Smart Energy and Stochastic Optimization

◇

WORKSHOP

An Overview of Decomposition/Coordination Methods in Multistage Stochastic Optimization

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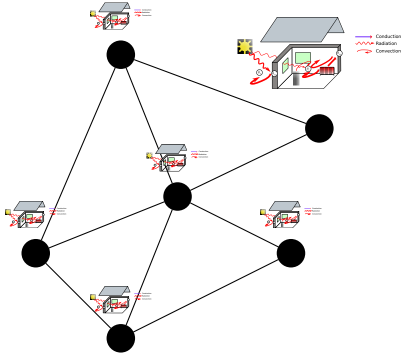
Motivation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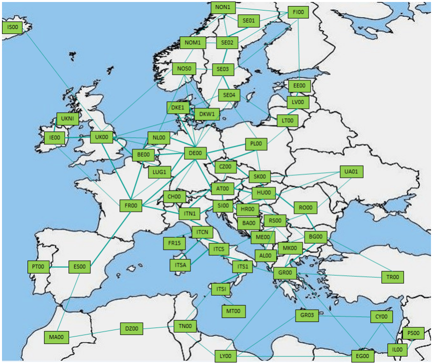
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Lecture outline

Decomposition and coordination

The three dimensions of stochastic optimization problems

A bird's eye view of decomposition methods: the cube

A brief insight into three decomposition methods

Scenario decomposition methods

Spatial (price/resource) decomposition methods

Time decomposition methods

Summary and research agenda

Outline of the presentation

Decomposition and coordination

A brief insight into three decomposition methods

Summary and research agenda

Outline of the presentation

Temporal, scenario and spatial structures in multistage stochastic optimization problems

In multistage stochastic optimization problems,
the control variable

$$\mathbf{U}_t^i(\omega)$$

is indexed by

- Time/stages $t \in \mathcal{T}$ $(= [\![0, T-1]\!])$
- Scenarios $\omega \in \Omega$
- Space/units/agents $i \in \mathcal{I}$

The letter U comes from the Russian word *upravlenie* for control

Let us fix problem and notations

$$\min_{\mathbf{U}} \overbrace{\mathbb{E}\Big[\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} L_t^i(\mathbf{H}_t^i,\mathbf{U}_t^i,\mathbf{W}_{t+1})\Big]}^{\text{additive costs}} \qquad \text{subject to}$$

dynamics constraints

$$\underbrace{\mathbf{H}_{t+1}^i}_{\text{history}} = g_t^i(\mathbf{H}_t^i,\mathbf{U}_t^i,\ \underbrace{\mathbf{W}_{t+1}}_{\text{uncertainty}}\ )\ ,\ \ \mathbf{H}_0^i = \mathbf{W}_0$$

measurability constraints (nonanticipativity of the control $\mathbf{U}_t^i$)

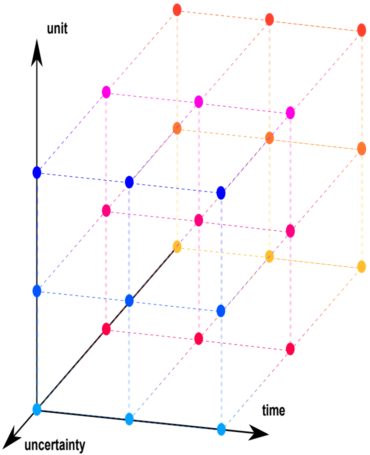
$$\sigma\big(\mathbf{U}_t^i\big) \subset \sigma(\mathbf{W}_0,\ldots,\mathbf{W}_t) \iff \mathbf{U}_t^i = \mathbb{E}\big[\mathbf{U}_t^i \,\big|\, \mathbf{W}_0,\ldots,\mathbf{W}_t\big]$$

spatially coupling constraints

$$\sum_{i\in\mathcal{I}} \Theta_t^i(\mathbf{H}_t^i,\mathbf{U}_t^i,\mathbf{W}_{t+1}) = 0$$

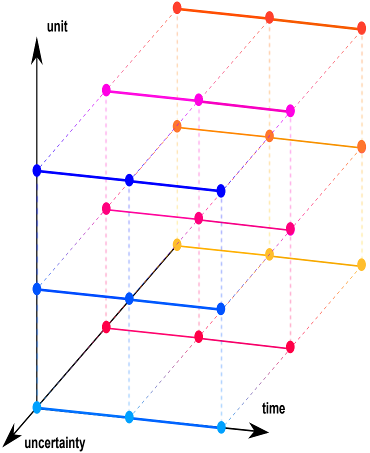
Outline of the presentation

Couplings for stochastic problems



$$\min \mathbb{E} \sum_i \sum_t L_t^i\big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$$

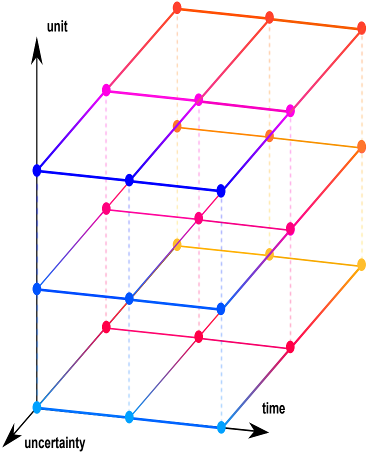
Couplings for stochastic problems: in time



$$\min \mathbb{E} \sum_i \sum_t L_t^i\big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$$

s.t. $\mathbf{H}_{t+1}^i = \big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$

Couplings for stochastic problems: in uncertainty

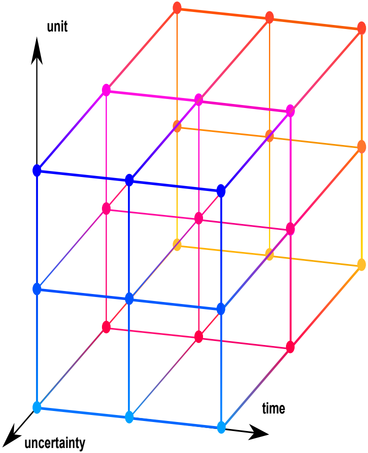


$$\min \mathbb{E} \sum_i \sum_t L_t^i\big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$$

$$\text{s.t.} \quad \mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\big[\mathbf{U}_t^i \,\big|\, \mathbf{W}_0, \dots, \mathbf{W}_t\big]$$

Couplings for stochastic problems: in space



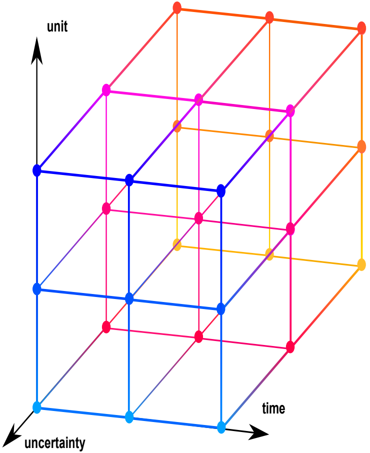
$$\min \mathbb{E} \sum_i \sum_t L_t^i\big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$$

$$\text{s.t.} \quad \mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\big[\mathbf{U}_t^i \,\big|\, \mathbf{W}_0, \ldots, \mathbf{W}_t\big]$$

$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Can we decouple stochastic optimization problems?



$$\min \mathbb{E} \sum_{i} \sum_{t} L_t^i\big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$$

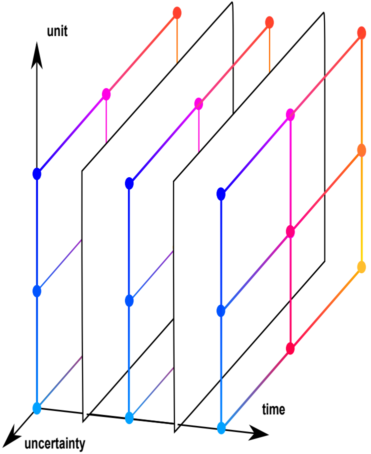
s.t. $$\mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\big[\mathbf{U}_t^i \,\big|\, \mathbf{W}_0, \ldots, \mathbf{W}_t\big]$$

$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Decomposition-coordination: divide and conquer

Sequential decomposition in time



$$\min \mathbb{E} \sum_i \sum_t L_t^i \big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$$

$$\text{s.t.} \quad \mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\big[\mathbf{U}_t^i \,\big|\, \mathbf{W}_0, \ldots, \mathbf{W}_t\big]$$

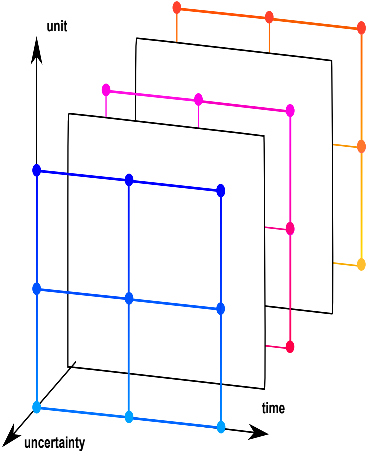
$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Dynamic Programming

[Bellman, 1957] [a]

[a] R. E. Bellman. *Dynamic Programming*. Princeton University Press, Princeton, N.J., 1957

Parallel decomposition in uncertainty/scenarios



$$\min \mathbb{E} \sum_i \sum_t L_t^i\big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$$

$$\text{s.t.} \quad \mathbf{H}_{t+1}^i = \big(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}\big)$$

$$\mathbf{U}_t^i = \mathbb{E}\big[\mathbf{U}_t^i \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\big]$$

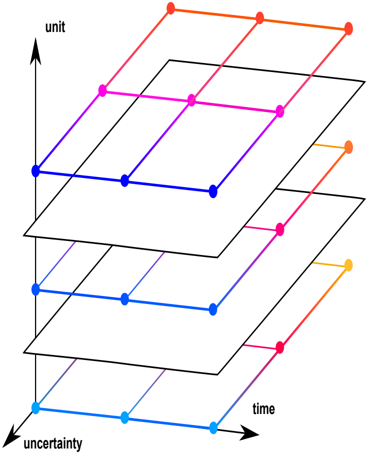
$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Progressive Hedging

[Rockafellar and Wets, 1991][a]

[a] R. Rockafellar and R. J.-B. Wets. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of operations research*, 16(1): 119–147, 1991

Parallel decomposition in space/units



$$\min \mathbb{E} \sum_i \sum_t L_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\text{s.t. } \mathbf{H}_{t+1}^i = (\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i = \mathbb{E}\left[\mathbf{U}_t^i \,\middle|\, \mathbf{W}_0, \ldots, \mathbf{W}_t\right]$$

$$\sum_i \Theta_t^i(\mathbf{H}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) = 0$$

Price/ Resource decompositions[a]

[a] [Carpentier, Cohen, and Culioli, 1995] Stochastic optimal control and decomposition-coordination methods *In: Recent Developments in Optimization, Roland Durier and Christian Michelot (Eds.), Springer-Verlag, Berlin*, 1995

Decomposition-coordination: divide and conquer

- Temporal decomposition
  - A state is an information summary
  - Time coordination realized through Dynamic Programming, by value functions (of the state)
  - Hard nonanticipativity constraints
- Scenario decomposition
  - Along each scenario, subproblems are deterministic (powerful algorithms)
  - Scenario coordination realized through Progressive Hedging, by updating nonanticipativity multipliers
  - Soft nonanticipativity constraints
- Spatial decomposition
  - By prices (multipliers of the spatial coupling constraint)
  - By resources (splitting the spatial coupling constraint)

Outline of the presentation

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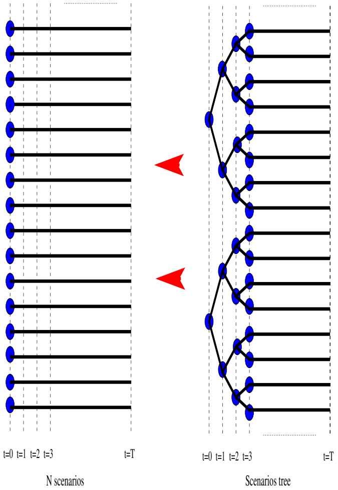
A brief insight into three decomposition methods

Summary and research agenda

Outline of the presentation

Moving from tree to fan (and scenarios)

Equivalent formulations of the nonanticipativity constraints



- On a (scenario) tree, the nonanticipativity constraints

$$\sigma(\mathbf{U}_t) \subset \sigma(\mathbf{W}_0, \ldots, \mathbf{W}_t)$$

are "hardwired"

- On a fan, the nonanticipativity constraints write as linear equality constraints

$$\mathbf{U}_t = \mathbb{E}\left[\mathbf{U}_t \mid \mathbf{W}_0, \ldots, \mathbf{W}_t\right]$$

Progressive Hedging stands as a scenario decomposition method

[Rockafellar and Wets, 1991] dualize the nonanticipativity constraints

$$\mathbf{U}_t = \mathbb{E}\big[\mathbf{U}_t \,\big|\, \mathbf{W}_0, \ldots, \mathbf{W}_t\big]$$

with (random processes) multipliers, *information price system*

> *In summary, the price systems [...] are the ones that would charge for hindsight everything it might be worth. They do therefore truly embody the value of information in the uncertain environment.*

- When the criterion is strongly convex, one uses a Lagrangian relaxation (algorithm "à la Uzawa") to obtain a scenario decomposition
- When the criterion is linear, Rockafellar-Wets (91) propose to use an augmented Lagrangian, and obtain the Progressive Hedging algorithm

Data: step $\rho > 0$, initial multipliers $\{\lambda_s^{(0)}\}_{s\in\mathcal{S}}$
and mean first decision $\bar{\mathbf{u}}^{(0)}$;
Result: optimal first decision $\mathbf{u}$;
repeat

forall *scenarios* $s \in \mathcal{S}$ **do**

Solve deterministic minimization problem for scenario s,
with a penalization $+\lambda_s^{(k)}\left(\mathbf{u}_s^{(k+1)} - \bar{\mathbf{u}}^{(k)}\right)$,
and obtain optimal first decision $\mathbf{u}_s^{(k+1)}$;

Update the mean first decisions by

$$\bar{\mathbf{u}}^{(k+1)} = \sum_{s\in\mathcal{S}} \pi_s \mathbf{u}_s^{(k+1)} \; ;$$

Update the multiplier by

$$\lambda_s^{(k+1)} = \lambda_s^{(k)} + \rho\Big(\mathbf{u}_s^{(k+1)} - \bar{\mathbf{u}}^{(k+1)}\Big) \; , \;\; \forall s \in \mathcal{S} \; ;$$

until $\mathbf{u}_s^{(k+1)} - \sum_{s'\in\mathcal{S}} \pi_{s'} \mathbf{u}_{s'}^{(k+1)} = 0 \; , \;\; \forall s \in \mathcal{S}$;

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We consider an additive model

Consider the following minimization problem

$$\min_{u \in \mathcal{U}_{\mathrm{ad}} \subset \mathcal{U}} J(u) \quad \text{subject to} \quad \Theta(u) - \theta = 0 \in \mathcal{V}$$

for which exists a decomposition of the space $\mathcal{U} = \mathcal{U}^1 \times \ldots \times \mathcal{U}^N$, so that $u \in \mathcal{U}$ writes $u = \big(u^1, \ldots, u^N\big)$ with $u^i \in \mathcal{U}^i$, and also

- $\mathcal{U}_{\mathrm{ad}} = \mathcal{U}_{\mathrm{ad}}^1 \times \cdots \times \mathcal{U}_{\mathrm{ad}}^N$ $\qquad \mathcal{U}_{\mathrm{ad}}^i \subset \mathcal{U}^i$
- $J(u) = J^1(u^1) + \cdots + J^N(u^N)$ $\qquad u^i \in \mathcal{U}^i$
- $\Theta(u) = \Theta^1(u^1) + \cdots + \Theta^N(u^N)$ $\qquad u^i \in \mathcal{U}^i$

Then the problem displays the following additive structure

$$\min_{\substack{u^1 \in \mathcal{U}_{\mathrm{ad}}^1 \\ \vdots \\ u^N \in \mathcal{U}_{\mathrm{ad}}^N}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0$$

Additive model — Price decomposition I

$$\min_{u \in \mathcal{U}_{\mathrm{ad}}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0$$

1. Form the Lagrangian of the problem
   We assume that a saddle point exists,
   so that solving the initial problem is equivalent to

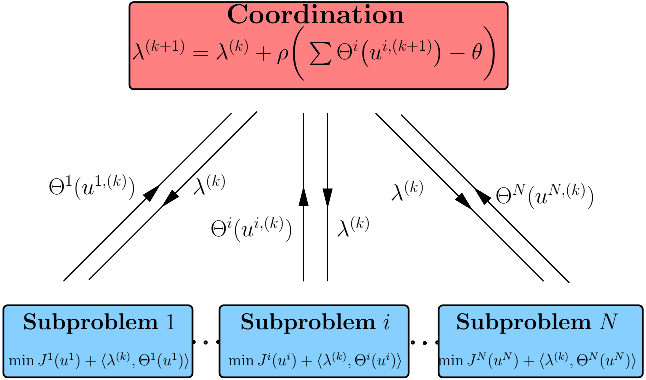
$$\max_{\lambda \in \mathcal{V}} \min_{u \in \mathcal{U}_{\mathrm{ad}}} \sum_{i=1}^{N} \Big(J^i(u^i) + \big\langle \lambda , \Theta^i(u^i) \big\rangle \Big) - \big\langle \lambda , \theta \big\rangle$$

2. Solve this problem by the Uzawa algorithm

$$u^{i,(k+1)} \in \arg\min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) + \big\langle \lambda^{(k)} , \Theta^i(u^i) \big\rangle \ , \quad i = 1 \ldots , N$$

$$\lambda^{(k+1)} = \lambda^{(k)} + \rho \Big(\sum_{i=1}^{N} \Theta^i \Big(u^{i,(k+1)} \Big) - \theta \Big)$$

Additive model — Price decomposition II



Additive model — Resource allocation I

$$\min_{u \in \mathcal{U}_{\mathrm{ad}}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0$$

1. Write the constraint in a equivalent manner by introducing new variables $v = (v^1, \ldots, v^N)$ (the so-called "allocation")

$$\sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0 \quad \Leftrightarrow \quad \Theta^i(u^i) - v^i = 0 \ \text{and} \ \sum_{i=1}^{N} v^i = \theta$$

and minimize the criterion w.r.t. u and v

$$\min_{v \in \mathcal{V}^N} \sum_{i=1}^{N} \Big(\min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) \ \text{s.t.} \ \Theta^i(u^i) - v^i = 0 \Big) \ \text{s.t.} \ \sum_{i=1}^{N} v^i = \theta$$

Additive model — Resource allocation II

$$\min_{v \in \mathcal{V}^N} \sum_{i=1}^{N} \underbrace{\left(\min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) \ \text{ s.t. } \ \Theta^i(u^i) - v^i = 0 \right)}_{G^i(v^i)} \ \text{ s.t. } \ \sum_{i=1}^{N} v^i = \theta$$

$$\Updownarrow$$

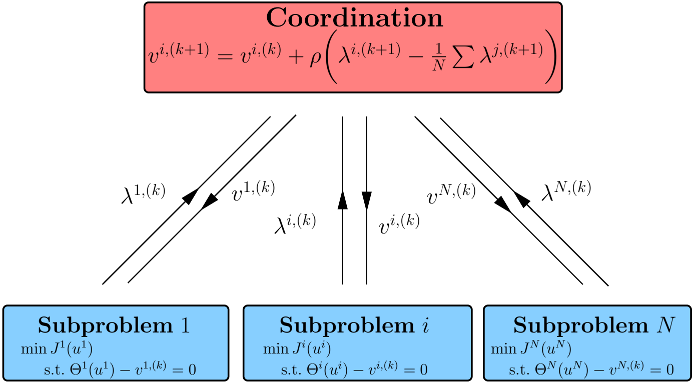
$$\min_{v \in \mathcal{V}^N} \sum_{i=1}^{N} G^i(v^i) \quad \text{s.t.} \quad \sum_{i=1}^{N} v^i = \theta$$

2. Solve the last problem using a projected gradient method

$$G^i(v^{i,(k)}) = \min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) \ \text{ s.t. } \ \Theta^i(u^i) - v^{i,(k)} = 0 \quad \rightsquigarrow \quad \lambda^{i,(k+1)}$$

$$v^{i,(k+1)} = v^{i,(k)} + \rho \left(\lambda^{i,(k+1)} - \frac{1}{N} \sum_{j=1}^{N} \lambda^{j,(k+1)} \right)$$

Additive model — Resource allocation III



Preparing Pierre Carpentier's talk

We can also use price/resource decomposition to bound a minimization problem

$$
\begin{aligned}
V_0^* = \inf_{u^1 \in \mathcal{U}^1_{\mathrm{ad}}, \cdots, u^N \in \mathcal{U}^N_{\mathrm{ad}}} & \sum_{i=1}^{N} J^i(u^i) \\
\text{s.t.} \quad & \underbrace{\big(\Theta^1(u^1), \cdots, \Theta^N(u^N)\big) \in S}_{\text{coupling constraint}}
\end{aligned}
$$

- $u^i \in \mathcal{U}^i$ be a local decision variable
- $J^i : \mathcal{U}^i \to \mathbb{R}, \; i \in [\![1, N]\!]$ be a local objective function
- $\mathcal{U}^i_{\mathrm{ad}}$ be a subset of the local decision set $\mathcal{U}^i$
- $\Theta^i : \mathcal{U}^i \to \mathcal{C}^i$ be a local constraint mapping
- S be a subset of $\mathcal{C} = \mathcal{C}^1 \times \cdots \times \mathcal{C}^N$

We denote by S^o the polar cone of S

$$
S^o = \big\{ p \in \mathcal{C}^\star \mid \langle p, r \rangle \leq 0 \,, \;\; \forall r \in S \big\}
$$

Price and resource local value functions

For each $i \in [\![1, N]\!]$,

- for any price $p^i \in (\mathcal{C}^i)^\star$, we define the local price value

$$\underline{V}_0^i[p^i] = \inf_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) + \left\langle p^i , \Theta^i(u^i) \right\rangle$$

- for any resource $r^i \in \mathcal{C}^i$, we define the local resource value

$$\overline{V}_0^i[r^i] = \inf_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

Proposition (upper and lower bounds for optimal value)

- *For any admissible price* $p = (p^1, \cdots, p^N) \in S^o$
- *For any admissible resource* $r = (r^1, \cdots, r^N) \in S$

$$\sum_{i=1}^{N} \underline{V}_0^i[p^i] \ \leq \ V_0^\ast \ \leq \ \sum_{i=1}^{N} \overline{V}_0^i[r^i]$$

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A brief insight into three decomposition methods
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- Spatial (price/resource) decomposition methods
- Time decomposition methods

Summary and research agenda

Brief literature survey on dynamic programming

	Bellman	Puterman	Bertsekas Schreve	Evstignev	Witsenhausen (standard form)
	1957	1994	1996	1976	1973
State	X	X	X	–	$(\omega, U_{1:t-1})$
Dynamics	$f(X,U,W)$	$P^u_{x,x'}$	$f(X,U,W)$	–	$X_t = (X_{t-1}, U_t)$
Uncertainties	Indep.	–	ρ	$(\Omega, \mathcal{F})$	$(\Omega, \mathcal{F})$
Cost	$\sum_t$	$\sum_t$	$\sum_t$	$j(\omega, U)$	$j(\omega, U)$
Controls	$\gamma(X)$	$\gamma(X)$ $\gamma(H)$	$\gamma(X)$ $\gamma(H)$	$\mathcal{F}_t$-meas.	$\gamma(x_t)$ $\mathcal{I}_t$-meas.
History	–	$(X, U, \ldots)_t$	$(W, U, \ldots)_t$	–	X_t

We introduce the history

- The timeline is

$$w_0 \rightsquigarrow u_0 \rightsquigarrow w_1 \rightsquigarrow u_1 \rightsquigarrow \quad \dots \quad \rightsquigarrow w_{T-1} \rightsquigarrow u_{T-1} \rightsquigarrow w_T$$

- and the history is

$$\overbrace{h_t}^{\text{history}} = (\overbrace{w_0}^{\text{uncertainty}}, \overbrace{u_0}^{\text{control}}, \overbrace{w_1}^{\text{uncertainty}}, u_1, \dots, u_{t-1}, w_t)$$

$$\in \mathcal{H}_t = \mathcal{W}_0 \times \prod_{s=0}^{t-1} (\underbrace{\mathcal{U}_s}_{\substack{\text{control}\\ \text{space}}} \times \underbrace{\mathcal{W}_{s+1}}_{\substack{\text{uncertainty}\\ \text{space}}})$$

History is the largest state

The history follows the dynamics

$$
\begin{aligned}
h_{t+1} &= (\overbrace{w_0, u_0, w_1, u_1, \ldots, u_{t-1}, w_t}^{\text{history } h_t}, u_t, w_{t+1}) \\
&= \Big(h_t, \underbrace{u_t}_{\text{control}}, \underbrace{w_{t+1}}_{\text{uncertainty}} \Big)
\end{aligned}
$$

We formulate a sequence of minimization problems over increasing history spaces

- Once given
  - a (measurable) criterion $j : \mathcal{H}_T \to \overline{\mathbb{R}}$
  - a sequence of stochastic kernels $\rho_{t:t+1} : \mathcal{H}_t \to \Delta(\mathcal{W}_{t+1})$
- we define, for any history h_t, a minimization problem

$$\underbrace{V_t(h_t)}_{\substack{\text{value}\\ \text{function}}} = \inf_{\substack{\gamma,\text{history}\\ \text{feedbacks}}} \int_{\mathcal{H}_T} \overbrace{j(h'_T)}^{\text{criterion}} \underbrace{\rho^{\gamma}_{t:T}(h_t, \mathrm{d}h'_T)}_{\substack{\text{controlled}\\ \text{stochastic kernel}}}$$

There is a Bellman equation involving value functions over increasing history spaces without white noise assumption

$$V_T = j$$
$$V_t = \mathcal{B}_{t+1:t} V_{t+1}$$

where the Bellman operator $\mathcal{B}_{t+1:t}$ is given by

$$\big(\mathcal{B}_{t+1:t}\varphi\big)(h_t) = \inf_{u_t \in \mathcal{U}_t} \int_{\mathcal{W}_{t+1}} \varphi(h_t, u_t, w_{t+1}) \rho_{t:t+1}(h_t, dw_{t+1})$$

Preparing Jean-Philippe Chancelier's talk

Towards state reduction by time blocks

- History h_t is itself a canonical state variable, which lives in the history space $\mathcal{H}_t = \mathcal{W}_0 \times \prod_{s=0}^{t-1}(\mathcal{U}_s \times \mathcal{W}_{s+1})$
- However the size of this canonical state increases with t, which is a nasty feature for dynamic programming
- We will now, but only at some specified times r in $0 = t_0 < t_1 < \cdots < t_N = T$
  - introduce "state" spaces $\mathcal{X}_r$
  - and then reduce the history with a mapping $\theta_r : \mathcal{H}_r \to \mathcal{X}_r$
  - to obtain a compressed "state" variable $\theta_r(h_r) = x_r \in \mathcal{X}_r$
- As an application, we will handle stochastic independence between time blocks but possible dependence *within* time blocks

Graphical sketch of state reduction

The triplet $(\theta_r, \theta_t, f_{r:t})$ is a state reduction across $(r : t)$ if

- the following diagram, for the dynamics, is commutative

$$
\begin{array}{ccc}
\mathcal{H}_r \times \mathcal{H}_{r+1:t} & \xrightarrow{I_d} & \mathcal{H}_t \\
\downarrow \theta_r \qquad \downarrow I_d & & \downarrow \theta_t \\
\mathcal{X}_r \times \mathcal{H}_{r+1:t} & \xrightarrow{f_{r:t}} & \mathcal{X}_t
\end{array}
$$

- the following diagram, for the stochastic kernels, is commutative

$$
\begin{array}{ccc}
\mathcal{H}_r \times \mathcal{H}_{r+1:s-1} & \xrightarrow{\rho_{s-1:s}} & \Delta(\mathcal{W}_s) \\
\downarrow \theta_r \qquad \downarrow I_d & \nearrow \tilde{\rho}_{s-1:s} & \\
\mathcal{X}_r \times \mathcal{H}_{r+1:s-1} & &
\end{array}
$$

Bellman operator across $(r : t)$

$\mathcal{B}_{r:t} : \mathbb{L}^0_+(\mathcal{H}_r, \mathfrak{H}_r) \to \mathbb{L}^0_+(\mathcal{H}_t, \mathfrak{H}_t)$ is defined by

$$\mathcal{B}_{r:t} = \mathcal{B}_{t+1:t} \circ \cdots \circ \mathcal{B}_{r:r-1} \ ,$$

where the one time step operators $\mathcal{B}_{s:s-1}$ are

$$\big(\mathcal{B}_{s:s-1}\varphi\big)(h_{s-1}) = \inf_{u_{s-1} \in \mathcal{U}_{s-1}} \int_{\mathcal{W}_s} \varphi(h_{s-1}, u_{s-1}, w_s) \rho_{s-1:s}(h_{s-1}, dw_s)$$

Graphical sketch of Bellman operator reduction

Supposing a state reduction across $(r:t)$, and denoting by $\theta_r^\star : \mathbb{L}_+^0(\mathcal{X}_r, \mathfrak{X}_r) \to \mathbb{L}_+^0(\mathcal{H}_r, \mathfrak{H}_r)$ the operator defined by

$$\theta_r^\star(\tilde{\varphi}_r) = \tilde{\varphi}_r \circ \theta_r \ , \ \ \forall \tilde{\varphi}_r \in \mathbb{L}_+^0(\mathcal{X}_r, \mathfrak{X}_r) \ ,$$

then there exists a reduced Bellman operator across $(r:t)$ such that

$$\theta_t^\star \circ \tilde{\mathcal{B}}_{r:t} = \mathcal{B}_{r:t} \circ \theta_r^\star \ ,$$

that is, the following diagram is commutative

$$\begin{array}{ccc}
\mathbb{L}_+^0(\mathcal{H}_r, \mathfrak{H}_r) & \xrightarrow{\mathcal{B}_{r:t}} & \mathbb{L}_+^0(\mathcal{H}_t, \mathfrak{H}_t) \\
\theta_r^\star \Big\uparrow & & \Big\uparrow \theta_t^\star \\
\mathbb{L}_+^0(\mathcal{X}_r, \mathfrak{X}_r) & \xrightarrow{\tilde{\mathcal{B}}_{r:t}} & \mathbb{L}_+^0(\mathcal{X}_t, \mathfrak{X}_t)
\end{array}$$

Outline of the presentation

Decomposition and coordination

A brief insight into three decomposition methods

Summary and research agenda

We have sketched three main decomposition methods in multistage stochastic optimization

- time: Dynamic Programming
- scenario: Progressive Hedging
- space: decomposition by prices or by resources

Numerical walls are well-known

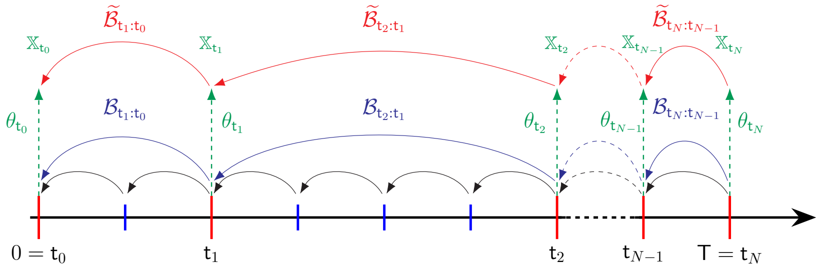
- in dynamic programming,
  the bottleneck is the dimension of the state
- in stochastic programming,
  the bottleneck is the number of stages

Here is our research agenda for stochastic decomposition

- Designing risk criteria compatible with decomposition
- Combining different decomposition methods
  - time: Dynamic Programming
  - scenario: Progressive Hedging
  - space: decomposition by prices or by resources
- to produce blends and tackle large scale energy applications
  - time blocks + prices/resources
    (talk of Jean-Philippe Chancelier)
    - dynamic programming across time blocks
      + prices/resources decomposition by time block
    - application to two time scales battery management
  - time + space
    (talk of Pierre Carpentier)
    - nodal decomposition by prices or by resources
      + dynamic programming within node
    - application to large scale microgrid management

Time block decomposition

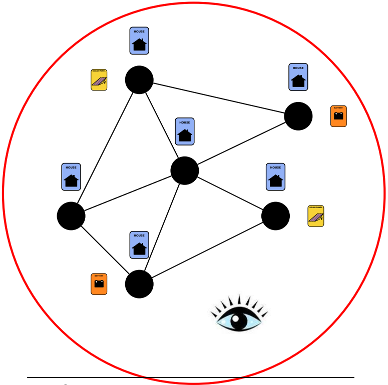
[Carpentier, Chancelier, De Lara, Martin, and Rigaut, 2023][1]

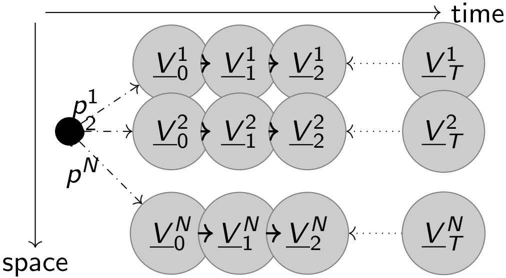


[1] P. Carpentier, J.-P. Chancelier, M. De Lara, T. Martin, and T. Rigaut. Time Block Decomposition of Multistage Stochastic Optimization Problems. *Journal of Convex Analysis*, 30(2), 2023

Mix of spatial and temporal decompositions

[Carpentier, Chancelier, De Lara, and Pacaud, 2020][2]
[Pacaud, De Lara, Chancelier, and Carpentier, 2022][3]





[2] P. Carpentier, J.-P. Chancelier, M. De Lara, and F. Pacaud. Mixed spatial and temporal decompositions for large-scale multistage stochastic optimization problems. *Journal of Optimization Theory and Applications*, 186(3):985–1005, 2020

[3] F. Pacaud, M. De Lara, J.-P. Chancelier, and P. Carpentier. Distributed multistage optimization of large-scale microgrids under stochasticity. *IEEE Transactions on Power Systems*, 37(1):204–211, 2022

Bibliography

R. E. Bellman. *Dynamic Programming*. Princeton University Press, Princeton, N.J., 1957.

P. Carpentier, G. Cohen, and J.-C. Culioli. Stochastic optimal control and decomposition-coordination methods — Part I: Theory. *In: Recent Developments in Optimization, Roland Durier and Christian Michelot (Eds.), LNEMS 429:72–87, Springer-Verlag, Berlin*, 1995.

P. Carpentier, J.-P. Chancelier, M. De Lara, and F. Pacaud. Mixed spatial and temporal decompositions for large-scale multistage stochastic optimization problems. *Journal of Optimization Theory and Applications*, 186(3): 985–1005, 2020.

P. Carpentier, J.-P. Chancelier, M. De Lara, T. Martin, and T. Rigaut. Time Block Decomposition of Multistage Stochastic Optimization Problems. *Journal of Convex Analysis*, 30(2), 2023.

F. Pacaud, M. De Lara, J.-P. Chancelier, and P. Carpentier. Distributed multistage optimization of large-scale microgrids under stochasticity. *IEEE Transactions on Power Systems*, 37(1):204–211, 2022. doi: 10.1109/TPWRS.2021.3087775.

R. Rockafellar and R. J.-B. Wets. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of operations research*, 16(1): 119–147, 1991.