Precedence, subsystem, memory relations between agents in Witsenhausen’s intrinsic model for discrete stochastic control

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A linear control-oriented stochastic model

Controls: $u_1 \in \mathbb{U}_1 = \mathbb{R}$ and $u_2 \in \mathbb{U}_2 = \mathbb{R}$

Random issue: $\omega = (x_0, v) \in \Omega = \mathbb{R} \times \mathbb{R}$

State equations $$\begin{cases} x_1 &= x_0 + u_1 \\ x_2 &= x_1 - u_2 \,. \end{cases}$$

Output equations $$\begin{cases} y_0 &= x_0 \\ y_1 &= x_1 + v \,. \end{cases}$$

H. S. Witsenhausen. A counterexample in stochastic optimal control. *SIAM J. Control*, 6(1):131–147, 1968.

A LQG problem with linear solution

x_0 and v are Gaussian independent.

$$\inf \mathbb{E}(k^2 u_1^2 + x_2^2) \,,$$

$$\begin{cases} x_1 &= x_0 + u_1 \\ x_2 &= x_1 - u_2 \end{cases}$$

$$\begin{cases} u_1 & \text{measurable w.r.t. } y_0 = x_0 \\ u_2 & \text{measurable w.r.t. } (y_0, y_1) = (x_0, x_1 + v) \,. \end{cases}$$

Solution $u_1 = v_1(y_0)$, $u_2 = v_2(y_0, y_1)$, where v_1 and v_2 are affine functions.

Classical information pattern

Still LQG but... nonlinear solution!

x_0 and v are Gaussian independent.

$$\inf \mathbb{E}(k^2 u_1^2 + x_2^2) \,,$$

$$\begin{cases} x_1 &= x_0 + u_1 \\ x_2 &= x_1 - u_2 \end{cases}$$

$$\begin{cases} u_1 & \text{measurable w.r.t. } y_0 = x_0 \\ u_2 & \text{measurable w.r.t. } y_1 = x_1 + v \,. \end{cases}$$

Solution $u_1 = v_1(y_0)$, $u_2 = v_2(y_0, y_1)$ *is known to exist and to be (highly) nonlinear!*

The Witsenhausen counterexample

Problems raised by nonclassical information patterns

- *Classical information pattern =*
  sequential and memory of past knowledge
  agent 1 observes y_0
  agent 2 observes y_0 and y_1
  Stochastic dynamic programming, HJB...
- *Nonclassical information pattern*
  Non convex optimization problems.
  Signaling/dual effect of control:
  direct cost minimization *versus*
  indirect effect on output available for control.
  Interaction between information/observation and control.

Plan

- The issues: outside sequentiality and perfect memory
- Witsenhausen's intrinsic model
- Causality
- Precedence, subsystem and memory-communication relations
- A typology of systems
- Sequential systems
- Partially nested systems

THE ISSUES: OUTSIDE SEQUENTIALITY AND PERFECT MEMORY

Control-oriented works on dynamic games

dynamic equation: state + decision variables of the players + random variables

output variables for a player: functions of the state, decision and random variables

information structure: each decision variable is any desired (measurable) function of the output variables generated for that player up to that time

Tacit assumptions: sequentiality and perfect memory

H. S. Witsenhausen. On information structures, feedback and causality. *SIAM J. Control*, 9(2):149–160, May 1971.

“Such a setup assumes that the time order in which the various decisions variables are selected is fixed in advance. It assumes that each player acts as if he had responsability only for one station. It assumes that this station has perfect memory. ”

“For large complex systems these tacit assumptions are unlikely to hold. (. . .) The order in which the various agents of the various organizations will have to act cannot always be predicted, and the information available to different agents, even of the same organization, may be noncomparable in the sense that, of two agents, neither one knows everything his colleague knows. ”

WITSENHAUSEN’S INTRINSIC MODEL

Agents decisions

A: finite set representing agents (decision makers)

Each agent $\alpha \in A$ takes one decision $u_\alpha \in \mathbb{U}_\alpha$

$(\mathbb{U}_\alpha, \mathcal{U}_\alpha)$: measurable control set $\mathbb{U}_\alpha$ with σ-algebra $\mathcal{U}_\alpha$

(Discrete time dynamics case: several different decision makers, one for each period)

$$\underbrace{\mathbb{U}_A \stackrel{\mathrm{def}}{=} \prod_{\beta \in A} \mathbb{U}_\beta}_{decisions\ set}, \quad \mathcal{U}_A \stackrel{\mathrm{def}}{=} \bigotimes_{\beta \in A} \mathcal{U}_\beta .$$

Randomness

$(\Omega, \mathcal{F})$: measurable sample space of random issues, with σ-algebra $\mathcal{F}$

NO probability measure (as long as one does not evaluate mathematical expectations)

$$\mathbb{H} \stackrel{\text{def}}{=} \mathbb{U}_A \times \Omega \quad \text{and} \quad \mathcal{H} \stackrel{\text{def}}{=} \mathcal{U}_A \otimes \mathcal{F} \, .$$

Information structure

The *information field* $\mathcal{I}_\alpha$ *of agent* α *is a subfield of* $\mathcal{U}_A \otimes \mathcal{F}$

(may depend upon other agents decisions and upon realizations of the sample space Ω)

The collection $(A, (\Omega, \mathcal{F}), (\mathbb{U}_\alpha, \mathcal{U}_\alpha, \mathcal{I}_\alpha)_{\alpha \in A})$ is an

information structure

or a

(stochastic control) system

Decision process = closed-loop equations

$$\Lambda_\alpha \stackrel{\text{def}}{=} \{\lambda_\alpha : \mathbb{U}_A \times \Omega \to \mathbb{U}_\alpha \mid \lambda_\alpha^{-1}(\mathcal{U}_\alpha) \subset \mathcal{I}_\alpha\}$$

Any $\lambda_\alpha \in \Lambda_\alpha$ represents a *possible policy* (or *control law*, *control design*) of agent α.
For any possible policy
$\lambda = (\lambda_\alpha)_{\alpha \in A} \in \Lambda_A \stackrel{\text{def}}{=} \prod_{\alpha \in A} \Lambda_\alpha$, the problem of decision process is to find, for any $\omega \in \Omega$, solutions $u \in \mathbb{U}_A$ (dependent upon ω) satisfying the *closed-loop equations*:

$$u_\alpha = \lambda_\alpha((u_\beta, \beta \in A), \omega), \quad \forall \alpha \in A.$$

Two agents, two decisions, two random issues

- Agents: $A = \{a, b\}$.
- Decision sets:
  $\mathbb{U}_a = \{a_1, a_2\}, \mathcal{U}_a = \{\emptyset, \mathbb{U}_a, \{a_1\}, \{a_2\}\}$,
  $\mathbb{U}_b = \{b_1, b_2\}, \mathcal{U}_b = \{\emptyset, \mathbb{U}_b, \{b_1\}, \{b_2\}\}$.
- Sample space:
  $\Omega = \{\omega_-, \omega_+\}, \mathcal{F} = \{\emptyset, \Omega, \{\omega_-\}, \{\omega_+\}\}$.
- $\mathbb{H} = \mathbb{U}_a \times \mathbb{U}_b \times \Omega = \{a_1, a_2\} \times \{b_1, b_2\} \times \{\omega_-, \omega_+\}$,
  $\mathcal{H} = 2^{\mathbb{H}}$.

Static team

- $\mathcal{I}_a = \{\emptyset, \mathbb{H}\}$:
  agent a knows nothing, neither on agents a and b decisions, nor on the random issue.
- $\mathcal{I}_b = \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \Omega, \{\omega_-\}, \{\omega_+\}\}$:
  agent b knows the random issue, but not agents a and b decisions.

There are no interactions between agents, just a dependence upon random issues:
this is an example of static team.

More generally,
$\mathcal{I}_a \subset \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathcal{F}$,
$\mathcal{I}_b \subset \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \mathcal{F}$.

A sequential example

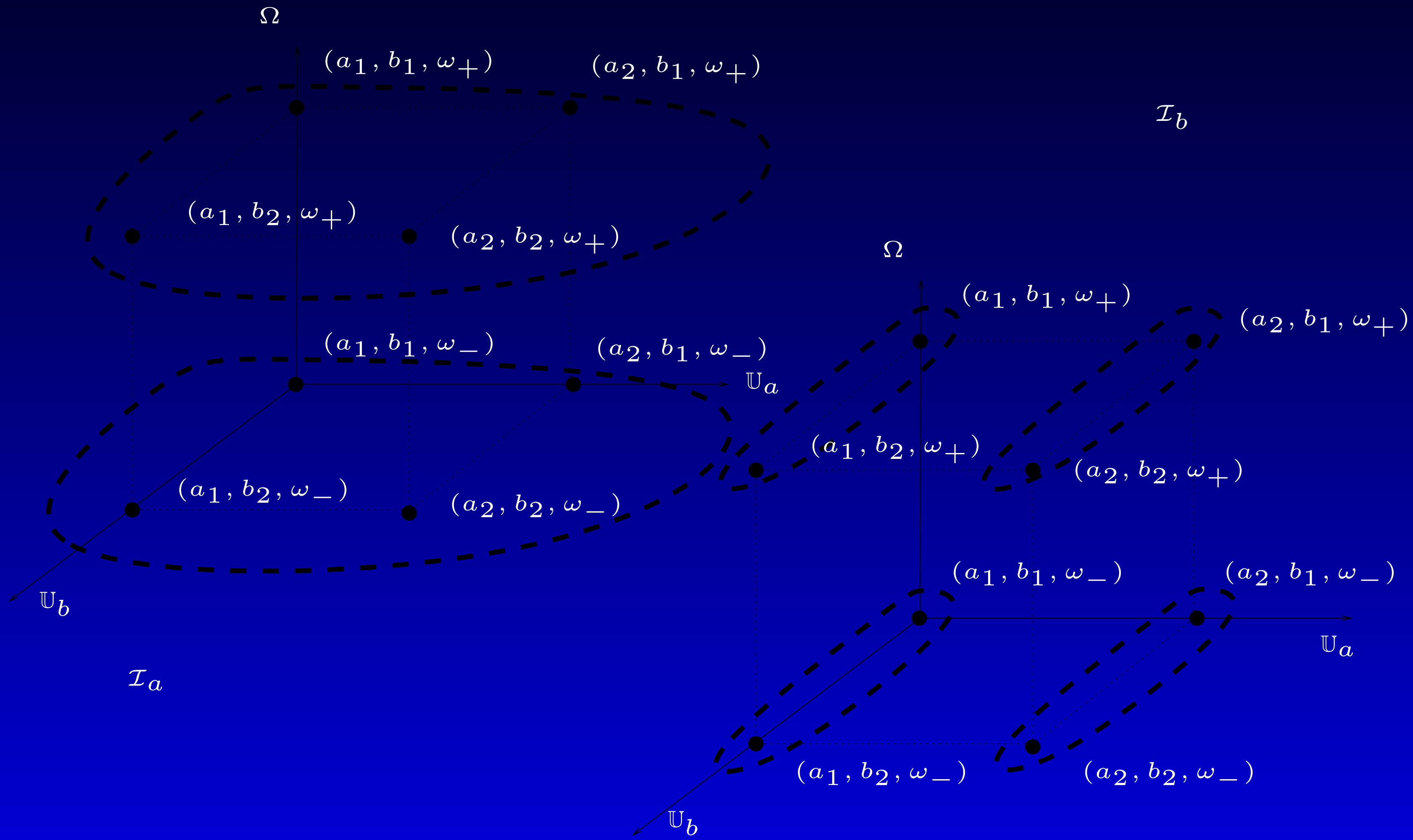

Figure 1: *A sequential information structure*

A sequential example

$$\mathcal{I}_a = \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \Omega, \{\omega_-\}, \{\omega_+\}\}$$

$$\mathcal{I}_b = \{\emptyset, \mathbb{U}_a, \{a_1\}, \{a_2\}\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \Omega, \{\omega_-\}, \{\omega_+\}\}.$$

The system is sequential:

1. agent a observes the random issue and takes his decision in function;
2. agent b observes both agent a's decision and the random issue and takes his decision in function.

Memory-communication relationship

$\mathcal{I}_a = \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \Omega, \{\omega_-\}, \{\omega_+\}\}$

$\mathcal{I}_b =$
$\{\emptyset, \mathbb{U}_a, \{a_1\}, \{a_2\}\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \Omega, \{\omega_-\}, \{\omega_+\}\}.$

$\mathcal{I}_a \subset \mathcal{I}_b$ may be interpreted in different ways.

- One may say that agent a *communicates* his own information to agent b.
- If agent a is an individual at time $t = 0$, while agent b is the same individual at time $t = 1$, one may say that the information is not forgotten with time: *memory of past knowledge*.

An example with deadlock

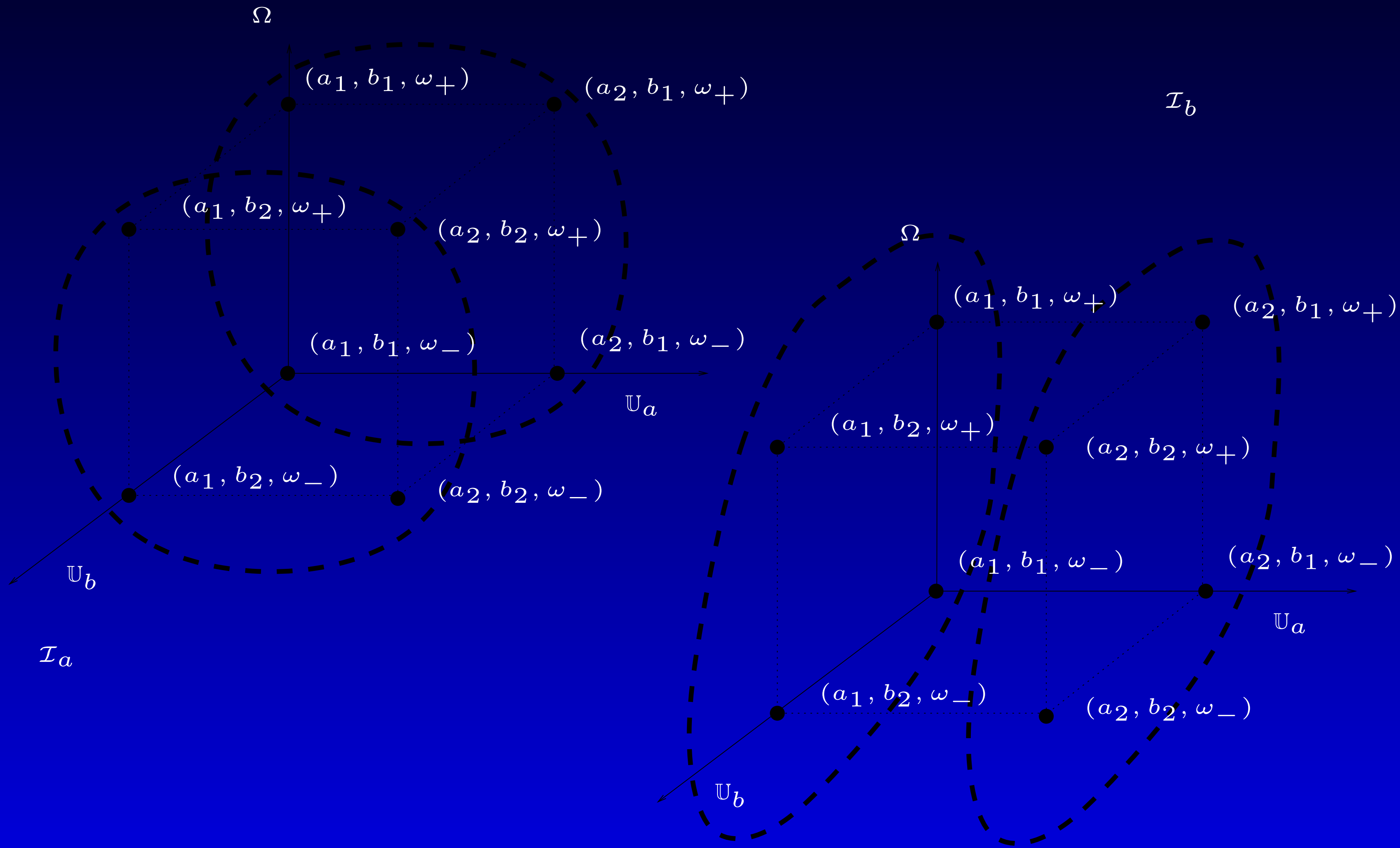

Figure 2: *An information structure with deadlock*

An example with deadlock

$$\mathcal{I}_a = \{\emptyset, \mathbb{U}_a\} \otimes \{\emptyset, \mathbb{U}_b, \{b_1\}, \{b_2\}\} \otimes \{\emptyset, \Omega\},$$
$$\mathcal{I}_b = \{\emptyset, \mathbb{U}_a, \{a_1\}, \{a_2\}\} \otimes \{\emptyset, \mathbb{U}_b\} \otimes \{\emptyset, \Omega\}.$$

Agent a observes only agent b's decision, while agent b observes only agent a's decision: this corresponds to a *deadlock* situation where the decision process may have no solution or multiple solutions.
The system is not sequential.

A causal nonsequential example

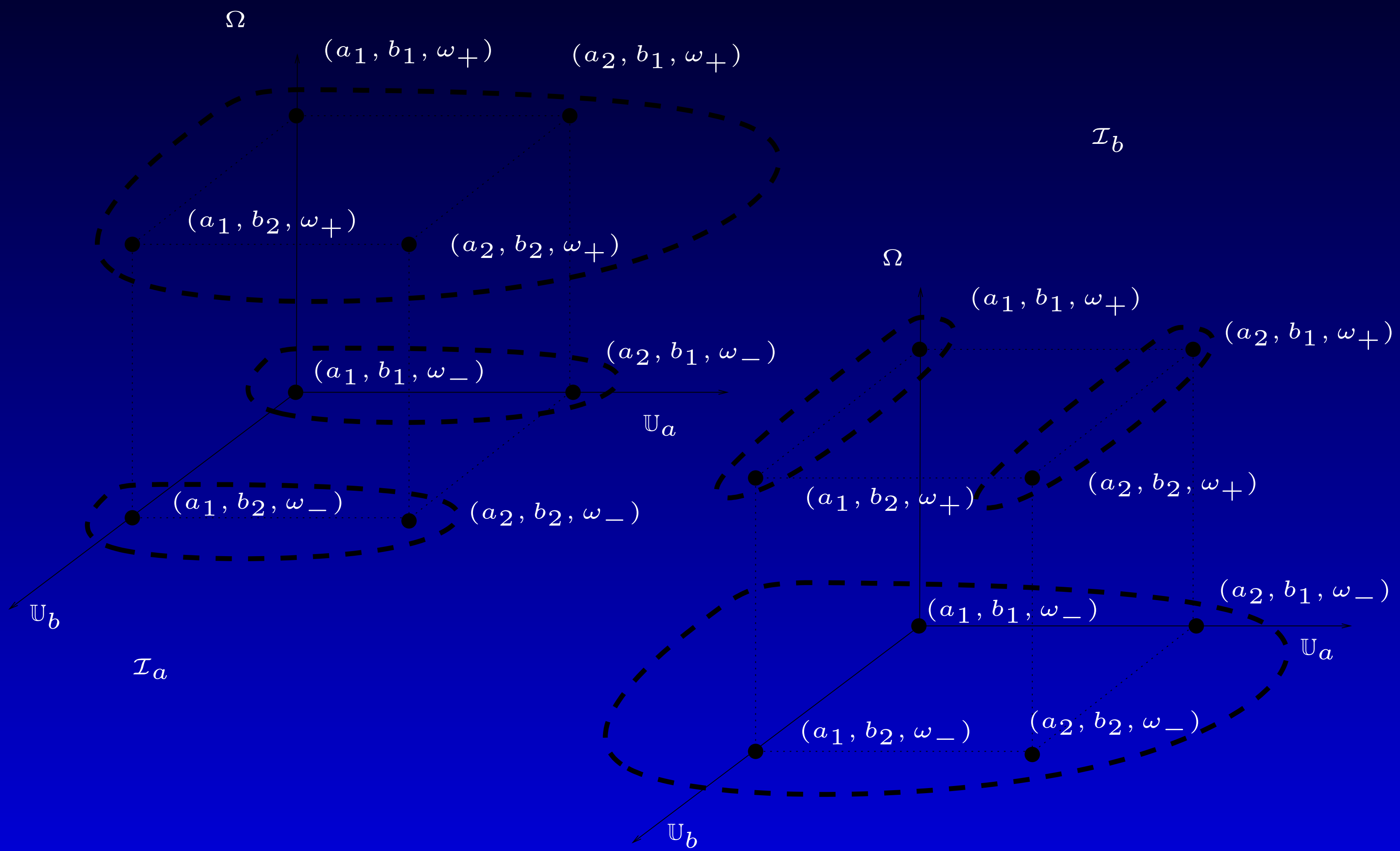

Figure 3: *A nonsequential information structure*

Multi-agents stochastic control problems

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CAUSALITY

Solvability property

The *solvability property (S)* holds when, for any $\lambda \in \Lambda_A$ and any $\omega \in \Omega$, there exists one and only one $u \in \mathbb{U}_A$ satisfying the closed-loop equations

$$u_\alpha = \lambda_\alpha((u_\beta, \beta \in A), \omega)\,, \quad \forall \alpha \in A\,.$$

This defines a mapping $M^\lambda : \Omega \to \mathbb{U}_A$.

The *solvability/measurability property (SM)* holds when, in addition, the mapping $M^\lambda : \Omega \to \mathbb{U}_A$ is measurable from $(\Omega, \mathcal{F})$ to $(\mathbb{U}_A, \mathcal{U}_A)$.

Witsenhausen's causality property (C)

Let n =card(A). For $k \in \{1, \dots, n\}$, let S_A^k denote the set of injective mappings from $\{1, \dots, k\}$ to A.

Thus $S_A \overset{\text{def}}{=} S_A^n$ is the set of *total orderings* of A. We also put $S_A^0 = \emptyset$.

For $0 \leq i \leq j \leq n$, let $T_i^j : S_A^j \longrightarrow S_A^i$ denote the *truncation map* which restricts any $\sigma \in S_A^j$ to the domain $\{1, \dots, i\}$, or to $\emptyset$ if $i = 0$.

Witsenhausen's causality property (C)

An information structure is said to possess *causality property (C)* (or a system is said to be *causal*) if there exists (at least) one mapping $\varphi : \mathbb{U}_A \times \Omega \to S_A$, with the property that

$$\forall k \in \{1, \ldots, n\}, \quad \forall s \in S_A^k,$$

$$\mathcal{I}_{s(k)} \cap (T_k^n \circ \varphi)^{-1}(s) \subset \mathcal{U}(\{s(1), \ldots, s(k-1)\}) \otimes \mathcal{F}.$$

Witsenhausen: Causality (C) $\Rightarrow$ Solvability (S)

PRECEDENCE, SUBSYSTEM AND MEMORY-COMMUNICATION BINARY RELATIONS BETWEEN AGENTS

Precedence binary relation between agents

Ho and Chu in [HC72, HC74] for the multi-agent LQG problem:

$$z_i = H_i \xi + \sum_j D_{ij} u_j \,, \quad \forall i$$

j is related to i if $D_{ij} \neq 0$.

Extension of precedence to non necessarily LQG problems was given in [BCC$^+$05], but not within the framework of Witsenhausen's intrinsic model.

Cylindrical extensions

Subset $B \subset A$ of all agents

$$\mathcal{U}(B) \stackrel{\text{def}}{=} \bigotimes_{\beta \in B} \mathcal{U}_\beta \otimes \bigotimes_{\beta \notin B} \{\emptyset, \mathbb{U}_\beta\} \subset \mathcal{U}_A = \mathcal{U}(A)\,.$$

Properties:

$$\mathcal{U}(B \cap C) = \mathcal{U}(B) \cap \mathcal{U}(C)$$

$$\mathcal{U}(B \cup C) = \mathcal{U}(B) \vee \mathcal{U}(C)$$

Precedence binary relation between agents

Consider an agent $\alpha \in A$ and define

$[\alpha] \stackrel{\text{def}}{=}$ intersection of subsets $B \subset A$ such that $\mathcal{I}_\alpha \subset \mathcal{U}(B) \otimes \mathcal{F}$:

$$\mathcal{I}_\alpha \subset \mathcal{U}([\alpha]) \otimes \mathcal{F}$$

Precedence binary relation $\mathfrak{P}$ on A:

$$\beta \,\mathfrak{P}\, \alpha \iff \beta \in [\alpha]$$

β is a *precedent* of α: agent β influences the observation available to agent α.

Self information

The precedence relation is generally not reflexive: $\alpha \in [\alpha]$ means that agent α decisions influence its own observation

Witsenhausen defines *absence of self information*: no agent is a precedent of himself

Witsenhausen:

$$\textbf{Causality (C)} \Downarrow \textbf{absence of self information}$$

Subsystem binary relation between agents (Witsenhausen, 1975)

Information of the subset $B \subset A$ of agents:

$$\mathcal{I}_B \stackrel{\text{def}}{=} \bigvee_{\beta \in B} \mathcal{I}_\beta \, .$$

A subset B of A is a *subsystem* if $\mathcal{I}_B \subset \mathcal{U}(B) \otimes \mathcal{F}$.

The information received by agents in B depends upon actions of nature and actions of members of B only.

Subsystems form the closed sets of a topology

Connected components are dynamically decoupled subsystems; a static coupling remains through the common dependence upon the random sample ω.

The *closure* $\langle B \rangle$ of a subset $B \subset A$ is the smallest subsystem containing B.

The *subsystem generated by agent* α is the closure $\langle \alpha \rangle$ of the singleton $\{\alpha\}$.

The corresponding *subsystem* binary relation $\mathfrak{S}$ between agents is as follows:

$$\forall (\alpha, \beta) \in A^2 , \quad \beta \mathfrak{S} \alpha \iff \beta \in \langle \alpha \rangle \, .$$

Memory-communication binary relation

Inspired by [HC72], generalized in [BCC+05]

$\|\alpha\| \stackrel{\text{def}}{=}$ union of subsets $B \subset A$ such that $\mathcal{I}_B \subset \mathcal{I}_\alpha$:

$$B \subset \|\alpha\| \iff \mathcal{I}_B \stackrel{\text{def}}{=} \bigvee_{\beta \in B} \mathcal{I}_\beta \subset \mathcal{I}_\alpha \,.$$

Memory-communication binary relation $\mathfrak{M}$ on A by

$$\forall (\alpha, \beta) \in A^2 \,, \quad \beta \,\mathfrak{M}\, \alpha \iff \beta \in \|\alpha\| \iff \mathcal{I}_\beta \subset \mathcal{I}_\alpha \,.$$

When $\beta \in \|\alpha\|$, the observations available to agent β are part of those available to agent α.

A TYPOLOGY OF SYSTEMS

Static teams, systems without self information

A system is a *static team* if $[\alpha] = \emptyset$, for all $\alpha \in A$, *i.e.* $[A] = \emptyset$. Equivalently, the precedence relation $\mathfrak{P}$ is empty or the subsystem relation $\mathfrak{S}$ is reduced to the equality relation $\mathbf{1}_A$.

A system *without self information* is one for which $\alpha \notin [\alpha]$, whatever the agent $\alpha \in A$. Equivalently, the complementary relation $\neg\mathfrak{P}$ of the precedence relation is reflexive.

Sequential systems

A system is *sequential* if there exists an ordering $(\alpha_0, \dots, \alpha_{n-1})$ of the agents A such that $[\alpha_0] = \emptyset$ and

$$\forall k = 1, \dots, n-1 \,, \quad [\alpha_k] \subset \{\alpha_0, \dots, \alpha_{k-1}\} \,.$$

Agents are labelled so that
agent α_i is a precedent of agent α_j
only if
index i is before index j ($i < j$).

Classical systems

A system is *classical* if it is sequential and, in addition, $\alpha_k \in \|\alpha_{k+1}\|$ for $k = 0, \ldots, n-2$.

Equivalently, there exists an ordering $(\alpha_0, \ldots, \alpha_{n-1})$ of A such that $[\alpha_0] = \emptyset$ and, for $k = 1, \ldots, n-1$,

$$[\alpha_k] \subset \{\alpha_0, \ldots, \alpha_{k-1}\} \subset \{\alpha_0, \ldots, \alpha_{k-1}, \alpha_k\} \subset \|\alpha_k\| \, .$$

This corresponds to *memory of past knowledge*.

Agents are labelled so that agent α_k
influences *at most* agents $\alpha_0, \ldots, \alpha_{k-1}$
and
knows *at least* what he and $\alpha_0, \ldots, \alpha_{k-1}$ know

Strictly classical systems

$\mathcal{U}(\{\alpha_k\}) \otimes \mathcal{F} \subset \mathcal{I}_{\alpha_{k+1}}$ means that agent α_{k+1} knows the actions of agent α_k.

A system is *strictly classical* if it is classical with $\mathcal{U}(\{\alpha_k\}) \otimes \mathcal{F} \subset \mathcal{I}_{\alpha_{k+1}}$.

strictly classical

=

sequential

+

memory of past knowledge

+

memory of past actions

=

sequential + *perfect recall*.

Subsystem = reflexive and transitive closure of precedence

Theorem 1 *The precedence relation $\mathfrak{P}$ is included in the subsystem relation $\mathfrak{S}$.*

The subsystem relation $\mathfrak{S}$ is the reflexive and transitive closure $\mathfrak{P}^*$ of the precedence relation $\mathfrak{P}$.

The following inclusions and equalities hold true:

$$\mathfrak{P} \subset \underbrace{\mathfrak{P}^{\infty}}_{transitive\ closure} \subset \mathfrak{S} = \underbrace{\mathfrak{P}^{*} = \mathfrak{P}^{\infty} \cup \mathbf{1}_A}_{reflexive\ and\ transitive\ closure} .$$

To form the subsystem generated by an agent, take this agent, add its precedents, the precedents of its precedents, and so on…

SEQUENTIAL SYSTEMS

Characterizations of sequential systems

The equivalence of the two first assertions in the following Theorem is due to Witsenhausen in [Wit75].

The others are new. They will prove useful in the sequel for characterizing partially nested systems without self information.

Theorem 2 *The following assertions are equivalent:*

1. *the system is sequential;*
2. *the system is without self information and the pre-order* $\mathfrak{S}$ *is an order;*
3. *the relation* $\mathfrak{P}$ *is acyclic (that is,* $\forall \alpha \in A$*,* $\forall n \geq 1$*,* $\alpha \notin [\alpha]^n$*);*
4. *the precedence directed graph* $G(\mathfrak{P})$ *built from* $\mathfrak{P}$ *is a forest;*
5. *the relation* $\neg(\mathfrak{P}^\infty)$ *is reflexive.*

Sequentiality for transitive precedence binary relations

Corollary 3 *If the precedence binary relation $\mathfrak{P}$ is transitive, that is if $\mathfrak{P} = \mathfrak{P}^{\infty}$, sequentiality is equivalent to absence of self information.*

PARTIALLY NESTED SYSTEMS

Partially nested systems and stochastic dynamic programming

H. S. Witsenhausen. On policy independence of conditional expectations. *Information and Control*, 28(1):65–75, 1975.

"If an observer of a stochastic control system observes both the decision taken by an agent in the system and the data that was available for this decision, then the conclusions that the observer can draw do not depend on the functional relation (policy, control law) used by this agent to reach his decision. "

Partially nested systems

We here generalize the quasiclassical systems to non necessarily sequential systems (the terminology is taken from [HC72, HC74] who introduced it for LQG systems).

A *partially nested* system is one for which the graph of precedence is included in the graph of communication:

$$\mathfrak{P} \subset \mathfrak{M}.$$

For such a system, any agent has access to the information of those agents which are its precedents (and thus influence its own information).

Causal partially nested systems are sequential

[BCC+05] **For a partially nested system, the precedence relation $\mathfrak{P}$ is transitive**

Theorem 4 *A partially nested system without self-information is sequential. As a consequence, a causal partially nested system is sequential.*

When any agent has access to the information of those agents which are its precedents and self-information is excluded, agents can be labelled so that agent 1 takes a decision depending at most upon the random issue, agent 2 takes a decision depending at most upon the random issue and agent 1's decision, . . .

A summary table of results

		Binary relations between agents				
		subsystem $\mathfrak{S}$		precedence $\mathfrak{P}$		memory $\mathfrak{M}$
Properties		pre-order		$\mathfrak{S} = \mathfrak{P}^{\infty} \cup \mathbf{1}_A$		pre-order
no self information	$\iff$			$\neg\mathfrak{P}$ reflexive		
static team	$\iff$	$\mathfrak{S} = \mathbf{1}_A$	or	$\mathfrak{P} = \emptyset$		
sequential	$\iff$	order	or	$\neg\mathfrak{P}^{\infty}$ reflexive		
sequential	$\iff$	order	or	acyclic		
classical	$\Rightarrow$			acyclic	and	$\mathfrak{M} \supset \mathfrak{P}$
quasiclassical	$\iff$			acyclic	and	$\mathfrak{M} \supset \mathfrak{P}$
closed under precedence	$\iff$			$\mathfrak{P} = \mathfrak{P}^{\infty}$		
partially nested	$\iff$					$\mathfrak{M} \supset \mathfrak{P}$
partially nested	$\Rightarrow$			$\mathfrak{P} = \mathfrak{P}^{\infty}$		

CONCLUSION

Strictly partially nested systems are good candidates for applying stochastic dynamic programming to team problems.

With our result above, it appears that strictly partially nested systems without self information belong to strictly classical systems, for which stochastic dynamic programming is well established…

Among systems without self information, the typology of systems is more or less reduced to the distinction between classical and nonclassical ones…

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[HC72] Y. C. Ho and K. C. Chu. Team decision