Adaptive Strategies for The Open-Pit Mine Optimal Scheduling Problem

Nathanael BEEKER, Michel DE LARA
CERMICS, Université Paris-Est, France

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Chuquicamata is an example of open-pit mine

Outline of the presentation

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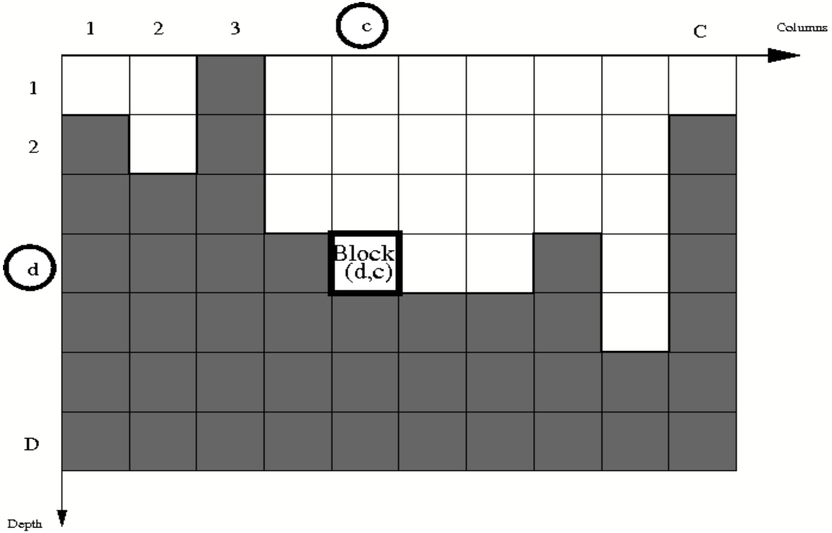
Mines are described by means of a block model



Blocks are extracted sequentially

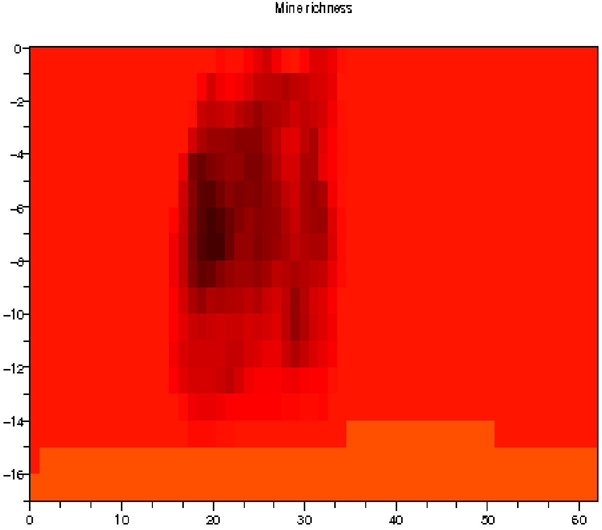
- Time $t = t_0, \dots, T$ is discrete: $T - t_0 + 1$ number of periods
- The set of blocks extracted during period t is denoted by $\mathcal{B}(t)$
- Blocks are extracted sequentially under the following hypothesis:
  - only blocks at the surface may be extracted
  - capacity constraints: no more than a given number of blocks can be extracted in one time unit (this number can be uncertain, due to equipment failure)

$$\text{cardinal}\, \mathcal{B}(t) \leq \quad \text{capacity}$$

  - slope constraints: a block cannot be extracted if the slope made with one of its neighbours is too high, due to physical requirements

Slope constraints are materialized in the Chuquicamata mine

Profit models capture economic data



A profit model is built upon a block model

Each block $b \in \mathbb{B}$ is a three-dimensional cuboid

- identified as $b = (c, d)$ by
  - its column $c \in \mathbb{C}$
  - its depth index $d \in \{1, \ldots, D\}$
- containing attributes $\omega_b(t) = \big(\omega_b^1(t), \cdots, \omega_b^I(t)\big) \in \mathbb{R}^I$
  - either intrinsic attributes (which do indeed depend upon block b)
    - ore grade (does not depend upon time t)
    - extraction cost (may depend upon time t)
    - density, volume, pollutants...
  - or external attributes
    - ore prices
    - capacity constraints
  - which gives its net value or worth

$$w_b(t) = \mathtt{Worth}(\omega_b(t)) = Price(t) \cdot Ore(b) - Cost(b, t)$$

An example of data file

```
X Y Z Density(t/m3) Tonelaje ValProc(US$) ValNOProc(US$)

4885 8250 735 2.7 72900 -437400 -72900
4915 8250 735 2.7 72900 -437400 -72900
4945 8250 735 2.7 72900 -437400 -72900
4975 8250 735 2.7 72900 -437400 -72900
5005 8250 735 2.7 72900 -437400 -72900
5035 8250 735 2.7 72900 -437400 -72900
5065 8250 735 2.7 72900 -437400 -72900
5125 8250 735 0.065 1755 -10530 -1755
5155 8250 735 0.187 5049 -30294 -5049
5185 8250 735 1.159 31293 -187758 -31293
5215 8250 735 2.267 61209 -367254 -61209
```

Open-pit mine sequencing optimization is the mathematical issue

- block model + profit model
- discount rate (for instance $r_f = 10\%$)

  1 \$ next year $\equiv \frac{1}{1+10\%}$ \$ seen from today
- the net present value (NPV) of an admissible extraction sequence $\big(\mathcal{B}(\cdot) := \mathcal{B}(t_0), \ldots, \mathcal{B}(T)\big)$ is the discounted sum of extracted block values

$$\sum_{t=t_0}^{T} \left(\frac{1}{1+r_f}\right)^t \sum_{b \in \mathcal{B}(t)} w_b(t)$$

open-pit mine optimal scheduling:
admissible extraction sequences which
maximize the net present value (NPV)

Outline of the presentation

State of the art in numerical methods

- Size of instances:
  Up to 5-10 millions blocks
  Several dozens of years of exploitation
- Exact methods: Integer linear programming
  Up to 1,000 blocks

Authors	Max size	Optimality	Time
Cacetta&Hill (2003)	209,664	2.5%	4h
Chicoisne et al. (2009)	5,000,000	3%	1h

Some algorithms may be seen as adaptive strategies

- Myopic strategy: best top block selection

$$b^{\star}(t) = \underset{\text{blocks } b \text{ at the surface}}{\arg\max} w_b(t)$$

- k-step look ahead strategy
  compute the NPV for all possible admissible selections of k blocks ahead, and choose the optimal one

$$\mathcal{B}^{\star}(t) = \underset{\text{admissible } \mathcal{B}(t) \text{ at time } t}{\arg\max} \sum_{b \in \mathcal{B}(t)} w_b(t)$$

- A strategy is designed off-line but is implemented on-line, depending on the "state" at time t (adaptive)

Multi-armed bandit

Structure of a "jobs dynamical model"

- Finite number of jobs $j = 1, \dots, J$
- To each job j is attached a local state x_j
- At each time t, a decision c consists in selecting one of the jobs in the set $\mathbb{U} = \{1, \dots, J\}$
- If the job j is selected at period t,
  - the local state $x_j(t)$ evolves according to a local dynamics $\mathtt{Dyn}_j$, giving the new state $x_j(t+1) = \mathtt{Dyn}_j\big(x_j(t), \omega(t)\big)$, where the random variables $\omega(t_0), \dots, \omega(T-1)$ are independent,
  - other local states $x_i(t)$ do not change: $x_i(t+1) = x_i(t)$ for $i \neq j$
  - a reward $\mathtt{Reward}_j\big(x_j(t), \omega(t)\big)$ is obtained

Structure of an index strategy

- Finite number of jobs $j = 1, \dots, J$
- To each job j is attached a local state x_j
- To each job j is attached an index function $\mathtt{Index}_j(x_j)$ which depends on the local state x_j
- At each time t, select the job with the highest index among all $\mathtt{Index}_j\big(x_j(t)\big)$, $j = 1, \dots, J$

An index strategy is an example of decomposition-coordination:

- decomposition: the global state is decomposed in J local states, each with its dynamics,
- coordination: the DM updates the maximum of the indexes and the arg max, and takes the decision accordingly

The Gittins index is specific

$$\mathtt{Index}_j(x_j^0) = \sup_{\text{stopping times } \tau} \mathbb{E}\left[\frac{\sum_{t'=0}^{\tau}\left(\frac{1}{1+r_f}\right)^{t'} \mathtt{Reward}_j\big(x_j(t),\omega(t)\big)}{\sum_{t'=0}^{\tau}\left(\frac{1}{1+r_f}\right)^{t'}}\right] ,$$

where

$$x_j(t'+1) = \mathtt{Dyn}_j\big(x_j(t'),\omega(t)\big) , \quad x_j(0) = x_j^0$$

The Gittins index proves optimal under specific settings

Assumptions:

- jobs dynamical model,
- criterion of the form

$$\mathbb{E}\left[\sum_{t=0}^{+\infty}(\frac{1}{1+r_f})^t \mathtt{Reward}_j\big(x_{j(t)}(t),\omega(t)\big)\right]$$

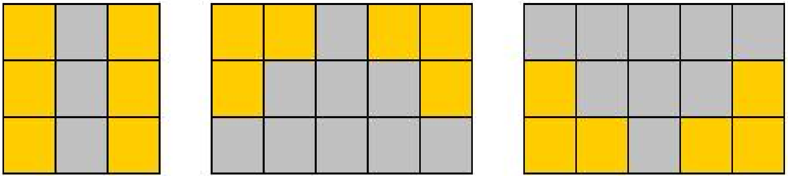
Conclusion: a Gittins index strategy is optimal

The open-pit mine scheduling problems shares characteristics with a job problem

- Jobs = columns
- Local state = depth of highest block
- Criterion = NPV
- Problem: slope restrictions, loss of the independence between jobs

Various index formulas can be imagined in mining

$$\mathtt{Index}_c(x)$$



An index lower bound can be obtained for OPMOS

- Define an index: Gittins index, top block value, etc.
- The index with slope constraints strategy selects the column with the highest index only among admissible columns

$$NPV(\text{ index with slope constraints }) \leq \max_{\text{all strategies}} NPV$$

A Gittins index upper bound can be obtained for OPMOS

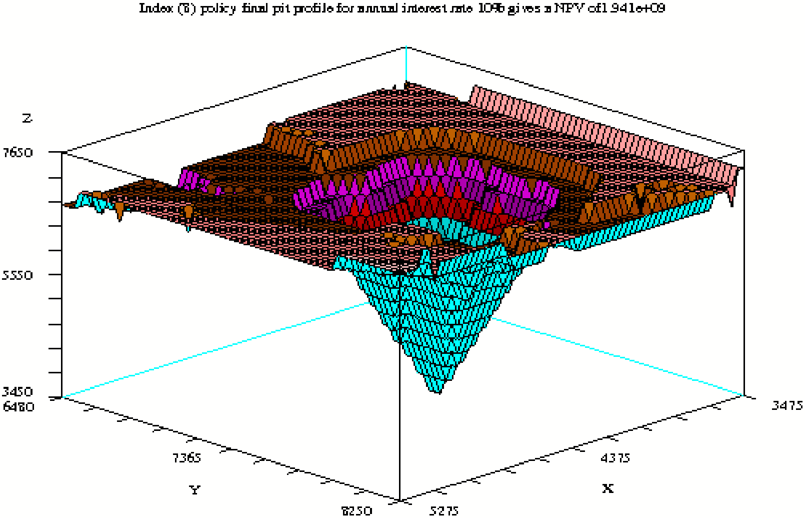
- Consider the optimal scheduling problem without slope constraints
- Since the space of strategies is enlarged, the optimal value for the scheduling problem without slope constraints is an upper bound for the original optimal scheduling problem with slope constraints
- In this case, we have a jobs dynamical model, with a criterion of the discounted form: therefore, the Gittins index strategy is optimal

$$\max_{\text{all strategies}} NPV \leq NPV(\text{ Gittins index without slope constraints })$$

Numerical results

Mine	Blocs	Index	TopoSort	LP	UBIndex
PM1	400	358.42 0.43s	432.14 307s	439.30 307s	521.53 0.08s
PM2	400	319.74 0.27s	438.60 375s	439.84 375s	674.24 0.07s
PM3	400	139.50 0.27s	149.06 362s	198.84 362s	318.72 0.07s
MM1	1125	744.78 1.12s	820.05 3627s	894.15 3627s	1138.23 0.14s
MM2	1125	346.72 1.14s	315.70 4257s	513.38 4257s	1108.86 0.14s
MarvSB	2800	157.8 2.48s	N.A.	N.A.	206.7 0.32s
Marvin	53000	392.9 1036.0s	N.A.	N.A.	488.5 15.1s

Ultimate profile after extraction with index strategy



Outline of the presentation

When the block model is uncertain, a new problem arises

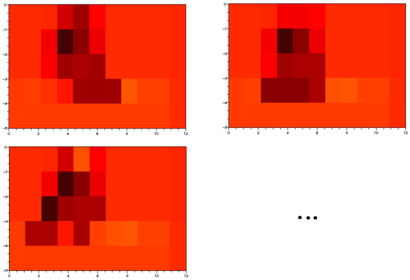
Uncertainty on

- Ore grade: geostatistics (Kriging)
- Prices: financials models
- Capacity constraints (failures)

open-pit mine optimal scheduling with uncertainty

Scenarii generation is a common practice

- In practice : generation of scenarii by conditional simulation
- A simulation technique developped by Journel (1983)
- A reference article by Ravenscroft (1992)



Sinces the nineties, scenarii-based strategies are designed

- Denby and Schofield (1995): deterministic optimization on each scenario, and combination by genetic algorithm
- Dimitrakopoulos and Ramazan (2004): "probability constraints"
- Dimitrakopoulos et al. (2007): maximum upside/minimum downside approach
- Boland et al. (2008): stochastic programming

Scenarii are collections of uncertain attributes

Collection of "uncertain" attributes at t:

$$\omega(t) := (\omega_b(t))_{b \in \mathbb{B}}$$

Attributes up to t:

$$\omega^t := (\omega(t_0), \ldots, \omega(t))$$

Scenario:

$$\omega(\cdot) := (\omega(t_0), \ldots, \omega(T))$$

Set of scenarii

$$\Omega \subset \mathbb{R}^{N \cdot (T - t_0) \cdot I}$$

Deterministic case when Ω is a singleton

Uncertain attributes are numerous in mining

Attributes $\omega_b(t) = \left(\omega_b^1(t), \cdots, \omega_b^l(t)\right) \in \mathbb{R}^l$

- either intrinsic attributes (which do indeed depend upon block b)
  - ore grade (does not depend upon time t)
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  - density, volume, pollutants...
- or external attributes
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A probabilistic formalism for offline information

Probability distribution on the set Ω of scenarii:

$$D(\omega_b(t), b \in \mathbb{B}, t = t_0, \ldots, T)$$

Example: the Gaussian case

- $(Ore(b))_{b \in \mathbb{B}}$ Gaussian vector of size N
  of mean vector $\mu = (\mathbb{E}[Ore(b)])_{b \in \mathbb{B}}$
  and covariance matrix $\Sigma = (Cov(Ore(b), Ore(b')))_{b,b' \in \mathbb{B}}$
- constant price $Price(t) = Price$ and cost $Cost(b,t) = Cost$
- no capacity failure
- then $w_b(t), b \in \mathbb{B}, t \in [t_0, \ldots, T]$ is
  Gaussian vector of size $N \cdot (T - t_0)$
  with mean vector and covariance matrix calculated with μ and Σ

Online information feeds decision

- Decision variable $u(t) = \mathcal{B}(t) \in \mathbb{B}\big(t, \omega(\cdot)\big) \subset \mathbb{B}$ corresponds to the blocks removed during period $t = t_0, \ldots, T$
- Via $\mathbb{B}\big(t, \omega(\cdot)\big)$, we can possibly capture slope and capacity constraints
- Past decisions

$$u^t = (u(t_0), \ldots, u(t)) \quad \text{and} \quad u(\cdot) = (u(t_0), \ldots, u(T))$$

- Information: $\mathcal{I}_t$ is a σ-algebra on the history space $\Omega \times \mathbb{U}^{T-t_0+1}$

Information patterns describe the interplay between information and decision

- Blind information pattern:

$$\mathcal{I}_t = \{\Omega, \varnothing\} \otimes \{\mathbb{U}^{T-t_0+1}, \varnothing\}$$

- Clairvoyant information pattern:

$$\mathcal{I}_t = \sigma(\{w_b(s), b \in \mathbb{B}, s = t_0, \dots, T\} \otimes \{\mathbb{U}^{T-t_0+1}, \varnothing\})$$

- Blocks that have been removed up to period t are

$$X(t, u^{t-1}) = \bigcup_{s=t_0}^{t} u(t) \subset \mathbb{B}$$

Cumulative information pattern is

$$\mathcal{I}_t = \sigma\{\omega_b(s), u(s) | s = t_0, \dots, t-1, b \in X(u^{s-1}, s)\}$$

Decision rules is the appropriate notion of solution

- A decision rule is a mapping

$$\texttt{rule} : \{t_0, \ldots, T\} \times \Omega \times \mathbb{U}^{T-t_0+1} \rightarrow \mathbb{U}$$

  such that $\texttt{rule}(t, \cdot)$ is measurable with respect to $\mathcal{I}_t$
- An admissible rule generates admissible decisions $u(t)$, which satisfy slopes and capacity constraints
  The set of admissible rules is denoted $\mathcal{D}^{ad}$

What is a policy?

Richard Bellman autobiography, Eye of the Hurricane

> *Again the intriguing thought: A solution is not merely a set of functions of time, or a set of numbers, but a rule telling the decisionmaker what to do; a policy.*
> *'Do thus-and-thus if you find yourself in this portion of state space with this amount of time left.'*

Controls may be open-loop or closed-loop

- Open-loop strategy: `rule` only depends on t
- Closed-loop strategy: `rule` depends on the information (may be computationaly out of reach because of the size of information)
- Open-loop with feedback optimization (OLFO)
  - Compute the *information state*, for instance the conditional law

$$D^t = D(\omega_b(t), b \in \mathbb{B}, t = t_0, \ldots, T | \text{knowledge at time } t)$$

  - Find a control sequence $(u(t), \ldots, u(T))$ that minimizes in open-loop the chosen criterion, using the information state
  - Apply the first decision $u(t)$ of the sequence

An objective function captures the economic goal

Objective or gain function $\texttt{Crit}(u(\cdot), \omega(\cdot))$ to be maximized

- Stochastic final pit

$$\texttt{Crit}(u(\cdot), \omega(\cdot)) = \sum_{t=t_0}^{T} \sum_{b \in u(t)} w_b(t)$$

- Net Present Value

$$\texttt{Crit}(u(\cdot), \omega(\cdot)) = \sum_{t=t_0}^{T} \rho(t) \sum_{b \in u(t)} w_b(t)$$

- Inclusion of processing costs, storage...

Under uncertainty, many criteria are possible and they reflect risk attitudes

Unlike deterministic problem,
there are many ways to agregate uncertainties

- Expected criterion: $\max_{\mathtt{rule} \in \mathcal{D}^{ad}} \mathbb{E}^{\mathbb{P}}[\mathtt{Crit}(u(\cdot), \omega(\cdot))]$
- Robust optimization: $\max_{\mathtt{rule} \in \mathcal{D}^{ad}} \min_{\omega(\cdot) \in \Omega} \mathtt{Crit}(u(\cdot), \omega(\cdot))$
- Multiprior approach (different probabilities $\mathbb{P}$ forming a set $\mathfrak{P}$): $\max_{\mathtt{rule} \in \mathcal{D}^{ad}} \min_{\mathbb{P} \in \mathfrak{P}} \mathbb{E}^{\mathbb{P}}[\mathtt{Crit}(u(\cdot), \omega(\cdot))]$
- Given two parameters $\alpha \in \mathbb{R}$ and $p \in [0, 1]$, we define the probability constraint $\mathbb{P}[\mathtt{Crit}(u(\cdot), \omega(\cdot)) \leq \alpha] \leq p$
  Expected optimization problem under risk constraint:
  $\max_{\mathtt{rule} \in \mathcal{D}^{ad}} \mathbb{E}^{\mathbb{P}}[\mathtt{Crit}(u(\cdot), w(\cdot))]$ s.t. $\mathbb{P}[\mathtt{Crit}(u(\cdot), \omega(\cdot)) \leq \alpha] \leq p$

Scenarii-based strategy design generates open-loop control

Schematic representation of scenarii-based strategy design:

$$\text{distribution } D \xrightarrow{\textit{sample}} (\omega_j(\cdot))_{j \in \mathcal{J}} \xrightarrow{\textit{compute}} u(\cdot)$$

This is an open-loop control

OLFO provides an adaptive suboptimal strategy

We propose an adaptive suboptimal strategy:

$$D^{t_0} \xrightarrow{sample} (\omega_j(\cdot))_{j\in\mathcal{J}^{t_0}} \xrightarrow{compute} (u^{t_0}(t_0),\dots,u^{t_0}(T)) \xrightarrow{select} u^{t_0}(t_0)$$

$$\hookrightarrow D^{t_0+1} \xrightarrow{sample} (\omega_j(\cdot))_{j\in\mathcal{J}^{t_0+1}} \xrightarrow{compute} (u^{t_0+1}(t_0+1),\dots,u^{t_0+1}(T)) \xrightarrow{select} u^{t_0+2}(t_0+$$

$$\dots$$

$$\hookrightarrow D^{T} \xrightarrow{sample} (\omega_j(\cdot))_{j\in\mathcal{J}^{T}} \xrightarrow{compute} (u^{T}(T)) \xrightarrow{select} u^{T}(T)$$

Index methods naturally extend to the stochastic case

Index methods naturally extend themself to the stochastic case with the local state replaced by the information state

$$\mathtt{Index}_u(D^t)$$

- Cumulative information pattern
  In the Gaussian case, use of the (discrete) Kalman filter

$$D^t \longrightarrow D^{t+1}$$

Conclusion

- Review of existing methods for the deterministic problem
- Development and implementation of index strategies
- Bibliographical review of literature on uncertainty
- Developement of a theoretical framework for the uncertain case