Contributions of Mathematical Optimization to Energy Management

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Outline of the presentation

Examples of recent optimization works with energy companies

Mathematical optimization for energy management

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Optimal dam management under "tourism" chance constraint

Jean-Christophe Alais, Pierre Carpentier, Michel De Lara. Multi-usage hydropower single dam management: chance-constrained optimization and stochastic viability. In Energy Systems, Volume 8, Issue 1, pp 7–30, February 2017

Optimal management of hydropower assets

setec

La collaboration a abouti au logiciel Hydroptim® qui utilise des algorithmes d'optimisation sous contraintes afin de déterminer la gestion optimale d'actifs hydroélectriques, à court, moyen ou long terme

Design of a pumping station for energy transfer

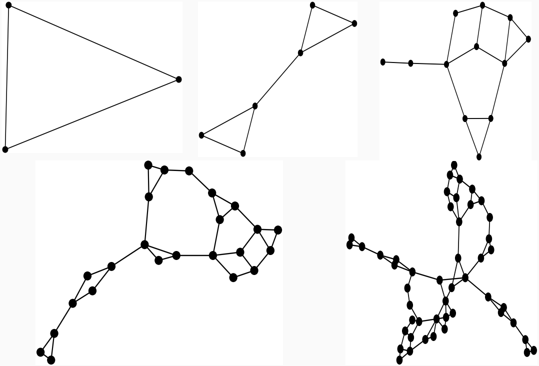
- R & D SunHydrO project
  (FUI, public and private funding)
- design of a pumping station
  for energy transfer (10–12 MW)
- optimal trading on reserve markets

Report on Optimization methods for smart grids

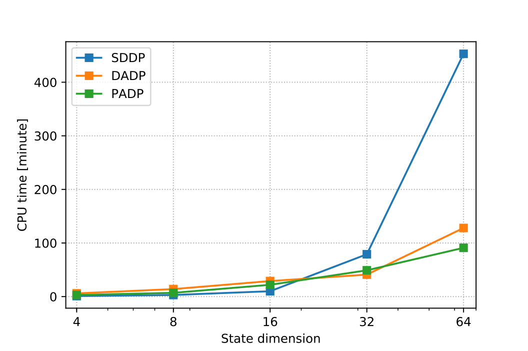
Michel De Lara, Pierre Carpentier, Jean-Philippe Chancelier, Vincent Leclère. Optimization Methods for the Smart Grid, report commissioned by Conseil Français de l'Energie, october 2014

Domestic district energy management

efficacity



# Nodes $\|\mathcal{N}\|$	3	6	12	24	48
dim $\mathbb{X}_t$	4	8	16	32	64
SDDP CPU time	1'	3'	10'	79'	453'
SDDP iterations	30	100	180	500	1500
DADP CPU time	6'	14'	29'	41'	128'
DADP iterations	27	34	30	19	29
PADP CPU time	3'	7'	22'	49'	91'
PADP iterations	11	12	20	19	20



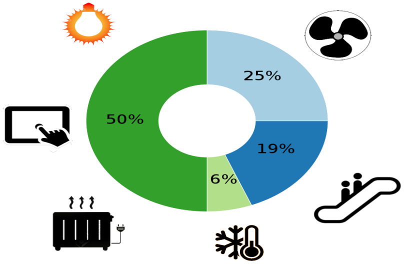
# Nodes $\|\mathcal{N}\|$	3	6	12	24	48
SDDP LB	225.2	455.9	889.7	1752.8	3310.3
DADP LB	213.7	447.3	896.7	1787.0	3396.4
PADP UB	252.1	528.5	1052.3	2100.7	4016.6

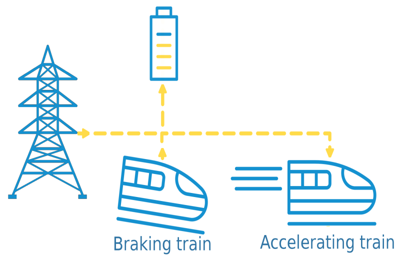
Table 1: Upper and lower bounds for the optimal expected total cost

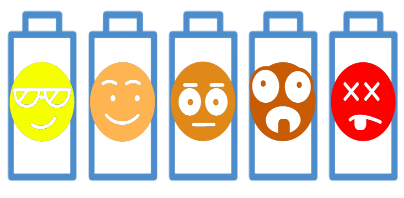
# Nodes $\|\mathcal{N}\|$	3	6	12	24	48
SDDP value	226 $\pm$ 0.6	471 $\pm$ 0.8	936 $\pm$ 1.1	1859 $\pm$ 1.6	3550 $\pm$ 2.3
DADP value	228 $\pm$ 0.6	464 $\pm$ 0.8	923 $\pm$ 1.2	1839 $\pm$ 1.6	3490 $\pm$ 2.3
(SDDP-DADP)/SDDP	- 0.8 %	+ 1.5 %	+1.4%	+1.1%	+1.7%
PADP value	229 $\pm$ 0.6	471 $\pm$ 0.8	931 $\pm$ 1.1	1856 $\pm$ 1.6	3508 $\pm$ 2.2
(SDDP-PADP)/SDDP	-1.3%	0.0%	+0.5%	+0.2%	+1.2%

Table 2: Simulation results for policies induced by SDDP, DADP and PADP

Subway station energy management



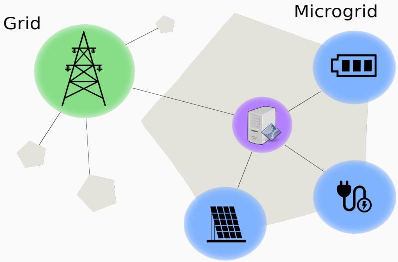


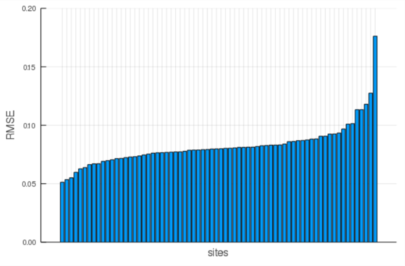


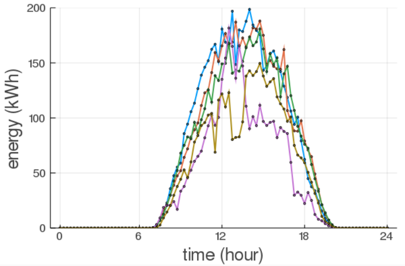
Tristan Rigaut, Pierre Carpentier, Jean-Philippe Chancelier, Michel De Lara, Julien Waeytens. Stochastic Optimization of Braking Energy Storage and Ventilation in a Subway Station. In IEEE Transactions on Power Systems, Volume 34, Issue 2, pp 1256–1263, March 2019

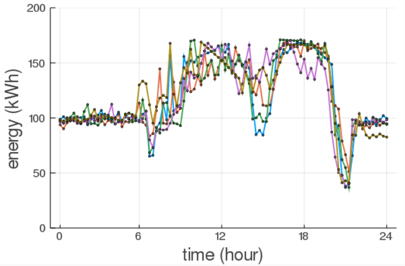
Domestic district energy management

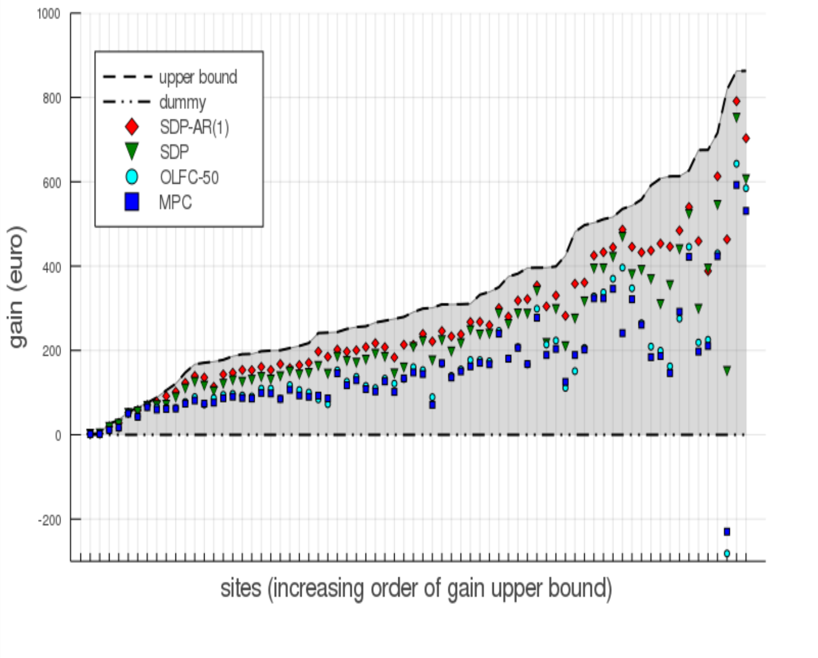
Domestic district energy management










1000
800
600
400
200
0
-200
gain (euro)
upper bound
dummy
SDP-AR(1)
SDP
OLFC-50
MPC
sites (increasing order of gain upper bound)

Domestic district energy management

	Performance score	Offline time (seconds)	Online time (seconds)
MPC	0.487	0.00	$9.82\ 10^{-4}$
OLFC-10	0.506	0.00	$1.14\ 10^{-2}$
OLFC-50	0.513	0.00	$8.62\ 10^{-2}$
OLFC-100	0.510	0.00	$1.87\ 10^{-1}$
SDP	0.691	2.67	$3.09\ 10^{-4}$
SDP-AR(1)	0.794	38.1	$4.44\ 10^{-4}$
SDP-AR(2)	0.795	468	$5.55\ 10^{-4}$
Upper bound	**1.0**	-	-

Mathematical optimization for energy management

Outline of the presentation

Examples of recent optimization works with energy companies

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Mathematical optimization for energy management

- Optimal economic dispatch (deterministic)
- Sensitivity analysis using convexity
- Optimal economic dispatch (deterministic, linear)
- Optimal economic dispatch (stochastic, linear)

Optimal economic dispatch

- Energy demand is $d \geq 0$
- Production technologies:
  - stock energies:
    fossil fuels, gas, coal, nuclear power, hydropower (dam, pump-hydro storage)
  - flow energies:
    hydropower (run-of-river), wind power (onshore, offshore), solar photovoltaics
- Suppose that each technology $j = 1, \ldots, N$
  - has capacity $\overline{q}^j > 0$
  - costs $\mathcal{C}^j(q^j)$ to produce the quantity $q^j \in [0, \overline{q}^j]$
- What energy mix $q = (q^1, \ldots, q^N)$ meets the demand at least cost?

Optimal mix

An optimal mix is a vector $q^* = (q^{*1}, \ldots, q^{*N})$ that solves the minimization problem

$$\min_{(q^1,\ldots,q^N)\in\mathbb{R}_+^N} \sum_{j=1}^{N} \mathcal{C}^j(q^j)$$

$$q^j \leq \overline{q}^j \ , \ \ j = 1, \ldots, N$$

$$\sum_{j=1}^{N} q^j \geq d$$

The value function is the optimal cost as a function of data

- The (optimal) value function j is defined by

$$j(\overline{q}^1, \ldots, \overline{q}^N, d) = \inf_{(q^1, \ldots, q^N) \in \mathbb{R}_+^N} \sum_{j=1}^{N} \mathcal{C}^j(q^j)$$

$$q^j \leq \overline{q}^j \ , \ \ j = 1, \ldots, N$$

$$\sum_{j=1}^{N} q^j \geq d$$

  for all $(\overline{q}^1, \ldots, \overline{q}^N, d) \in \mathbb{R}_+^{N+1}$

- The optimal cost decreases when the capacities $(\overline{q}^1, \ldots, \overline{q}^N)$ increase, and increases when the demand d increases, but can we assess the marginal variations?

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Convexity of cost functions induces convexity of the value function

- If the cost functions $\mathcal{C}^j : \mathbb{R}_+ \to \mathbb{R}$ are convex, for $j = 1, \ldots, N$ (that is, increasing marginal costs)
- then the value function j is convex

$$j(\overline{q}^1, \ldots, \overline{q}^N, d) = \inf_{(q^1, \ldots, q^N) \in \mathbb{R}_+^N} \sum_{j=1}^{N} \mathcal{C}^j(q^j)$$

$$q^j \leq \overline{q}^j \ , \ \ j = 1, \ldots, N$$

$$\sum_{j=1}^{N} q^j \geq d$$

- and convex analysis provides duality tools for a sensitivity analysis

Perturbation analysis

- Quantities $q = (q^{1,} \dots, q^N) \in \mathbb{R}^N$
- Perturbations $\varepsilon = (\varepsilon^1, \dots, \varepsilon^N, \varepsilon^{N+1}) \in \mathbb{R}^{N+1}$, as there are $N+1$ scalar constraints
- Multipliers $\lambda = (\lambda^1, \dots, \lambda^N, \lambda^{N+1}) \in \mathbb{R}^{N+1}$ of the perturbations

We introduce the perturbation scheme

$$H(q, \varepsilon) = \sum_{j=1}^{N} \mathcal{C}^j(q^j) + \sum_{j=1}^{N} \delta_{0 \leq q^j \leq \overline{q}^j + \varepsilon^j} + \delta_{\sum_{j=1}^{N} q^j \geq d + \varepsilon^{N+1}}$$

from which we calculate the Lagrangian

Calculating the Lagrangian

$$
\begin{aligned}
\mathcal{L}(q,\lambda) &= \inf_{\varepsilon \in \mathbb{R}^{N+1}} \Big(H(q,\varepsilon) + \sum_{j=1}^{N+1} \lambda^j \varepsilon^j \Big) \\
&= \sum_{j=1}^{N} \mathcal{C}^j(q^j) + \sum_{j=1}^{N} \inf_{\varepsilon^j \in \mathbb{R}} \Big(\delta_{0 \le q^j \le \overline{q}^j + \varepsilon^j} + \lambda^j \varepsilon^j \Big) \\
&+ \inf_{\varepsilon^{N+1} \in \mathbb{R}} \Big(\delta_{\sum_{j=1}^{N} q^j \ge d + \varepsilon^{N+1}} + \lambda^{N+1} \varepsilon^{N+1} \Big) \\
&= \begin{cases} -\infty & \text{if } \lambda \notin \mathbb{R}_+^N \times \mathbb{R}_- \\ \sum_{j=1}^{N} \mathcal{C}^j(q^j) + \sum_{j=1}^{N} \lambda^j (q^j - \overline{q}^j) + \delta_{0 \le q^j \le \overline{q}^j} & \\ + \lambda^{N+1}(d - \sum_{j=1}^{N} q^j) + \delta_{\sum_{j=1}^{N} q^j \ge d} & \text{if } \lambda \in \mathbb{R}_+^N \times \mathbb{R}_- \end{cases}
\end{aligned}
$$

The Lagrangian is, for $\lambda \in \mathbb{R}_+^N \times \mathbb{R}_-$, $0 \le q^j \le \overline{q}^j$, $\sum_{j=1}^{N} q^j \ge d$, given by

$$
\mathcal{L}(q,\lambda) = \sum_{j=1}^{N} \Big(\mathcal{C}^j(q^j) + \lambda^j (q^j - \overline{q}^j) - \lambda^{N+1} q^j \Big) + \lambda^{N+1} d
$$

and $-\infty$ elsewhere

Karush-Kuhn-Tucker (KKT) condition

The couple $(q^*, \lambda^*) \in \mathbb{R}_+^N \times \mathbb{R}^{N+1}$ satisfies the KKT condition if (q^*, λ^*) is a saddle point of the Lagrangian $\mathcal{L}$, that is,

$$q^* \in \arg\min_{q \in \mathbb{R}_+^N} \mathcal{L}(q, \lambda^*)$$

$$\lambda^* \in \arg\max_{\lambda \in \mathbb{R}^{N+1}} \mathcal{L}(q^*, \lambda)$$

- Provide sufficient conditions depending on the data $\overline{q}^1, \ldots, \overline{q}^N, d$ under which such a saddle point (q^*, λ^*) exists
- Show that, once the multipliers λ^* are fixed, the quantities q^* can be calculated in a decentralized fashion

Back to the Lagrangian

The Lagrangian is, for $\lambda \in \mathbb{R}_+^N \times \mathbb{R}_-$, $0 \leq q^j \leq \overline{q}^j$, $\sum_{j=1}^N q^j \geq d$, given by

$$\mathcal{L}(q, \lambda) = \sum_{j=1}^{N} \left(\mathcal{C}^j(q^j) + \lambda^j (q^j - \overline{q}^j) - \lambda^{N+1} q^j \right) + \lambda^{N+1} d$$

so that

$$q^* \in \underset{q \in \mathbb{R}_+^N}{\arg\min}\, \mathcal{L}(q, \lambda^*) \iff q^{*j} \in \underset{q \in \mathbb{R}_+}{\arg\min} \left(\mathcal{C}^j(q^j) + \lambda^j (q^j - \overline{q}^j) - \lambda^{N+1} q^j \right)$$

$$j = 1, \dots, N$$

$$\lambda^* \in \underset{\lambda \in \mathbb{R}^{N+1}}{\arg\max}\, \mathcal{L}(q^*, \lambda) \iff \begin{cases} \lambda^j (q^j - \overline{q}^j) = 0 & j = 1, \dots, N \\ \lambda^{N+1} (d - \sum_{j=1}^N q^j) = 0 & \end{cases}$$

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The dual problem is a linear program

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