e}} = K$ and, for $d = D, \ldots, 0$, by computing recursively

$$
\begin{aligned}
V_d^{\mathrm{e}}(x) = \min_{(\mathbf{X}_{d+1}, \mathbf{U}_d)} \ & \mathbb{E}\left[L_d(x, \mathbf{U}_d, \mathbf{W}_d) + V_{d+1}^{\mathrm{e}}(\mathbf{X}_{d+1})\right] \\
\text{s.t} \quad & \mathbf{X}_{d+1} = f_d(x, \mathbf{U}_d, \mathbf{W}_d) \\
& \sigma(\mathbf{U}_{d,m}) \subset \sigma(\mathbf{W}_{d,0}, \ldots, \mathbf{W}_{d,m}) \ , \ \ \forall m \in \{0, \ldots, M\}
\end{aligned}
$$

Time Block Independence Assumption

$\{\mathbf{W}_d\}_{d=0,\ldots,D}$ is a sequence of daily independent random vectors

Proposition ([Carpentier et al, JCA 2023])

Under the Time Block Independence Assumption, the value $V_0^{\mathrm{e}}(x_0)$ of the Bellman value function at time $d = 0$ is the optimal value $V^{\mathrm{e}}(x_0)$ of $\mathcal{P}^{\mathrm{e}}$

We introduce price/resource daily decompositions

The main practical difficulty is the large number of stages ($D \times M = 350,400$)! To overcome this, we appeal to decomposition methods.

Decomposition is done on the dynamics $\mathbf{X}_{d+1} = f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d)$

1. Resource decomposition: we choose resources (targets) $\mathbf{R}_{d+1}$ and we split the dynamic constraints in

$$\mathbf{X}_{d+1} = \mathbf{R}_{d+1} \ , \ \ \mathbf{R}_{d+1} = f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d)$$

2. Price decomposition: we choose prices (weights) $\mathbf{\Lambda}_{d+1}$ and we dualize the dynamic constraints

$$\left\langle \mathbf{\Lambda}_{d+1}, \ f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d) - \mathbf{X}_{d+1} \right\rangle$$

Relaxation of the stochastic control problem

A new difficulty arises, in resource decomposition

The optimization subproblems in the resource decomposition method involve equality constraints between random variables

$$\mathbf{R}_{d+1} = f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d)$$

which are almost always impossible to satisfy for a given resource

To solve this new difficulty, we relax the optimization problem $\mathcal{P}^e$ by rewriting the dynamic constraints as inequality constraints

$$\mathbf{X}_{d+1} \leq f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d)$$

that is, we enlarge the admissible set of the problem

Relaxation of the stochastic control problem

We consider the following relaxed optimization problem

$$\mathcal{P}^{\mathrm{i}}: \quad V^{\mathrm{i}}(x_0) = \min_{(\mathbf{X}_{0:D+1}, \mathbf{U}_{0:D})} \mathbb{E}\left(\sum_{d=0}^{D} L_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d) + K(\mathbf{X}_{D+1})\right)$$

$$\text{s.t} \quad \mathbf{X}_{d+1} \leq f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d)$$

$$\sigma(\mathbf{U}_{d,m}) \subset \sigma\big(\mathbf{W}_{d',m'}, \, (d', m') \leq (d, m)\big)$$

$$\sigma(\mathbf{X}_{d+1}) \subset \sigma\big(\mathbf{W}_{d',m'}, \, (d', m') \leq (d, M)\big)$$

and the associated sequence of Bellman value functions

$$V_d^{\mathrm{i}}(x) = \min_{(\mathbf{X}_{d+1}, \mathbf{U}_d)} \mathbb{E}\left[L_d(x, \mathbf{U}_d, \mathbf{W}_d) + V_{d+1}^{\mathrm{i}}(\mathbf{X}_{d+1})\right]$$

$$\text{s.t} \quad \mathbf{X}_{d+1} \leq f_d(x, \mathbf{U}_d, \mathbf{W}_d)$$

$$\sigma(\mathbf{U}_{d,m}) \subset \sigma(\mathbf{W}_{d,0}, \ldots, \mathbf{W}_{d,m})$$

$$\sigma(\mathbf{X}_{d+1}) \subset \sigma(\mathbf{W}_{d,0}, \ldots, \mathbf{W}_{d,M})$$

Monotonicity-inducing Assumtion

Assumption 1 (Monotonicity-inducing)

1. *The final cost function K is nonincreasing on its effective domain:*

$$\forall (x, x') \in (\mathrm{dom} K)^2 \ , \ \ x \leq x' \implies K(x) \geq K(x')$$

2. $\forall d \in [\![0, D]\!]$, *the effective domain of V^e_d*
is induced by the effective domain of the instantaneous cost L_d

$$\mathrm{dom}\, V^e_d = \{x \in \mathbb{X} \,|\, \exists\, \mathbf{U} \text{ s.t. } \mathbb{E}[L_d(x, \mathbf{u}_d, \mathbf{w}_d)] < +\infty\}$$

3. $\forall d \in [\![0, D]\!]$, $\forall (x', x) \in (\mathrm{dom}\, V^{\mathrm{e}}_d)^2$ *with $x' \geq x$,*
for any admissible control $\mathbf{u}_d$ s.t. $\mathbb{E}\left[L_d(x, \mathbf{u}_d, \mathbf{w}_d)\right] < +\infty$,
there exists an admissible control random $\widetilde{\mathbf{u}}_d$ s.t.

$$f_d(x', \widetilde{\mathbf{u}}_d, \mathbf{w}_d) \in \mathrm{dom}\, V^{\mathrm{e}}_{d+1} \text{ and } f_d(x', \widetilde{\mathbf{u}}_d, \mathbf{w}_d) \geq f_d(x, \mathbf{u}_d, \mathbf{w}_d)$$
$$L_d(x', \widetilde{\mathbf{u}}_d, \mathbf{w}_d) \leq L_d(x, \mathbf{u}_d, \mathbf{w}_d)$$

Equivalence between the initial and the relaxed problem

Proposition ([Rigaut et al, 2023])

Under the monotonicity-inducing Assumtion 1, the value functions V_d^{e} are nonincreasing on their effective domains

$$\forall (x', x) \in (\mathrm{dom}\, V_d^{\mathrm{e}})^2 \ , \ \ x \leq x' \quad \Longrightarrow \quad V_d^{\mathrm{e}}(x) \geq V_d^{\mathrm{e}}(x')$$

Proposition ([Rigaut et al, 2023])

Under the monotonicity-inducing Assumtion 1,

$$V_d^{\mathrm{i}} = V_d^{\mathrm{e}} \ , \ \ \forall d \in \{0, \ldots, D+1\}$$

Proposition ([Rigaut et al, 2023])

The battery management problem satisfies the monotonicity-inducing Assumtion 1.

Lecture outline

Two-time-scale battery management problem

Resource and price decomposition methods

- Time blocks and resource decomposition
- Time blocks and price decomposition
- Producing fast time-scale policies

Managing a battery over 20 years

We introduce price/resource daily decompositions

We present two decomposition algorithms to compute upper and lower bounds of the Bellman value functions V_d^{i}

Decomposition is done on the dynamics $\mathbf{X}_{d+1} \leq f_d(x, \mathbf{U}_d, \mathbf{W}_d)$

1. Resource decomposition: choosing deterministic resources (targets) r_{d+1} and splitting the dynamic constraints in

$$\mathbf{X}_{d+1} = r_{d+1} \ , \ \ r_{d+1} \leq f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d)$$

   gives upper bounds of the Bellman value functions V_d^{i}

2. Price decomposition: choosing deterministic prices (weights) $\lambda_{d+1} \leq 0$ and dualizing the dynamic constraints

$$\langle \lambda_{d+1} \mid \ f_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_d) - \mathbf{X}_{d+1} \rangle$$

   gives lower bounds of the Bellman value functions V_d^{i}

Lecture outline

Resource decomposition mechanism

$$
\begin{aligned}
V_d^{\mathrm{i}}(x_d) = & \min_{(\mathbf{X}_{d+1},\mathbf{U}_d)} \ \mathbb{E}\Big[L_d(x_d,\mathbf{U}_d,\mathbf{W}_d) + V_{d+1}^{\mathrm{i}}(\mathbf{X}_{d+1})\Big] \\
& \quad \text{s.t} \quad \mathbf{X}_{d+1} \leq f_d(x_d,\mathbf{U}_d,\mathbf{W}_d) && \text{(Bellman equation)} \\
= & \min_{\mathbf{R}_{d+1}} \bigg(\min_{\mathbf{U}_d} \ \mathbb{E}\Big[L_d(x_d,\mathbf{U}_d,\mathbf{W}_d) + V_{d+1}^{\mathrm{i}}(\mathbf{R}_{d+1})\Big]\bigg) \\
& \quad \text{s.t} \quad \mathbf{R}_{d+1} \leq f_d(x_d,\mathbf{U}_d,\mathbf{W}_d) && \text{(stochastic resource)} \\
\leq & \min_{r_{d+1}} \bigg(\min_{\mathbf{U}_d} \ \mathbb{E}\Big[L_d(x_d,\mathbf{U}_d,\mathbf{W}_d) + V_{d+1}^{\mathrm{i}}(r_{d+1})\Big]\bigg) \\
& \quad \text{s.t} \quad r_{d+1} \leq f_d(x_d,\mathbf{U}_d,\mathbf{W}_d) && \text{(deterministic resource)} \\
= & \min_{r_{d+1}} \bigg(\underbrace{\min_{\mathbf{U}_d} \ \Big(\mathbb{E}\Big[L_d(x_d,\mathbf{U}_d,\mathbf{W}_d)\Big] \text{ s.t } r_{d+1} \leq f_d(x_d,\mathbf{U}_d,\mathbf{W}_d)\Big)}_{L_d^{\mathrm{R}}(x_d,r_{d+1})} + V_{d+1}^{\mathrm{i}}(r_{d+1})\bigg)
\end{aligned}
$$

Relaxed deterministic resource decomposition

We introduce a relaxed deterministic resource intraday problem

$$L_d^{\mathrm{R}}(x_d, r_{d+1}) = \min_{\mathbf{U}_d} \; \mathbb{E}\Big[L_d(x_d, \mathbf{U}_d, \mathbf{W}_d)\Big]$$

$$\text{s.t} \quad f_d(x_d, \mathbf{U}_d, \mathbf{W}_d) \geq r_{d+1}$$

$$\sigma(\mathbf{U}_{d,m}) \subset \sigma(\mathbf{W}_{d,0:m})$$

and the associated Bellman recursion

$$\overline{V}_d^{\mathrm{R}}(x_d) = \min_{r_{d+1}} \; L_d^{\mathrm{R}}(x_d, r_{d+1}) + \overline{V}_{d+1}^{\mathrm{R}}(r_{d+1})$$

Proposition ([Rigaut et al, 2023])

The Bellman value functions $\overline{V}_d^{\mathrm{R}}$ are upper bounds to the Bellman value functions V_d^{i} of Problem $\mathcal{P}^{\mathrm{i}}$

$$\overline{V}_d^{\mathrm{R}} \geq V_d^{\mathrm{i}} \; , \;\; \forall d \in \{0, \ldots, D+1\}$$

Efficiency of deterministic resource decomposition

$$\overline{V}^{\mathrm{R}}_d(x_d) = \min_{r_{d+1}} \overbrace{\underbrace{L^{\mathrm{R}}_d(x_d, r_{d+1})}_{\text{Hard to compute}} + \overline{V}^{\mathrm{R}}_{d+1}(r_{d+1})}^{\text{Easy to compute by dynamic programming}}$$

It is challenging to compute the intraday function value $L^{\mathrm{R}}_d(x_d, r_{d+1})$ for each couple (x_d, r_{d+1}) and each day d, but

- we can exploit periodicity of the problem, that is, compute the functions L^{R}_d for I typical days and not for all the D days
- for some components of the state, the intraday function L^{R}_d depends on $x_d - r_{d+1}$ rather than (x_d, r_{d+1})
- we can parallelize the computation of L^{R}_d on several days

Note that we can use any suitable method to solve the multistage intraday problems L^{R}_d (SDP, SDDP, scenario tree methods, PH,...)

Lecture outline

Price decomposition mechanism

$$
\begin{aligned}
V_d^{\mathrm{i}}(x_d) &= \min_{(\mathbf{X}_{d+1},\mathbf{U}_d)} \mathbb{E}\Big[L_d(x_d,\mathbf{U}_d,\mathbf{W}_d) + V_{d+1}^{\mathrm{i}}(\mathbf{X}_{d+1})\Big] \\
&\qquad \text{s.t} \quad \mathbf{X}_{d+1} \leq f_d(x_d,\mathbf{U}_d,\mathbf{W}_d) && \text{(Bellman equation)} \\
&\geq \max_{\mathbf{\Lambda}_{d+1}\leq 0} \min_{(\mathbf{X}_{d+1},\mathbf{U}_d)} \mathbb{E}\Big[L_d(x_d,\mathbf{U}_d,\mathbf{W}_d) + V_{d+1}^{\mathrm{i}}(\mathbf{X}_{d+1}) \\
&\qquad + \langle \mathbf{\Lambda}_{d+1}, f_d(x_d,\mathbf{U}_d,\mathbf{W}_d) - \mathbf{X}_{d+1}\rangle\Big] && \text{(duality)} \\
&= \max_{\mathbf{\Lambda}_{d+1}\leq 0} \min_{\mathbf{U}_d} \mathbb{E}\Big[L_d(x_d,\mathbf{U}_d,\mathbf{W}_d) + \langle \mathbf{\Lambda}_{d+1}, f_d(x_d,\mathbf{U}_d,\mathbf{W}_d)\rangle \\
&\qquad + \min_{\mathbf{X}_{d+1}} \big(-\langle \mathbf{\Lambda}_{d+1}, \mathbf{X}_{d+1}\rangle + V_{d+1}^{\mathrm{i}}(\mathbf{X}_{d+1})\big)\Big] && \text{(Fenchel)} \\
&\geq \max_{\lambda_{d+1}\leq 0} \min_{\mathbf{U}_d} \mathbb{E}\Big[L_d(x_d,\mathbf{U}_d,\mathbf{W}_d) + \langle \lambda_{d+1}, f_d(x_d,\mathbf{U}_d,\mathbf{W}_d)\rangle\Big] \\
&\qquad - \big(V_{d+1}^{e}\big)^{\star}(\lambda_{d+1}) && \text{(deterministic price)} \\
&= \max_{\lambda_{d+1}\leq 0} \Bigg(\underbrace{\min_{\mathbf{U}_d} \mathbb{E}\Big[L_d(x_d,\mathbf{U}_d,\mathbf{W}_d) + \langle \lambda_{d+1}, f_d(x_d,\mathbf{U}_d,\mathbf{W}_d)\rangle\Big]}_{L_d^{\mathrm{P}}(x_d,\lambda_{d+1})} - \big(V_{d+1}^{\mathrm{i}}\big)^{\star}(\lambda_{d+1})\Bigg)
\end{aligned}
$$

Relaxed deterministic price decomposition

We introduce a relaxed deterministic price intraday problem

$$L_d^{\mathrm{P}}(x_d, \lambda_{d+1}) = \min_{\mathbf{U}_d} \mathbb{E}\Big[L_d(x_d, \mathbf{U}_d, \mathbf{W}_d) + \langle \lambda_{d+1}, f_d(x_d, \mathbf{U}_d, \mathbf{W}_d)\rangle\Big]$$

$$\text{s.t.} \quad \sigma(\mathbf{U}_{d,m}) \subset \sigma(\mathbf{W}_{d,0:m})$$

and the associated Bellman recursion

$$\underline{V}_d^{\mathrm{P}}(x_d) = \max_{\lambda_{d+1} \leq 0} L_d^{\mathrm{P}}(x_d, \lambda_{d+1}) - \big(\underline{V}_{d+1}^{\mathrm{P}}\big)^{\star}(\lambda_{d+1})$$

Proposition ([Rigaut et al, 2023])

The Bellman value functions $\underline{V}_d^{\mathrm{P}}$ are lower bounds to the Bellman value functions V_d^{i} of Problem $\mathcal{P}^{\mathrm{i}}$

$$\underline{V}_d^{\mathrm{P}} \leq V_d^{\mathrm{i}}\ , \ \ \forall d \in \{0, \dots, D+1\}$$

Efficiency of deterministic price decomposition

$$\overbrace{\underline{V}_d^{\mathrm{P}}(x_d) = \max_{\lambda_{d+1} \leq 0} \underbrace{L_d^{\mathrm{P}}(x_d, \lambda_{d+1})}_{\text{Hard to compute}} - \big(\underline{V}_{d+1}^{\mathrm{P}}\big)^{\star}(\lambda_{d+1})}^{\text{Easy to compute by dynamic programming}}$$

It is challenging to compute the intraday function value $L_d^{\mathrm{P}}(x_d, \lambda_{d+1})$ for each couple (x_d, λ_{d+1}) and each day d, but

- we can exploit periodicity of the problem, that is, compute the functions L_d^{P} for I typical days and not for all the D days
- we can parallelize the computation of L_d^{P} on several days
- we can use any suitable method to solve the multistage intraday problems L_d^{P} (SDP, SDDP, scenario tree methods, PH,...)

Lecture outline

Value functions $\underline{V}_d^{\mathrm{P}}$ and $\overline{V}_d^{\mathrm{R}}$ yield bounds

By resource and price decompositions, we have obtained Bellman functions that bound the Bellman value functions V_d^{i} of the relaxed problem $\mathcal{P}^{\mathrm{i}}$

$$\underline{V}_d^{\mathrm{P}} \leq V_d^{\mathrm{i}} \leq \overline{V}_d^{\mathrm{R}}$$

Under the monotonicity-inducing assumption we obtain bounds on the Bellman value functions V_d^{e} of the original problem $\mathcal{P}^{\mathrm{e}}$

$$\underline{V}_d^{\mathrm{P}} \leq V_d^{\mathrm{e}} \leq \overline{V}_d^{\mathrm{R}}$$

Finally, under the time block independence assumption, the resource and price Bellman value functions at the initial day bound the optimal value function of Problem $\mathcal{P}^{\mathrm{e}}$

$$\underline{V}_0^{\mathrm{P}} \leq V^{\mathrm{e}} \leq \overline{V}_0^{\mathrm{R}}$$

Value functions $\underline{V}_d^{\mathrm{P}}$ and $\overline{V}_d^{\mathrm{R}}$ yield admissible policies

Having at disposal the resource and price Bellman value functions $\underline{V}_d^{\mathrm{P}}$ and $\overline{V}_d^{\mathrm{R}}$, we can solve the following subproblems on all days d

$$\begin{aligned} \min_{\mathbf{U}_d} \ & \mathbb{E}\Big[L_d(x, \mathbf{U}_d, \mathbf{W}_d) + \widetilde{V}_{d+1}\big(f_d(x, \mathbf{U}_d, \mathbf{W}_d)\big)\Big] \\ \text{s.t} \quad & \sigma(\mathbf{U}_{d,m}) \subset \sigma(\mathbf{W}_{d,0:m}) \end{aligned}$$

with $\widetilde{V}_{d+1} = \underline{V}_{d+1}^{\mathrm{P}}$ or $\widetilde{V}_{d+1} = \overline{V}_{d+1}^{\mathrm{R}}$, and obtain resource and price policies at the fast time scale

Simulating the battery management problem along several noise scenarios by applying the resource and price policies, we compute the associated average costs, which are (statistical) upper bounds of the optimal cost of Problem $\mathcal{P}^{\mathrm{e}}$

Lecture outline

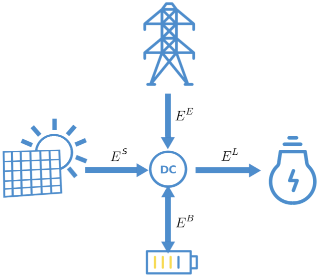
Two-time-scale battery management problem

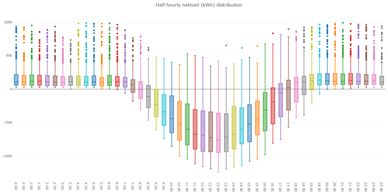
Resource and price decomposition methods

Managing a battery over 20 years

We present numerical results for one use case

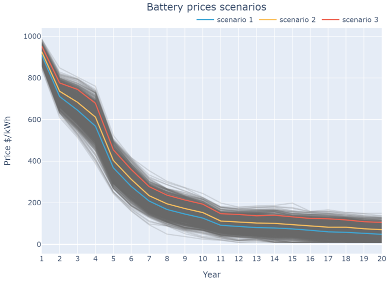
1. Net demand (demand minus solar production) from an industrial site
2. Managing battery charge, health and renewal on 20 years
   to show that resource and price decompositions scale





Managing battery charge, health an renewal

- 20 years, 7300 days, $350,400$ half hours, 4 periodicity classes
- Battery capacity between 0 and $1,500$ kWh
- Scenarios for batteries prices



SDP fail to solve such a problem over hundreds of thousands of stages!

Resource decomposition is numerically tractable

Resource decomposition

$$\overbrace{\overline{V}^{\mathrm{R}}_d(x_d) = \min_{r_{d+1}} \underbrace{L^{\mathrm{R}}_d(x_d, r_{d+1})}_{\text{Computing each } L^{\mathrm{R}}_d(\cdot,\cdot) \text{ takes 45 min}} + \overline{V}^{\mathrm{R}}_{d+1}(r_{d+1})}^{\text{Computing Bellman value functions by Dynamic Programming takes 25 min}}$$

- Complexity: 25 min + $D \times$ 45 min
- With I periodicity classes: 25 min + $I \times$ 45 min ($I \ll D$)
- With parallelization: 25 min + 45 min

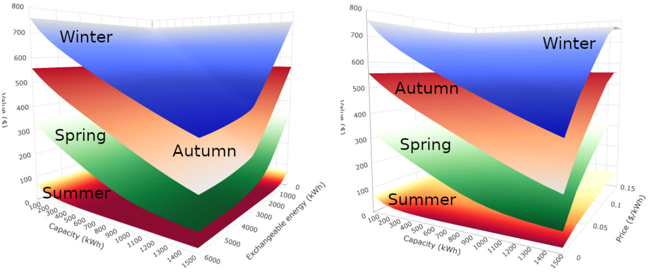
Price decomposition is numerically tractable

Price decomposition

$$\overbrace{\underline{V}_d^{\mathrm{P}}(x_d) = \max_{\lambda_{d+1} \leq 0} \quad \underbrace{L_d^{\mathrm{P}}(x_d, \lambda_{d+1})}_{\text{Computing } L_d^{\mathrm{P}}(\cdot,\cdot) \text{ takes 15 min}} \quad - \big(\underline{V}_{d+1}^{\mathrm{P}}\big)^{\star}(\lambda_{d+1})}^{\text{Computing Bellman value functions by Dynamic Programming takes 100 min}}$$

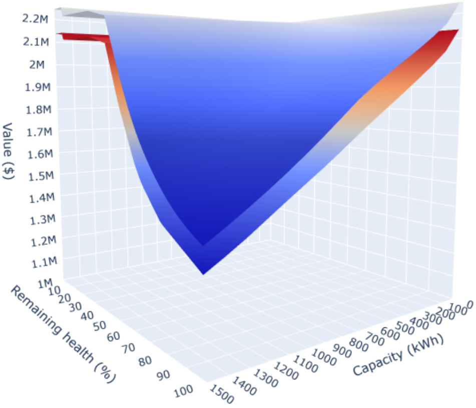
- Complexity: 100 min + $D \times$ 15 min
- With I periodicity classes:: 100 min + $I \times$ 15 min
- With parallelization: 100 min + 15 min

Intraday functions



Resource (left) and price (right) intraday functions for each trimester

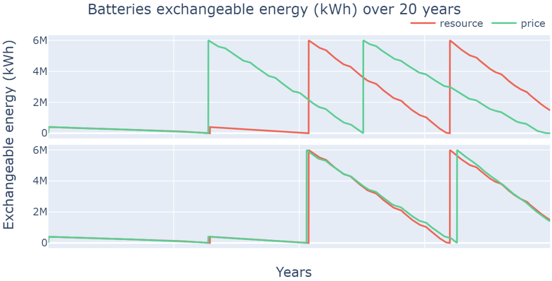
Bellman value functions



Resource and price Bellman value functions at initial day

Scenario simulation

We draw two battery price scenario and solar/demand scenario over 20 years and simulate the policies obtained by price and resource decomposition



Price decomposition slightly outperforms resource decomposition

	Scenario 1	**Scenario 2**
Total cost (resource)	2.757 M$	2.825 M$
Total cost (price)	2.722 M$	2.820 M$

Conclusions

1. We have solved problems with hundreds of thousands of time steps using the resource and price decomposition algorithms
2. We have designed control strategies for charging/aging/renewing batteries
3. We have used our algorithm to obtain results beyond the reach of algorithms that are sensitive to the number of time steps (SDP, SDDP)

Bibliography

P. Carpentier, J.-P. Chancelier, M. De Lara, T. Martin and T. Rigaut.
Time Block Decomposition of Multistage Stochastic Optimization Problem.
Journal of Convex Analysis, 30(2), 2023.

T. Rigaut, P. Carpentier, J.-P. Chancelier and M. De Lara.
Decomposition Methods for Dynamically Monotone Two-Time-Scale Stochastic 0ptimization Problem.
Preprint, 2023.

B. Heymann and P. Martinon.
Optimal battery aging: an adaptive weights dynamic programming algorithm.
Journal of Optimization Theory and Applications, 179(3):1043–1053, 2018.

T. Rigaut.
Time decomposition methods for optimal management of energy storage under stochasticity.
Thèse de Doctorat, Université Paris-Est, 2019.