Precautionary Effect and Variations of the Value of Information

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Outline of the presentation

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1. Problem statement: the precautionary effect
2. Second-period value of the information monotonicity
   - Second-period value of the information
   - Jones and Ostroy monotonicity result
   - Epstein functional
   - When is the difference of optimal payoffs convex in the prior?
3. Utility functions ensuring the precautionary effect
   - First-order condition characterization
   - Additive separable preferences
   - Risk neutral preferences
   - Risk averse preferences
4. Conclusion

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Sketch

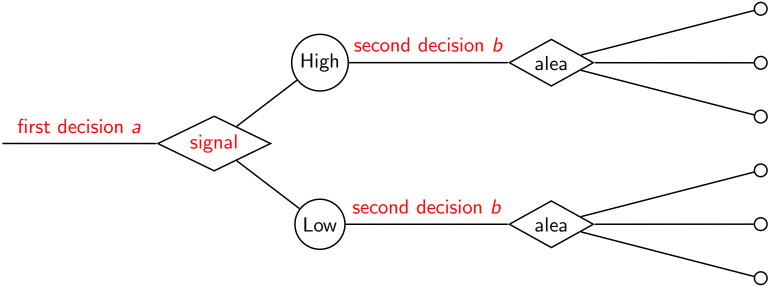

Figure: Decision with learning; agent takes decision a; a signal is revealed; agent takes decision b accordingly.

Global warming illustration

[Ulph and Ulph, 1997]

- a 2010 pollution emissions
- b 2030 pollution emissions
- random damages $C(a+b)x$

$$U(a,b,x) = \underbrace{u(a)+v(b)}_{benefits} - \underbrace{C(a+b)x}_{damage\ \ costs} \ .$$

Act vigorously now?
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Formal model

1. The initial decision a is a scalar belonging to an interval: $a \in \mathbb{I} \subset \mathbb{R}$.
2. The following and final decision b belongs to a set which may depend on a: $b \in \mathbb{B}(a) \subset \mathbb{B}$. This may materialize irreversibility due to the initial decision.
3. Uncertainty is represented by states of nature $\omega \in \Omega$ with prior $\mathbb{P}$, and by a random variable $X : \Omega \to \mathbb{X}$.
4. Partial information on X is provided by means of a signal (random variable) $Y : \Omega \to \mathbb{Y}$. Information allows for learning.
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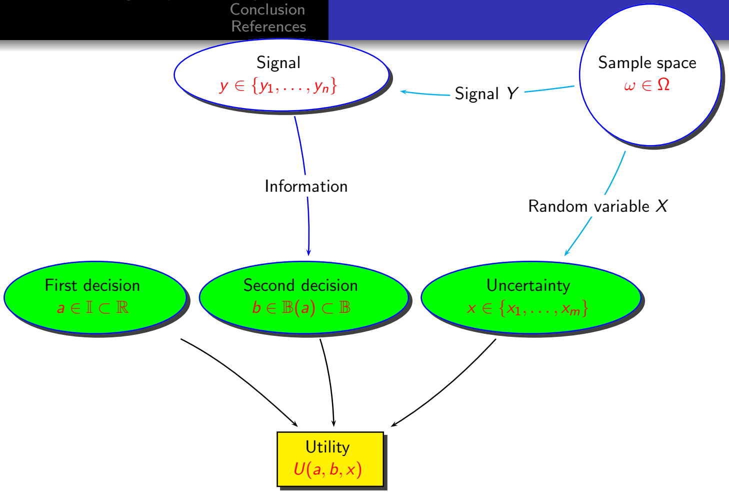
Signal
$y \in \{y_1, \ldots, y_n\}$
Signal Y
Sample space
$\omega \in \Omega$
Information
Random variable X
First decision
$a \in \mathbb{I} \subset \mathbb{R}$
Second decision
$b \in \mathbb{B}(a) \subset \mathbb{B}$
Uncertainty
$x \in \{x_1, \ldots, x_m\}$
Utility
$U(a, b, x)$

Precautionary effect

- The Y-informed expected utility maximizer solves

$$\max_a \mathbb{E}\Big[\max_{b \in \mathbb{B}(a)} \mathbb{E}[U(a,b,X) \mid Y]\Big] ,$$

with deterministic initial optimal solution $\bar{a}^Y$.

- The Y'-informed expected utility maximizer solves

$$\max_a \mathbb{E}\Big[\max_{b \in \mathbb{B}(a)} \mathbb{E}[U(a,b,X) \mid Y']\Big] .$$

The precautionary effect is said to hold whenever the optimal initial decision is lower with more information:

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Expected utility maximizer program

The evaluation of expected utility right after the first decision a has been taken is conditional on the signal Y and defined as follows:

$$\mathbb{V}^Y(a) := \mathbb{E}\Big[\max_{b \in \mathbb{B}(a)} \mathbb{E}[U\big(a, b, X\big) \mid Y]\Big] .$$

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Second-period value of the information monotonicity

Proposition ([Jones and Ostroy, 1984], [De Lara and Gilotte, 2009])

Assume that the programs $\max_a \mathbb{V}^{Y}(a)$ *and* $\max_a \mathbb{V}^{Y'}(a)$ *have unique optimal solutions* $\bar{a}^{Y}$ *and* $\bar{a}^{Y'}$. *Whenever the second-period value of the information is a decreasing function of the initial decision, namely*

$$a \mapsto \mathbb{V}^{Y}(a) - \mathbb{V}^{Y'}(a) \text{ is decreasing,}$$

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Epstein functional

- The random variable X is supposed to take its value in $\{x_1, \ldots, x_m\}$.
- Any prior ρ on $\{x_1, \ldots, x_m\}$ is identified with an element of the simplex $\mathcal{S}^{m-1}$.

Following [Epstein, 1980], let us define what we shall coin the Epstein functional by the maximal expected utility:

$$J(a, \rho) := \sup_{b \in \mathbb{B}(a)} \mathbb{E}_\rho\big[U(a, b, \cdot)\big] = \sup_{b \in \mathbb{B}(a)} \int_{\mathbb{X}} U(a, b, x)\, d\rho(x) ,$$

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Assume that

1. *for any* $a_1 \geq a_0$, $\rho \in \mathcal{S}^{m-1} \mapsto J(a_1, \rho) - J(a_0, \rho)$ *is convex (resp. concave),*
2. Y *is more informative than* Y' $(\sigma(Y) \supset \sigma(Y'))$.

Then the value of substituting Y *for* Y',
$\Delta\mathbb{V}^{YY'}(a) := \mathbb{V}^{Y}(a) - \mathbb{V}^{Y'}(a)$ *is increasing with* a
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A geometric property

Let set of maximal possible random rewards when the initial decision is a be defined by $\Lambda^-(a) :=$

$$\{f : \mathbb{X} \to \mathbb{R} \mid \exists b \in \mathbb{B}(a) \text{ such that } f(x) \leq U(a,b,x) \ , \quad \forall x \in \mathbb{X}\} \ .$$

Proposition

Let $a_1 > a_0$. If there exists a subset K of functions defined on $\mathbb{X}$ such that[a]

$$\Lambda^-(a_1) = \Lambda^-(a_0) + K \ ,$$

then $\rho \in \mathcal{S}^{m-1} \mapsto J(a_1,\rho) - J(a_0,\rho)$ is convex.

[a] For any subsets Λ_1 and Λ_2, $\Lambda_1 + \Lambda_2 = \{x_1 + x_2 \ , \ x_1 \in \Lambda_1 \text{ and } x_2 \in \Lambda_2\}$ is their so called direct sum, or *Minkowsky* sum.

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First-order condition characterization

Let $a_1 > a_0$. To any mapping $\phi : \mathbb{B}(a_0) \to \mathbb{B}(a_1)$ associate the following set of second decision minimizers

$$\mathbb{B}_\phi(a_1, a_0, x) := \arg\min_{b \in \mathbb{B}(a_0)} \Big(U(a_1, \phi(b), x) - U(a_0, b, x) \Big) \qquad (1)$$

and

$$\mathbb{B}_\phi(a_1, a_0) := \bigcap_{x \in \mathbb{X}} \mathbb{B}_\phi(a_0, x) \ .$$

When this latter set is not empty, there exists at least one second decision minimizer $b \in \mathbb{B}(a_0)$ of $U(a_1, \phi(b), x) - U(a_0, b, x)$ independent of the realization x of the random variable X.

First-order condition characterization

Let $a_1 > a_0$. To any mapping $\phi : \mathbb{B}(a_0) \to \mathbb{B}(a_1)$ associate the following set of second decision minimizers

$$\mathbb{B}_\phi(a_1, a_0, x) := \arg\min_{b \in \mathbb{B}(a_0)} \Big(U(a_1, \phi(b), x) - U(a_0, b, x) \Big) \qquad (1)$$

and

$$\mathbb{B}_\phi(a_1, a_0) := \bigcap_{x \in \mathbb{X}} \mathbb{B}_\phi(a_0, x) \, .$$

When this latter set is not empty, there exists at least one second decision minimizer $b \in \mathbb{B}(a_0)$ of $U(a_1, \phi(b), x) - U(a_0, b, x)$ independent of the realization x of the random variable X.

Proposition

Assume that

1. *the set of functions between second decision sets*

$$\Phi = \{\phi : \mathbb{B}(a_0) \to \mathbb{B}(a_1) \mid \mathbb{B}_\phi(a_0) \neq \emptyset\}$$

   is not empty,

2. *to any second decision $b_1 \in \mathbb{B}(a_1)$ can be associated at least one mapping $\phi \in \Phi$ and one second decision $b_0 \in \mathbb{B}_\phi(a_0)$ such that $b_1 = \phi(b_0)$.*

Then there exists a subset K of functions defined on $\mathbb{X}$ such that $\Lambda^-(a_1) = \Lambda^-(a_0) + K$.

Hence, the assumption of Proposition 3 is satisfied.

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Corollary

Assume that the second decision variable b belongs to $\mathbb{B} = \mathbb{R}^n$ and that the minimizers in (1) are characterized by first-order optimality condition.

Suppose that, to any vector $b_1 \in \mathbb{B}(a_1)$ can be associated at least one vector $b_0 \in \mathbb{B}(a_0)$ and one square matrix $M \in \mathbb{R}^{n \times n}$ such that

$$M \frac{\partial U}{\partial b}(a_1, b_1, x) - \frac{\partial U}{\partial b}(a_0, b_0, x) = 0 , \quad \forall x \in \mathbb{X} . \qquad (2)$$

If, in addition, we have $b_1 + M(b - b_0) \in \mathbb{B}(a_1)$ for all b in a neighbourhood of b_0 in $\mathbb{B}(a_0)$,[a]
then the assumptions of Proposition 4 are satisfied.

[a] This condition is meaningless if b_1 belongs to the interior of $\mathbb{B}(a_0)$. Hence this condition has to be verified only when an irreversibility constraint bites.

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Assume that the second decision variable b belongs to $\mathbb{B} = \mathbb{R}^n$ and that the minimizers in (1) are characterized by first-order optimality condition.
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[Salanié and Treich, 2007]

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If the utility U admits an invariant, then for any a and b, there exists a vector $d(a,b)$ and a matrix $M(a,b)$ such that

$$\frac{\partial^2 U}{\partial a \partial b}(a,b,x) + \frac{\partial^2 U}{\partial b^2}(a,b,x) d(a,b) = M(a,b) \frac{\partial U}{\partial b}(a,b,x) , \quad \forall x \in \mathbb{X} .$$

To be compared to: for any a_1, a_0, b_1, there exist a vector $\psi(a_1, a_0, b_1)$ and a matrix $M(a_1, a_0, b_1)$ such that

$$M(a_1, a_0, b_1) \frac{\partial U}{\partial b}(a_1, b_1, x) - \frac{\partial U}{\partial b}(a_0, \psi(a_1, a_0, b_1), x) = 0 , \quad \forall x \in \mathbb{X} ,$$

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Additive separable preferences

$$U(a, b, x) = u(a, x) + v(b, x)$$

[Arrow and Fisher, 1974]
[Henry, 1974]
[Epstein, 1980], highways and farms, the timing of orders for capital
[Freixas and Laffont, 1984]
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A solution (M, b_0) to

$$M\frac{\partial v}{\partial b}(b_1,x) = \frac{\partial v}{\partial b}(b_0,x) \ , \quad \forall x \in \mathbb{X} \ ,$$

is given by

$$M = 1 \text{ and } b_0 = b_1 \ .$$

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More an irreversibility than a learning problem

However, the irreversibility conditions that

- $b_1 \in \mathbb{B}(a_1) \Rightarrow b_0 \in \mathbb{B}(a_0)$
- $b_1 + \langle M, b - b_0 \rangle \in \mathbb{B}(a_1)$ for all b in a neighborhood of $b_0 \in \mathbb{B}(a_0)$

may prevent the precautionary effect to hold true.

Risk neutrality

[Epstein, 1980], a firm's demand for capital
$a = K \geq 0$, $b = L \geq 0$

$$U(a, b, x) = -ca + F(a, b)x - wb \; .$$

[Ulph and Ulph, 1997], global warming
a, b pollution emissions

$$U(a, b, x) = u(a) + v(b) - M(a + b)x \; .$$

Risk neutrality

$$U(a,b,x) = u(a,b) + v(a,b)x \ .$$

A solution $(M, b_0) \in \mathbb{R}^n \times \mathbb{R}^n$ to

$$M\frac{\partial v}{\partial b}(b_1, x) = \frac{\partial v}{\partial b}(b_0, x) \ , \quad \forall x \in \mathbb{X} \ ,$$

is given by

$$\begin{cases} M\frac{\partial u}{\partial b}(a_1, b_1) &= \frac{\partial u}{\partial b}(a_0, b_0) \\ M\frac{\partial v}{\partial b}(a_1, b_1) &= \frac{\partial v}{\partial b}(a_0, b_0) \ . \end{cases}$$

This is a system of $2n$ equations with $2n$ unknown (M, b_0).

Risk neutrality

$$U(a,b,x) = u(a,b) + v(a,b)x = u(a,b) + \sum_{i=1}^{p} v_i(a,b)x_i \ .$$

A solution $(M, b_0) \in \mathbb{R}^n \times \mathbb{R}^n$ is given by

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This is a system of $n + np$ equations with $n + n^2$ unknown (M, b_0).
When the dimension p of the noise is less than the dimension n of the second decision variable, the precautionary effect is possible.

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[Epstein, 1980], a firm's demand for capital

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A solution (M, b_0) to

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is given by $M = 1$ and

$$\frac{\partial F}{\partial b}(a_1, b_1) = \frac{\partial F}{\partial b}(a_0, b_0) \; .$$

A solution b_0 exists as soon as $b \mapsto \frac{\partial F}{\partial b}(a_0, b)$ can be inverted. The condition that $b_0 \in \mathbb{B}(a_0)$ depends on how $\frac{\partial F}{\partial b}(a, b)$ varies with a and b.

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[Ulph and Ulph, 1997], global warming

a, b pollution emissions

$$U(a, b, x) = u(a) + v(b) - C(a + b)x \ .$$

A solution (M, b_0) to

$$Mv'(b_1) - MC'(a_1 + b_1)x = v'(b_0) - C'(a_0 + b_0)x \ , \quad \forall x \in \mathbb{X} \ ,$$

is given by $M = v'(b_0)/v'(b_1)$ and

$$\frac{C'(a_0 + b_0)}{v'(b_0)} = \frac{C'(a_1 + b_1)}{v'(b_1)} \ .$$

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[Epstein, 1980], a consumption-savings problem

a, b savings with $\mathbb{B}(a) = [0, ra]$ and

$$U(a, b, x) = u_1(w - a) + \beta u_2(ra - b) + \beta^2 u_3(bx) .$$

A solution (M, b_0) to

$$M\beta x u_3'(b_1 x) - \beta x u_3'(b_0 x) = M u_2'(ra_1 - b_1) - u_2'(ra_0 - b_0) , \quad \forall x \in \mathbb{X} ,$$

implies that there must exist constants α, γ and δ such that u_3' satisfies an equation of the form

$$x u_3'(\alpha x) = \gamma x u_3'(x) + \delta , \quad \forall x \in \mathbb{X} .$$

A candidate is $u_3'(x) = x^{-\gamma}$.

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[Gollier, Jullien, and Treich, 2000] global warming

$$U(a,b,x) = u(a) + v\big(b - x(a+b)\big) \; .$$

A solution (M, b_0) to (2) is given by

$$Mv'\big(b_1 - x(a_1+b_1)\big) = v'\big(b_0 - x(a_0+b_0)\big) \; , \quad \forall x \in \mathbb{X} \; ,$$

implies that there must exist constants α, β and M such that v' satisfies an equation of the form

$$v'(\alpha x + \beta) = Mv'(x) \; , \quad \forall x \in \mathbb{X} \; .$$

In this case, $b_0 = b_1 \frac{a_0}{a_1}$.

Notice that the utility $v(x) = \frac{\gamma}{1-\gamma}\left[\eta + \frac{x}{\gamma}\right]^{1-\gamma}$ satisfies

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[Eeckhoudt, Gollier, and Treich, 2005], eating a cake with unknown size

$$U(a,b,x) = u(a) + v(b) + w(x-a-b) \ .$$

A solution (M, b_0) to

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implies that there must exist constants β, κ and M such that w' satisfies an equation of the form

$$w'(x+\beta) = Mv'(x) + \kappa \ , \quad \forall x \in \mathbb{X} \ .$$

We find that $\beta + a_1 + b_1 = a_0 + b_0$ with the compatibility condition $Mv'(b_1) - v'(b_0) + \kappa = 0$.

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Conclusion

- Monotonicity of the second-period value of the information as a function of initial decision as a first key to the 'precautionary effect'. Monotonicity related to convexity of variations of the Epstein functional.
- Geometric characterization of when a difference of optimal payoffs is convex in the prior.
- Direct characterization on the primitives of the economic model (which is not the case for Epstein condition).
- First-order condition characterization allows to treat cases in the literature and to extend their validity conditions.
- Irreversibility constraints may prevent the 'precautionary effect' to hold true.

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