Subdifferentiability in Convex and Stochastic Optimization Applied to Renewable Power Systems

PhD thesis supervised by Michel De Lara

Adrien Le Franc

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École des Ponts ParisTech

efficacity

Schneider Electric

Outline of the presentation

1. **EMSx: a numerical benchmark for energy management systems**

2. **Parametric multistage stochastic optimization for day-ahead power scheduling**

3. **Perspectives for numerical methods in generalized convexity**

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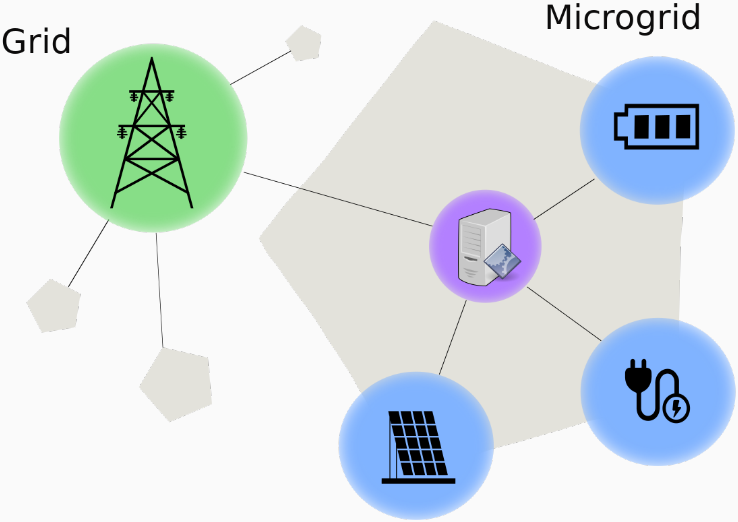
A PV-battery microgrid



Contribution of the section

- **Question**

How to evaluate an **Energy Management System** (EMS)
designed for operating a microgrid with
uncertain load and production
at **least expected cost**?

- **Our contribution**

We introduce EMSx, a **microgrid controller benchmark**
to compare (deterministic and stochastic) EMS techniques
on an **open** and **diversified** testbed

Outline of the section

1. **EMSx: a numerical benchmark for energy management systems**

   The EMSx dataset

   The EMSx mathematical framework: microgrid model

   The EMSx mathematical framework: controller assessment

   The EMSx software

   Numerical examples of controllers

Examples of daily scenarios from EMSx

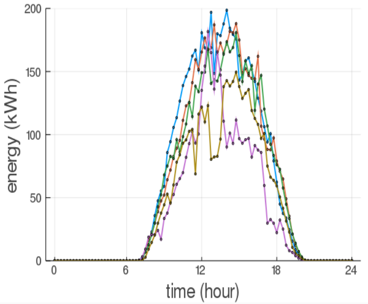


Figure 1: Examples of daily photovoltaic profiles

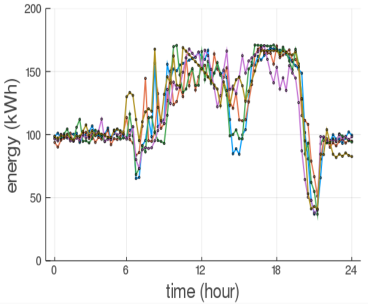


Figure 2: Examples of daily load profiles

Over 1 year of historical observations and forecasts collected by Schneider Electric on 70 industrial sites

Stochasticity of the net demand across sites

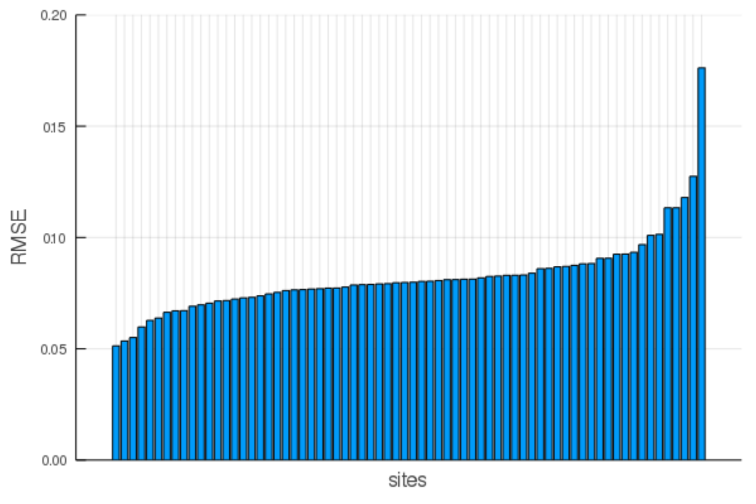


Figure 3: RMSE of the net demand forecasts for each of the 70 sites

Outline of the section

1. **EMSx: a numerical benchmark for energy management systems**

Time scale and variables

We make decisions at time steps $t \in \{0, 1, \ldots, T\}$
over one week ($\Delta_t = 15$ min , $T = 672$)

- $x_t \in [0, 1]$ state of charge of the battery
- $u_t \in [\underline{u}, \overline{u}]$ energy charged ($u_t \geq 0$)
  or discharged ($u_t \leq 0$) over $[t, t+1]$
- $w_{t+1} = (g_{t+1}, d_{t+1})$ generation and demand
  historical data over $[t, t+1]$
- $\hat{w}_{t,t+k} = (\hat{g}_{t,t+k}, \hat{d}_{t,t+k})$, $k \in \{1, \ldots, 96\}$
  generation and demand historical forecast at time t
  over $[t+k-1, t+k]$



Our microgrid management model

- state of charge ruled by the **dynamics**

$$x_{t+1} = f(x_t, u_t) = x_t + \frac{\rho_{\mathrm{c}}}{c} u_t^+ - \frac{1}{\rho_{\mathrm{d}} c} u_t^-$$

- controls restricted to the **admissibility set**

$$\mathcal{U}(x_t) = \{u_t \in \mathbb{R} \mid \underline{u} \leq u_t \leq \overline{u} \text{ and } 0 \leq f(x_t, u_t) \leq 1\}$$

- energy exchanges induce a **cost**

$$L_t(u_t, w_{t+1}) = p_t^{\mathrm{buy}} \cdot (d_{t+1} - g_{t+1} + u_t)^+ - p_t^{\mathrm{sell}} \cdot (d_{t+1} - g_{t+1} + u_t)^-$$

Outline of the section

Online information for decision making

- A **partial chronicle** is a sequence $h = (h_0, \ldots, h_{T-1})$ of vectors where for $t \in \{0, \ldots, T-1\}$

$$h_t = \begin{pmatrix} w_t, w_{t-1}, \ldots, w_{t-95} \\ \hat{w}_{t,t+1}, \ldots, \hat{w}_{t,t+96} \end{pmatrix} \in \mathbb{H} = \mathbb{R}^{2\times 96} \times \mathbb{R}^{2\times 96}$$

- A **controller** is a sequence $\phi = (\phi_0, \ldots, \phi_{T-1})$ of mappings where for $t \in \{0, \ldots, T-1\}$

$$\begin{aligned} \phi_t \;:\; & [0,1] \times \mathbb{H} \to \mathbb{R} \\ & (x_t, h_t) \mapsto \phi_t(x_t, h_t) \in \mathcal{U}(x_t) \end{aligned}$$

Management cost of a controller

For each site $i \in I = \{1, \ldots, 70\}$

- A controller ϕ^i applied to a partial chronicle $h \in \mathbb{H}^T$ yields a **management cost**

$$J^i(\phi^i, h) = \sum_{t=0}^{T-1} L_t^i(u_t, w_{t+1})$$

$$x_0 = 0$$

$$x_{t+1} = f^i(x_t, u_t)$$

$$\underbrace{u_t = \phi_t^i(x_t, h_t)}_{\text{nonanticipativity}}$$

- If we allow anticipative decisions we obtain a lower bound for management costs $\underline{J}^i(h) \leq J^i(\phi^i, h)$

Gain of a controller ϕ^i on site $i \in I$

We have a pool of **simulation chronicles** $\mathscr{S}^i \subset \mathbb{H}^T$

- We measure **gains** w.r.t. a **dummy controller** $\phi^{\mathrm{d}} = 0$ (which does not use the battery)

$$G^i(\phi^i) = \frac{1}{|\mathscr{S}^i|} \sum_{h \in \mathscr{S}^i} J^i(\phi^{\mathrm{d}}, h) - J^i(\phi^i, h)$$

- We define the **anticipative gain**

$$\overline{G}^i = \frac{1}{|\mathscr{S}^i|} \sum_{h \in \mathscr{S}^i} J^i(\phi^{\mathrm{d}}, h) - \underline{J}^i(h)$$

- We obtain an upper bound for gains $\overline{G}^i \geq G^i(\phi^i)$

Normalized score of a control technique $\{\phi^i\}_{i \in I}$

- We define the **normalized score** of a control technique $\{\phi^i\}_{i \in I}$

$$\mathcal{G}\left(\{\phi^i\}_{i \in I}\right) = \frac{1}{|I|} \sum_{i \in I} \frac{G^i(\phi^i)}{\overline{G}^i}$$

- A performing control technique gives

$$\underbrace{0 \leq}_{\text{if better that } \phi^{\mathbf{d}}} \quad \mathcal{G}\left(\{\phi^i\}_{i \in I}\right) \quad \underbrace{\leq 1}_{\text{always}}$$

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   Numerical examples of controllers

A Julia package: EMSx.jl

```julia
struct Information
    t::Int64
    soc::Float64
    pv::Array{Float64,1}
    forecast_pv::Array{Float64,1}
    load::Array{Float64,1}
    forecast_load::Array{Float64,1}
    price::Price
    battery::Battery
    site_id::String
end
```

The EMSx.jl built-in type Information gathers all the information available to the controller to make a decision

A Julia package: EMSx.jl

```julia
using EMSx

mutable struct DummyController <: EMSx.AbstractController end

EMSx.compute_control(controller::DummyController,
    information::EMSx.Information) = 0.

const controller = DummyController()

EMSx.simulate_sites(controller,
    "home/xxx/path_to_save_folder",
    "home/xxx/path_to_price",
    "home/xxx/path_to_metadata",
    "home/xxx/path_to_simulation_data")
```

Example of the implementation and simulation of a dummy controller with the EMSx.jl package

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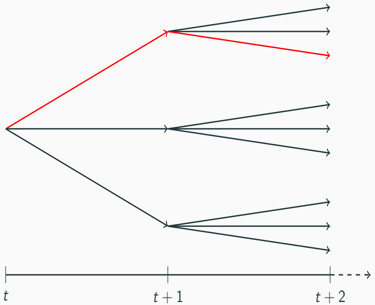
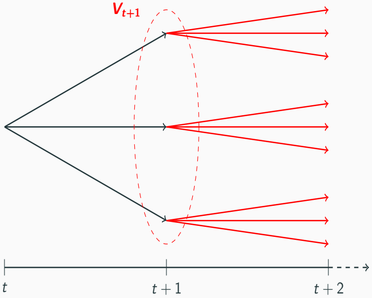
Standard controller design techniques



Look-ahead techniques:
MPC, OLFC

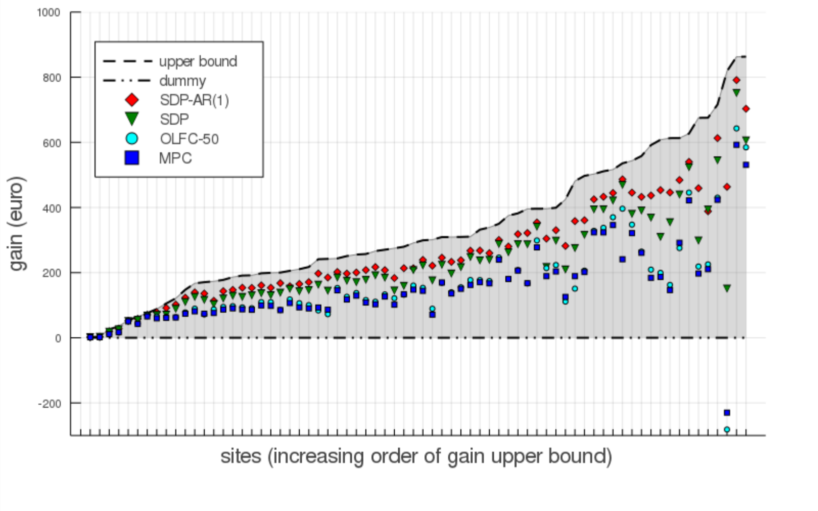


Cost-to-go techniques:
SDP, SDP-AR(k)

Normalized score per design technique

	Normalized score	**Offline time** (seconds)	**Online time** (seconds)
MPC	0.487	0.00	$9.82\ 10^{-4}$
OLFC-10	0.506	0.00	$1.14\ 10^{-2}$
OLFC-50	0.513	0.00	$8.62\ 10^{-2}$
OLFC-100	0.510	0.00	$1.87\ 10^{-1}$
SDP	0.691	2.67	$3.09\ 10^{-4}$
SDP-AR(1)	0.794	38.1	$4.44\ 10^{-4}$
SDP-AR(2)	0.795	468	$5.55\ 10^{-4}$
Upper bound	**1.0**	-	-

Detailed gain over the 70 sites



Outline of the presentation

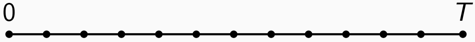
1. EMSx: a numerical benchmark for energy management systems

2. **Parametric multistage stochastic optimization for day-ahead power scheduling**

3. Perspectives for numerical methods in generalized convexity

A typical power scheduling example

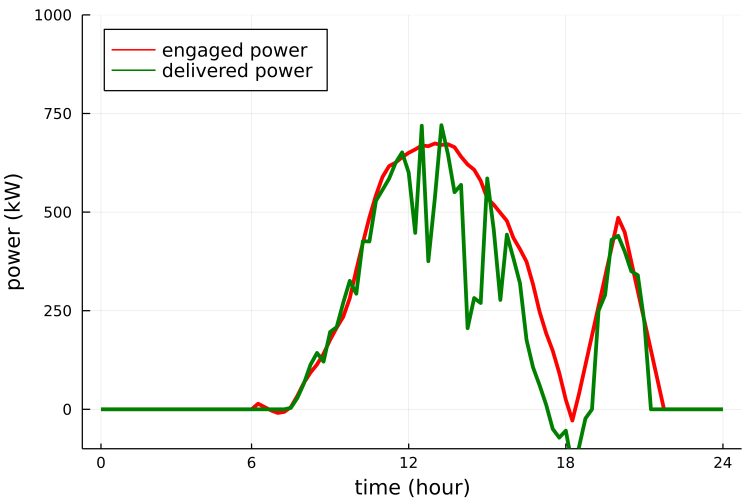
- We operate a solar plant over **one day** with discrete time steps $t \in \{0, 1, \ldots, T\}$



- For every operating day
  - In the **day-ahead** stage, we must supply a power production profile $p \in \mathbb{R}^T$
  - In the **intraday** stage, we manage the power plant and deliver a power profile $\tilde{p} \in \mathbb{R}^T$

Engaged power vs delivered power

The delivered power $\tilde{p}$ induces gains
and differences between p and $\tilde{p}$ induce penalties



Contribution of the section

- **Question**

  How can we optimize **day-ahead and intraday decisions** for operating a solar plant with **uncertain generated power** at **least expected cost**?

- **Our contribution**

  We introduce **parametric multistage stochastic optimization problems** for day-ahead power scheduling and study **differentiability properties** of **parametric value functions**

Outline of the section

2. Parametric multistage stochastic optimization for day-ahead power scheduling

Parametric multistage stochastic optimization

Differentiability of smooth parametric value functions

Lower smooth approximations

Lower polyhedral approximations

Numerical application

Our standard formulation

We consider a **multistage stochastic optimization problem** parametrized by $p = (p_0, \ldots, p_T) \in \mathbb{R}^{n_p \times (T+1)}$ formulated as

$$\Phi(p) = \inf_{\mathbf{U_0}, \ldots, \mathbf{U}_{T-1}} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}, p_t) + K(\mathbf{X}_T, p_T)\Big]$$

$$\mathbf{X}_0 = x_0$$

$$\mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}) \ , \ \ \forall t \in \{0, \ldots, T-1\}$$

$$\mathbf{U}_t \in \mathcal{U}_t(\mathbf{X}_t, p_t) \ , \ \ \forall t \in \{0, \ldots, T-1\}$$

$$\sigma(\mathbf{U}_t) \subseteq \sigma(\mathbf{W}_1, \ldots, \mathbf{W}_t) \ , \ \ \forall t \in \{0, \ldots, T-1\}$$

where $\mathbf{X}_t : \Omega \to \mathbb{R}^{n_x}$, $\mathbf{U}_t : \Omega \to \mathbb{R}^{n_u}$, $\mathbf{W}_t : \Omega \to \mathbb{R}^{n_w}$

Optimal solution via stochastic dynamic programming

Assumption (discrete white noise)

The sequence $\{\mathbf{W}_t\}_{t\in\{1,\dots,T\}}$ is ***stagewise independent****,*
and each noise variable $\mathbf{W}_t$ has a ***finite support***

For $t \in \{0, \dots, T\}$ and $x \in \mathbb{R}^{n_x}$
we define the **parametric value functions**

$$V_T(x, p) = K(x, p)$$

$$V_t(x, p) = \inf_{u \in \mathcal{U}_t(x, p_t)} \mathbb{E}\Big[L_t(x, u, \mathbf{W}_{t+1}, p_t) + V_{t+1}\big(f_t(x, u, \mathbf{W}_{t+1}), p\big)\Big]$$

Under the (discrete) white noise assumption $\quad \Phi(p) = V_0(x_0, p)$

Assumption

Assumption (convex multistage problem)

1. *the cost functions* $\{L_t\}_{t \in \{0,\dots,T-1\}}$ *are* ***jointly convex and lsc*** *w.r.t.* (x_t, u_t, p_t), *and are* ***proper***, *and the final cost* K *is* ***convex, proper, lsc***
2. *the dynamics* $\{f_t\}_{t \in \{0,\dots,T-1\}}$ *are* ***affine*** *w.r.t.* (x_t, u_t)
3. *the set-valued mappings* $\{\mathcal{U}_t\}_{t \in \{0,\dots,T-1\}}$ *are* ***closed, convex***, *have* ***nonempty domains*** *and* ***compact ranges***
4. *the problem satisfies a* ***relatively complete recourse-like assumption***

Convexity of parametric value functions

Proposition

Under the ***discrete white noise*** *assumption*
and the ***convex multistage problem*** *assumption,*
the parametric value functions $\{V_t\}_{t \in \{0,\dots,T\}}$ *are* ***convex, proper, lsc*** *w.r.t.* (x, p)

Outline of the section

2. Parametric multistage stochastic optimization for day-ahead power scheduling

Assumption

Assumption (smoothness)

1. *the cost functions* $\{L_t\}_{t \in \{0,\ldots,T-1\}}$ *and* K
   are ***differentiable w.r.t.*** p_t
2. *for all* $t \in \{0, \ldots, T-1\}$, *the set-valued mapping* $\mathcal{U}_t$
   takes the same set value for all $p_t \in \mathbb{R}^{n_p}$;
   in that case, we use the notation $\mathcal{U}_t(x)$ *instead of* $\mathcal{U}_t(x, p_t)$

Differentiable parametric value functions

Theorem (Le Franc [2021])

*Under the **discrete white noise** assumption,*
*the **convex multistage problem** assumption,*
*and the **smoothness** assumption,*
*the value functions $\{V_t\}_{t\in\{0,\dots,T\}}$ are **differentiable w.r.t. p**,*
and their gradients may be computed by backward induction, with

$$\nabla_p V_T(x,p) = \nabla_p K(x,p_T)\ , \ \ \forall (x,p) \in \mathrm{dom}(V_T)$$

and at stage $t \in \{0,\dots,T-1\}$, for $(x,p) \in \mathrm{dom}(V_t)$,
the solution set $\mathcal{U}_t^*(x,p_t)$ is nonempty, and for any $u^* \in \mathcal{U}_t^*(x,p_t)$,

$$\nabla_p V_t(x,p) = \mathbb{E}\Big[\nabla_p L_t(x,u^*,\mathbf{W}_{t+1},p_t) + \nabla_p V_{t+1}\big(f_t(x,u^*,\mathbf{W}_{t+1}),p\big)\Big]$$

Outline of the section

2. Parametric multistage stochastic optimization for day-ahead power scheduling

Parametric multistage stochastic optimization

Differentiability of smooth parametric value functions

Lower smooth approximations

Lower polyhedral approximations

Numerical application

Assumption

We consider a **parameter set** $\mathcal{P} \subseteq \mathbb{R}^{n_p \times (T+1)}$

Assumption (parameter set)

1. *the parameter set* $\mathcal{P}$ *is* ***nonempty, convex and compact***
2. *for all* $t \in \{0, \ldots, T-1\}$*,*
   the domain of the set-valued mapping $\mathcal{U}_t$
   is such that $\mathrm{dom}(\mathcal{U}_t) \subseteq \mathbb{R}^{n_x} \times \mathcal{P}_t$

where $\mathcal{P}_t = \mathrm{proj}_t(\mathcal{P}) \subseteq \mathbb{R}^{n_p}$, $\forall t \in \{0, \ldots, T\}$

Moreau envelopes of cost functions

Given values $(x, u, w) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_w}$
and a **regularization parameter** $\mu \in \mathbb{R}_+^*$, we introduce

$$L_t^\mu(x, u, w, p_t) = \inf_{p_t' \in \mathbb{R}^{n_p}} \Big(L_t(x, u, w, p_t') + \delta_{\mathrm{gr}(\mathcal{U}_t)}(x, u, p_t') + \delta_{\mathcal{P}_t}(p_t') + \frac{1}{2\mu} ||p_t - p_t'||_2^2 \Big) \ , \ \ \forall t \in \{0, \dots, T-1\} \ , \ \ \forall p_t \in \mathbb{R}^{n_p}$$

$$K^\mu(x, p_T) = \inf_{p_T' \in \mathbb{R}^{n_p}} \Big(K(x, p_T') + \delta_{\mathcal{P}_T}(p_T') + \frac{1}{2\mu} ||p_T - p_T'||_2^2 \Big) \ , \ \ \forall p_T \in \mathbb{R}^{n_p}$$

Lower smooth parametric value functions

$$\underset{\sim}{V}_T^\mu(x, p) = K^\mu(x, p_T) \ , \ \ \forall (x, p) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_p \times (T+1)}$$

$$\underset{\sim}{V}_t^\mu(x, p) = \inf_{u \in \mathrm{range}(\mathcal{U}_t)} \mathbb{E}\Big[L_t^\mu(x, u, \mathbf{W}_{t+1}, p_t) + \underset{\sim}{V}_{t+1}^\mu\big(f_t(x, u, \mathbf{W}_{t+1}), p\big)\Big]$$

$$\forall (x, p) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_p \times (T+1)} \ , \ \ \forall t \in \{0, \dots, T-1\}$$

Proposition (Le Franc [2021])

Under the ***discrete white noise*** *assumption,*
the ***convex multistage problem*** *assumption,*
and the ***parameter set*** *assumption,*
the lower smooth parametric value functions $\left\{\underset{\sim}{V}_t^\mu\right\}_{t \in \{0, \dots, T\}}$
are ***differentiable w.r.t.*** p*, and their gradients may be computed by backward induction*

Convergence result

$$\Phi^* = \inf_{p \in \mathcal{P}} \Phi(p)$$

Proposition (Le Franc [2021])

Under the same assumptions, if the sequence of regularization parameters $\{\mu_n\}_{n\in\mathbb{N}} \in (\mathbb{R}_+^*)^{\mathbb{N}}$ *is nonincreasing and such that* $\lim_{n\to+\infty} \mu_n = 0$*, then for any initial state* $x_0 \in \mathbb{R}^{n_x}$*, we have that*

$$\inf_{p\in\mathcal{P}} \underset{\sim}{V}_0^{\mu_n}(x_0, p) \leq \Phi^* \ , \ \ \forall n \in \mathbb{N} \ , \quad \textit{and} \quad \inf_{p\in\mathcal{P}} \underset{\sim}{V}_0^{\mu_n}(x_0, p) \xrightarrow[n\to+\infty]{} \Phi^*$$

Outline of the section

2. Parametric multistage stochastic optimization for day-ahead power scheduling

State extension

$$x_t^\sharp = \begin{pmatrix} x_t \\ p \end{pmatrix} \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_p \times (T+1)} , \ \forall t \in \{0, \dots, T\}$$

$$\Phi(p) = \inf_{\mathbf{U_0}, \dots, \mathbf{U}_{T-1}} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t^\sharp(\mathbf{X}_t^\sharp, \mathbf{U}_t, \mathbf{W}_{t+1}) + K^\sharp(\mathbf{X}_T^\sharp) \Big]$$

$$\mathbf{X}_0^\sharp = \begin{pmatrix} x_0 \\ p \end{pmatrix}$$

$$\mathbf{X}_{t+1}^\sharp = f_t^\sharp(\mathbf{X}_t^\sharp, \mathbf{U}_t, \mathbf{W}_{t+1}) , \ \forall t \in \{0, \dots T-1\}$$

$$\mathbf{U}_t \in \mathcal{U}_t^\sharp(\mathbf{X}_t^\sharp) , \ \forall t \in \{0, \dots, T-1\}$$

$$\sigma(\mathbf{U}_t) \subseteq \sigma(\mathbf{W}_1, \dots, \mathbf{W}_t) , \ \forall t \in \{0, \dots, T-1\}$$

Lower polyhedral value functions

- We introduce the state value functions

$$V_T^\sharp(x^\sharp) = K^\sharp(x^\sharp) \; , \;\; \forall x^\sharp \in \left(\mathbb{R}^{n_x} \times \mathbb{R}^{n_p \times (T+1)}\right)$$

$$V_t^\sharp(x^\sharp) = \inf_{u \in \mathcal{U}_t^\sharp(x^\sharp)} \mathbb{E}\Big[L_t^\sharp(x^\sharp, u, \mathbf{W}_{t+1}) + V_{t+1}^\sharp\big(f_t^\sharp(x^\sharp, u, \mathbf{W}_{t+1})\big)\Big]$$

$$\forall x^\sharp \in \left(\mathbb{R}^{n_x} \times \mathbb{R}^{n_p \times (T+1)}\right) \; , \;\; \forall t \in \{0, \dots, T-1\}$$

- We compute **polyhedral lower approximations** $\{\underline{V}_t^k\}_{t \in \{0,\dots,T\}}$ of $\{V_t^\sharp\}_{t \in \{0,\dots,T\}}$ by running $k \in \mathbb{N}$ forward-backward passes of the **SDDP algorithm**

- Since $\underline{V}_0^k$ is polyhedral, **linear programming** gives us a **subgradient** $(y, q) \in \partial \underline{V}_0^k\big((x_0, p)\big)$

Convergence result

Proposition (Le Franc [2021])

Let $(x_0, p) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_p \times (T+1)}$. If after $k \in \mathbb{N}^*$ forward-backward passes of the SDDP algorithm the approximation error of the value function $V_0^\sharp$ by the lower polyhedral approximation $\underline{V}_0^k$ is bounded by

$$V_0^\sharp\big((x_0, p)\big) - \underline{V}_0^k\big((x_0, p)\big) \leq \varepsilon$$

for some $\varepsilon \in \mathbb{R}_+$, then if we compute

$$\begin{cases} \phi = \underline{V}_0^k\big((x_0, p)\big) \\ (y, q) \in \partial \underline{V}_0^k\big((x_0, p)\big) \end{cases} \quad \text{we have that} \quad \begin{cases} |\Phi(p) - \phi| \leq \varepsilon \\ q \in \partial_\varepsilon \Phi(p) \end{cases}$$

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2. Parametric multistage stochastic optimization for day-ahead power scheduling

Parametric multistage stochastic optimization

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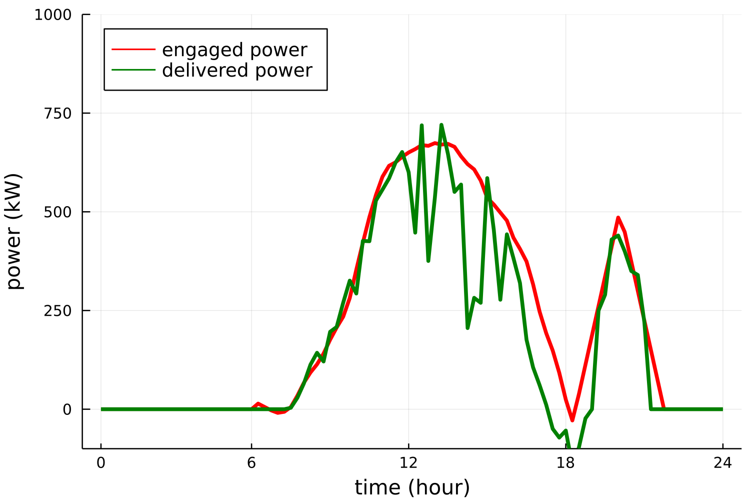
Lower polyhedral approximations

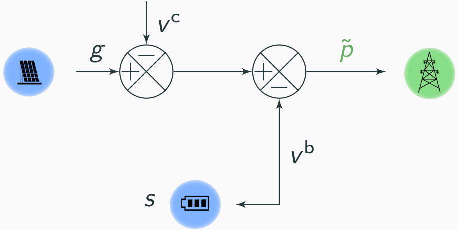
Numerical application

Back to our problem

The delivered power $\tilde{p}$ induces gains
and differences between p and $\tilde{p}$ induce penalties

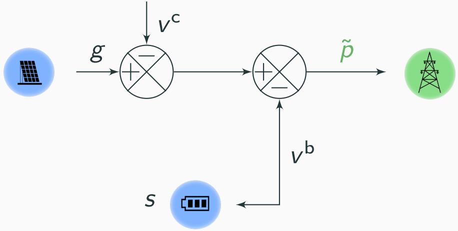


Schematic organization of the solar plant



- $g \in [0, \bar{p}]^T$ generated power (uncertainty)
- $v^{\mathrm{c}} \in [0, g]^T$ curtailed power (control)
- $s \in [0, \bar{s}]^{T+1}$ state of charge (state)
- $v^{\mathrm{b}} \in [\underline{v}, \overline{v}]^T$ battery power (control)
- $\tilde{p} = g - v_b - v_c$ **delivered power**

Schematic organization of the solar plant



- $g \in [0, \bar{p}]^T$ generated power (uncertainty) $\rightarrow$ AR(1) process
- $v^{\mathrm{c}} \in [0, g]^T$ curtailed power (control)
- $s \in [0, \bar{s}]^{T+1}$ state of charge (state)
- $v^{\mathrm{b}} \in [\underline{v}, \overline{v}]^T$ battery power (control)
- $\tilde{p} = g - v_b - v_c$ **delivered power**

Stochastic optimal control framework

- We introduce the the **state**, **control** and **noise** variables

$$x = \begin{pmatrix} s \\ g \end{pmatrix} \ , \ \ u = \begin{pmatrix} v^{\mathrm{b}} \\ v^{\mathrm{c}} \end{pmatrix} \ , \ \ w$$

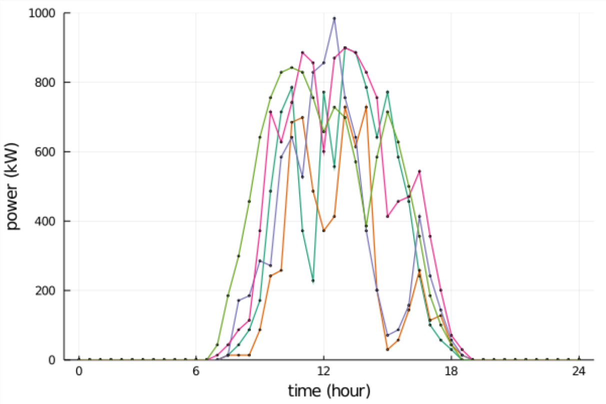
- The state process $\mathbf{X}$ is ruled by the dynamics

$$\mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}) = \begin{pmatrix} \mathbf{S}_t + \rho_c {\mathbf{V}_t^{\mathrm{b}}}^+ - \frac{1}{\rho_d} {\mathbf{V}_t^{\mathrm{b}}}^- \\ \alpha_t \mathbf{G}_t + \beta_t + \mathbf{W}_{t+1} \end{pmatrix} = \begin{pmatrix} \mathbf{S}_{t+1} \\ \mathbf{G}_{t+1} \end{pmatrix}$$

- The stage costs formulate as

$$L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}, p_t) = \underbrace{-c_t \widetilde{\mathbf{P}}_{t+1}}_{\text{delivery gain}} + \underbrace{\lambda c_t |\widetilde{\mathbf{P}}_{t+1} - p_t|}_{\text{penalty}}$$

Scenarios



We use one year of power data from Ausgrid to calibrate the weights (α_t, β_t) and the law of $\mathbf{W}_{t+1}$ for the generated power $\mathbf{G}_t$

Methods to compute an optimal profile $p^* \in \mathbb{R}^T$

We want to compute $p^* \in \arg\min_{p \in \mathcal{P}} \Phi(p)$

Generic method

input: $p^0 \in \mathcal{P}$

for $n = 1 \dots N$ **do**

- call a **a first order oracle** to estimate
  - $\rightarrow \Phi(p^n)$
  - $\rightarrow q^n$ as a (sub)gradient of Φ at p^n
- use an **iterative update rule** to compute p^{n+1} from $(p^n, q^n, \mathcal{P})$ and a step size $\alpha_n \in \mathbb{R}_+$

end

output: p^*

We define a method as **a first order oracle** + **an iterative algorithm**

Instances of methods

We have three methods

- μ**SDP+IPM:** $\begin{cases} \text{Lower smooth oracle} \\ \text{Interior Points Method} \end{cases}$ $\rightarrow$ the discretization of $\mathbb{R}^{n_x}, \mathbb{R}^{n_u}$ is critical
- k**SDDP+PSM:** $\begin{cases} \text{Lower polyhedral oracle} \\ \text{Projected Subgradient Method} \end{cases}$ $\rightarrow$ the value of $k \in \mathbb{N}$ is critical
- μ**SDP+PGD:** $\begin{cases} \text{Lower smooth oracle} \\ \text{Projected Gradient Descent} \end{cases}$ $\rightarrow$ same as μSDP+IPM

for **each method**, we try **several instances**
i.e. several discretizations of $\mathbb{R}^{n_x}, \mathbb{R}^{n_u}$ or several values of k

Evaluate a profile $p^* \in \mathbb{R}^T$

Given a profile $p^* \in \mathbb{R}^T$, we run the SDDP algorithm to compute

$$\underline{V}_T(x) = K(x) \ , \ \ \forall x \in \mathbb{R}^2$$

$$\underline{V}_t(x) = \inf_{u \in \mathcal{U}_t(x)} \mathbb{E}\Big[L_t(x, u, \mathbf{W}_{t+1}, p_t^*) + \underline{V}_{t+1}\big(f_t(x, u, \mathbf{W}_{t+1})\big) \Big]$$

$$\forall x \in \mathbb{R}^2 \ , \ \ \forall t \in \{0, \dots, T-1\}$$

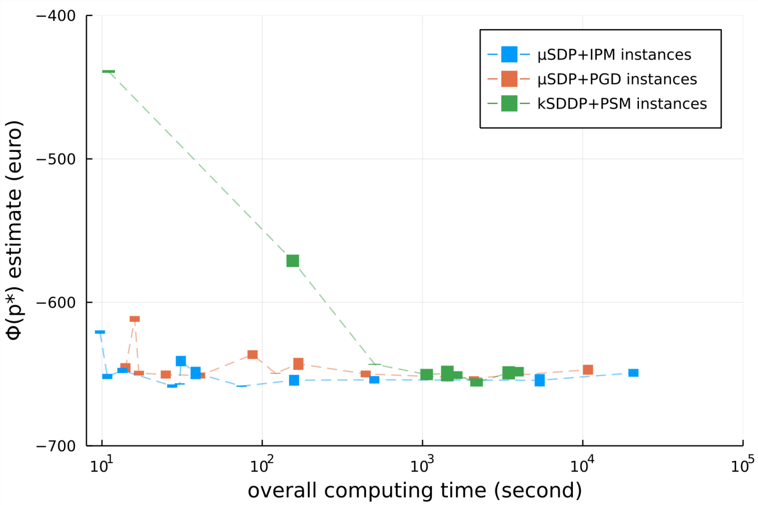
Then, we obtain a policy $\{\underline{\pi}_t\}_{t \in \{0,\dots,T-1\}}$ from $\{\underline{V}_t\}_{t \in \{0,\dots,T-1\}}$ and estimate the expected cost by sampling 25.000 scenarios

$$\overline{V}_0(x_0) = \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t\big(\mathbf{X}_t, \underline{\pi}_t(\mathbf{X}_t), \mathbf{W}_{t+1}, p_t^*\big) + K(\mathbf{X}_T) \Big]$$

We deduce

$$\underbrace{\underline{V}_0(x_0) \leq}_{\text{exact}} \ \Phi(p^*) \ \underbrace{\leq \overline{V}_0(x_0)}_{\text{statistical}}$$

Results: cost vs overall computing time



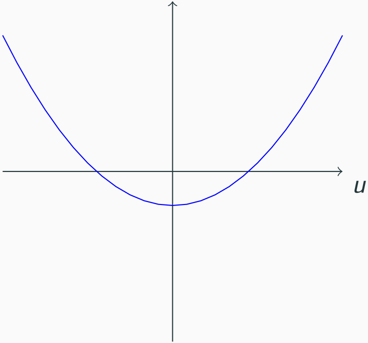
Outline of the presentation

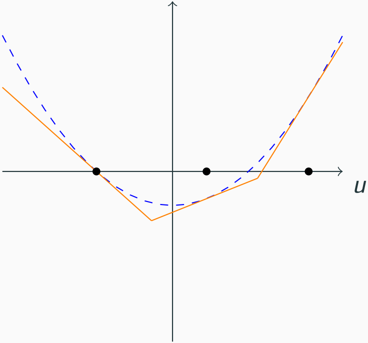
1. EMSx: a numerical benchmark for energy management systems

2. Parametric multistage stochastic optimization for day-ahead power scheduling

3. Perspectives for numerical methods in generalized convexity

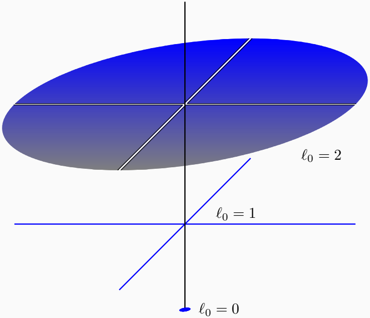
Fenchel conjugate, subdifferential, and polyhedral approximate of a convex lower semicontinuous function

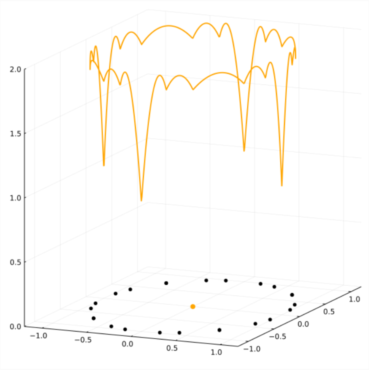




$$f(u) \geq \max_{\substack{v_i \in \partial f(u_i) \\ i \in I}} \Big(\langle u \, , v_i \rangle + \big(-f^\star(v_i) \big) \Big)$$

Beyond convex lower semicontinuous functions...





$$\ell_0(u) \geq \max_{\substack{v_i \in \partial_{\text{¢}} \ell_0(u_i) \\ i \in I}} \left(\text{¢}(u, v_i) + \left(-\ell_0^{\text{¢}}(v_i)\right) \right)$$

Contribution of the section

- **Question**

Can we leverage **generalized convexity** notions
to solve **nonconvex optimization problems**?

- **Our contribution**

We focus on **one-sided linear conjugacies**
to extend the **mirror descent algorithm**
and study its aplicability in **sparse optimization**

Outline of the section

3. Perspectives for numerical methods in generalized convexity

Background in one-sided linear (OSL) conjugacies

The mirror descent algorithm

The Capra coupling and the ℓ_0 pseudonorm

Perspectives for sparse optimization

Couplings and generalized Fenchel-Moreau conjugacies

$$\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\} \quad \text{and} \quad (+\infty) \dotplus (-\infty) = (-\infty) \dotplus (+\infty) = -\infty$$

Definition

Two sets $\mathbb{U}$ ("Primal") and $\mathbb{V}$ ("Dual") paired by
a **coupling function** $c : \mathbb{U} \times \mathbb{V} \to \overline{\mathbb{R}}$
give rise to the ***c*-Fenchel-Moreau conjugacy**

$$f \in \overline{\mathbb{R}}^{\mathbb{U}} \mapsto f^c \in \overline{\mathbb{R}}^{\mathbb{V}}$$

$$f^c(v) = \sup_{u \in \mathbb{U}} \Big(c(u,v) \dotplus \big(-f(u)\big) \Big) , \quad \forall v \in \mathbb{V}$$

Example: two vector spaces $\mathbb{U}$ and $\mathbb{V}$ paired with a bilinear form $\langle \cdot \, , \cdot \rangle$
give rise to the classic Fenchel conjugacy $f \in \overline{\mathbb{R}}^{\mathbb{U}} \mapsto f^\star \in \overline{\mathbb{R}}^{\mathbb{V}}$

Generalized c-biconjugate and c-convexity

- We also introduce the c'-**Fenchel-Moreau conjugacy**

$$g \in \overline{\mathbb{R}}^{\mathbb{V}} \mapsto g^{c'} \in \overline{\mathbb{R}}^{\mathbb{U}} , \quad g^{c'}(u) = \sup_{v \in \mathbb{V}} \Big(\underbrace{c(u,v)}_{=c'(v,u)} \dot{+} \big(-g(v)\big) \Big) , \quad \forall u \in \mathbb{U}$$

- This gives rise to the c-**Fenchel-Moreau biconjugate**

$$f \in \overline{\mathbb{R}}^{\mathbb{U}} \mapsto f^{cc'} \in \overline{\mathbb{R}}^{\mathbb{U}} , \quad f^{cc'}(u) = \big(f^{c}\big)^{c'}(u) , \quad \forall u \in \mathbb{U}$$

Definition

A function $f \in \overline{\mathbb{R}}^{\mathbb{U}}$ is c-**convex** if $f = f^{cc'}$, that is

$$f(u) = \sup_{v \in \mathbb{V}} \Big(c(u,v) \dot{+} \big(-f^{c}(v)\big) \Big) , \quad \forall u \in \mathbb{U}$$

Example: a proper function $f \in \overline{\mathbb{R}}^{\mathbb{U}}$ is $\langle\, , \rangle$-convex iff f is convex and lsc

Generalized c-subdifferential

Definition

The ***c*-subdifferential** of a function $f \in \overline{\mathbb{R}}^{\mathbb{U}}$ at $u \in \mathbb{U}$
with respect to the coupling c is the subset $\partial_c f(u) \subseteq \mathbb{V}$
defined equivalently either by

$$v \in \partial_c f(u) \iff f^c(v) = c(u,v) \dotplus \big(-f(u)\big)$$

or by

$$v \in \partial_c f(u) \iff c(u,v) \dotplus \big(-f(u)\big) \geq c(u',v) \dotplus \big(-f(u')\big) \ , \ \ \forall u' \in \mathbb{U}$$

One-sided linear (OSL) couplings

- Let $\mathbb{U}$ and $\mathbb{V}$ be two **vector spaces** paired by a bilinear form $\langle \cdot \, , \cdot \rangle$
- We suppose given a mapping $\theta : \mathbb{W} \to \mathbb{U}$ where $\mathbb{W}$ is **any set**

Definition

We define the **one-sided linear coupling (OSL)**

$$\mathbb{W} \overset{\star_\theta}{\longleftrightarrow} \mathbb{V}$$

between $\mathbb{W}$ and $\mathbb{V}$ by

$$\star_\theta(w, v) = \langle \theta(w) \, , v \rangle \; , \;\; \forall w \in \mathbb{W} \; , \;\; \forall v \in \mathbb{V}$$

Some properties of convex analysis can be extended
to $\star_\theta$-convex functions...

Outline of the section

The standard Bregman divergence

Definition

Let $\mathbb{W}$ and $\mathbb{V}$ be two vector spaces paired by a bilinear form $\langle \cdot \,, \cdot \rangle$
let $\kappa \in \overline{\mathbb{R}}^{\mathbb{W}}$ be a proper, closed, convex and differentiable (divergence generating) function.
We define the **Bregman divergence associated with κ** by

$$D_\kappa(w, w') = \kappa(w) - \kappa(w') - \langle w - w' \,, \nabla\kappa(w') \rangle \,, \quad \forall (w, w') \in \mathbb{W} \times \mathrm{dom}(\nabla\kappa)$$

D_κ is not a distance, but if κ is **strongly convex**

- $D_\kappa(w, w') \geq 0 \,, \;\; \forall (w, w') \in \mathbb{W} \times \mathrm{dom}(\nabla\kappa)$
- $D_\kappa(w, w') = 0 \iff w = w'$
- We have a "triangular inequality" that makes mirror descent work

The Bregman divergence with couplings

Definition

Let $\mathbb{W}$ and $\mathbb{V}$ be two sets and a **finite coupling** $\mathbb{W} \overset{c}{\longleftrightarrow} \mathbb{V}$
let $\kappa \in \overline{\mathbb{R}}^{\mathbb{W}}$ be a proper c-convex (divergence generating) function.
We define the **c-Bregman divergence associated with κ** by

$$D^c_\kappa(w, w', v') = \kappa(w) - \kappa(w') - c(w, v') + c(w', v') ,$$
$$\forall (w, w') \in \mathbb{W} \times \mathrm{dom}(\partial_c \kappa) , \ \forall v' \in \partial_c \kappa(w')$$

If moreover

- $\mathbb{V}$ is a vector space
- The coupling c is **OSL**
- The function κ is **c-strongly convex**

We retrieve some properties of the original Bregman divergence

The mirror descent algorithm with OSL couplings

We consider the optimization problem

$$\min_{w \in W} f(w)$$

- We initialize three sequences by

$$\begin{aligned} w_0 &\in W \\ v_0 &\in \partial_{\star_\theta} \left(\kappa + \delta_W\right)(w_0) \\ v_0^f &\in \partial_{\star_\theta} f(w_0) \end{aligned}$$

- We run $N \in \mathbb{N}$ steps with a step size $\alpha_n > 0$ and the update rules

$$\begin{aligned} w_{n+1} &\in \arg\min_{w \in W} \left(\kappa(w) + \left\langle \theta(w), \alpha_n v_n^f - v_n \right\rangle\right) \\ v_{n+1} &= v_n - \alpha_n v_n^f \\ v_{n+1}^f &\in \partial_{\star_\theta} f(w_{n+1}) \end{aligned}$$

Convergence result

Theorem (Le Franc [2021])

Under a suitable choice of divergence generating function κ we can bound the optimality gap by

$$\min_{0 \leq n \leq N-1} \left(f(w_n) - \inf_{w \in W} f(w) \right) \leq \frac{R^2 + \frac{G^2}{4} \sum_{n=0}^{N-1} \alpha_n^2}{\sum_{n=0}^{N-1} \alpha_n}$$

- R and G are constant values determined by the problem and by κ
- We retrieve the same convergence rule as in the standard mirror descent algorithm

Outline of the section

3. Perspectives for numerical methods in generalized convexity

Background in one-sided linear (OSL) conjugacies

The mirror descent algorithm

The Capra coupling and the ℓ_0 pseudonorm

Perspectives for sparse optimization

We introduce the Capra coupling

Definition (Chancelier and De Lara [2020])

Let $\|\!|\cdot|\!\|$ be a norm on $\mathbb{R}^d$ called the **source norm**

we define the **Capra** *coupling* $\mathbb{R}^d \overset{¢}{\longleftrightarrow} \mathbb{R}^d$ by

$$\forall v \in \mathbb{R}^d\ ,\ \ ¢(u,v) = \begin{cases} \frac{\langle u \,, v \rangle}{\|\!|u|\!\|} & \text{if } u \neq 0 \\ 0 & \text{if } u = 0 \end{cases}$$

The coupling Capra is

- **Constant Along Primal RAys (Capra)**
- **OSL** with $¢(u,v) = \langle n(u)\,, v \rangle\ ,\ \ \forall (u,v) \in (\mathbb{R}^d)^2$

$$\text{where} \quad n(u) = \begin{cases} \frac{u}{\|\!|u|\!\|} & \text{if } u \neq 0 \\ 0 & \text{if } u = 0 \end{cases}$$

The ℓ_0 pseudonorm

$$\ell_0(u) = \left|\left\{j \in \{1, \ldots, d\} \,\middle|\, u_j \neq 0\right\}\right| , \ \forall u \in \mathbb{R}^d$$

Proposition (Chancelier and De Lara [2021])

If both the source norm $|||\cdot|||$ and the dual norm $|||\cdot|||_\star$ are ***orthant-strictly monotonic****, we have that*

$$\ell_0 = \ell_0^{¢¢'}$$

$$\partial_{¢}\ell_0(u) \neq \emptyset , \ \forall u \in \mathbb{R}^d$$

that is, the pseudonorm ℓ_0 is ***Capra-convex*** *and* ***Capra-subdifferentiable*** *everywhere on $\mathbb{R}^d$*

Examples:
- $||(0,1)||_\infty = ||(1,1)||_\infty = 1$ hence ℓ_∞ is not OSM
- ℓ_2 is OSM

Explicit formulations for the Capra-subdifferential of ℓ_0

Proposition (Le Franc [2021])

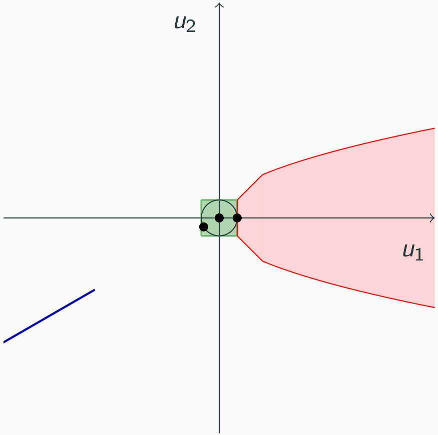
For the source norms $|||\cdot||| = \ell_p$, $p \in]1, \infty[$, *we have that*

$$\partial_{¢}\ell_0(0) = \mathbb{B}_{\ell_\infty}$$

and if $u \neq 0$, $l = \ell_0(u)$, $L = supp(u)$, *for* $v \in \mathbb{R}^d$, $|v_{\nu(1)}| \geq \ldots \geq |v_{\nu(d)}|$, $||v||^{\mathrm{tn}}_{(k,q)} = \left(|v_{\nu(1)}|^q + \ldots + |v^q_{\nu(k)}|\right)^{\frac{1}{q}}$ *and* $\frac{1}{p} + \frac{1}{q} = 1$

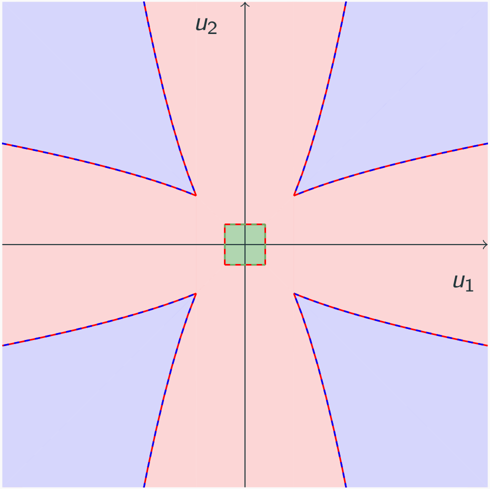
$$v \in \partial_{¢}\ell_0(u) \iff \begin{cases} v_L \in \mathcal{N}_{\mathbb{B}_{||\cdot||_p}}\left(\frac{u}{||u||_p}\right) \\ |v_j| \leq \min_{i \in L} |v_i| \,, \ \ \forall j \notin L \\ |v_{\nu(k+1)}|^q \geq \left(||v||^{\mathrm{tn}}_{(k,q)} + 1\right)^q - \left(||v||^{\mathrm{tn}}_{(k,q)}\right)^q \,, \ \ \forall k < l \\ |v_{\nu(l+1)}|^q \leq \left(||v||^{\mathrm{tn}}_{(l,q)} + 1\right)^q - \left(||v||^{\mathrm{tn}}_{(l,q)}\right)^q \end{cases}$$

Examples of sets $\partial_{\text{¢}}\ell_0(u)$ in $\mathbb{R}^2$ with the source norm $|||\cdot||| = \ell_2$



$$\partial_{\text{¢}}\ell_0(0,0)\ ,\ \ \partial_{\text{¢}}\ell_0(1,0)\ ,\ \ \partial_{\text{¢}}\ell_0(u_1,u_2)$$

Vizualisation of $\partial_{\c{c}}\ell_0$ in $\mathbb{R}^2$ with the source norm $|||\cdot||| = \ell_2$



$$\partial_{\c{c}}\ell_0(0) \bigcup \left\{ \bigcup_{\ell_0(u)=1} \partial_{\c{c}}\ell_0(u) \right\} \bigcup \left\{ \bigcup_{\ell_0(u)=2} \partial_{\c{c}}\ell_0(u) \right\}$$

Outline of the section

3. Perspectives for numerical methods in generalized convexity

Background in one-sided linear (OSL) conjugacies

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Perspectives for sparse optimization

Capra-convex sparse optimization problems

We consider problems of type

$$\min_{u \in U} \ell_0(u)$$

and we look for constraint sets $U \subseteq \mathbb{U}$ for which
we have a **Capra-convex (sparse) optimization problem**

Definition

We say that **the set $U \subseteq \mathbb{U}$ is Capra-convex** if the indicator function **δ_U is a Capra-convex function**

Which sets are Capra-convex ?

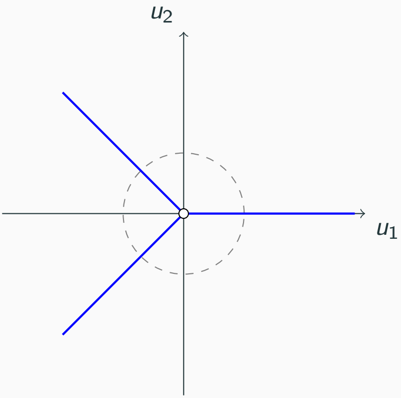
Characterization of Capra-convex sets

Proposition (Le Franc [2021])

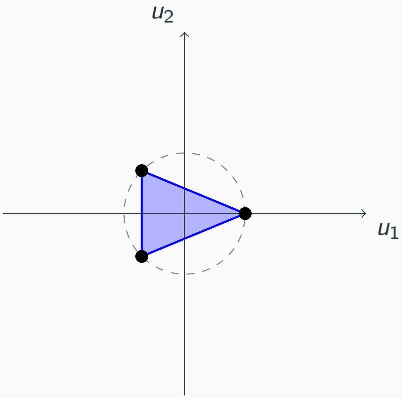
Let the source norm $|||\cdot||| = ||\cdot||_p$, $p \in]1, \infty[$ *and* $U \subseteq \mathbb{U}$ *be a nonempty set*

$$U \text{ is Capra-convex} \iff \begin{cases} U \text{ is a cone} \\ U \cup \{0\} \text{ is closed} \\ U \cap \{0\} = \overline{\mathrm{co}}\big(n(U)\big) \cap \{0\} \end{cases}$$

Example with $\|\cdot\| = \ell_2$: a non Capra-convex cone

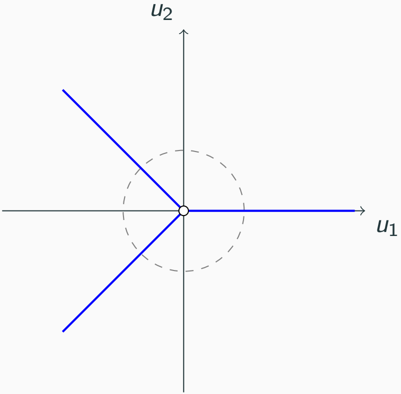


K



$\overline{\mathrm{co}}(n(K))$

Example with $\|\cdot\| = \ell_2$: a non Capra-convex cone



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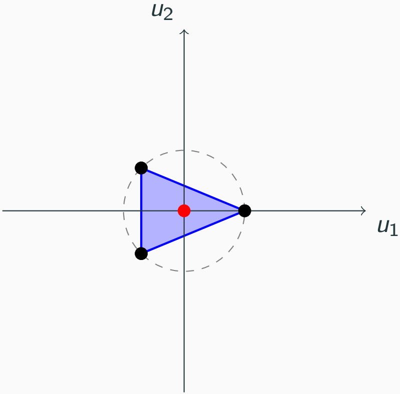
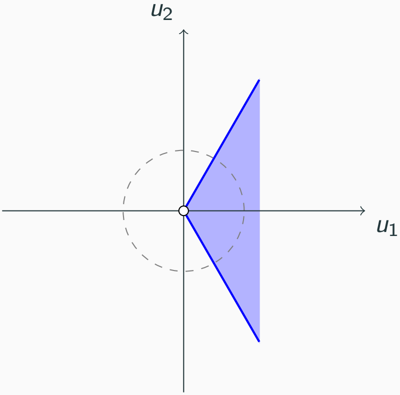


$\overline{\mathrm{co}}(n(K))$

$$K \cap \{0\} \neq \overline{\mathrm{co}}\big(n(K)\big) \cap \{0\}$$

Example with $\|\cdot\| = \ell_2$: a Capra-convex cone



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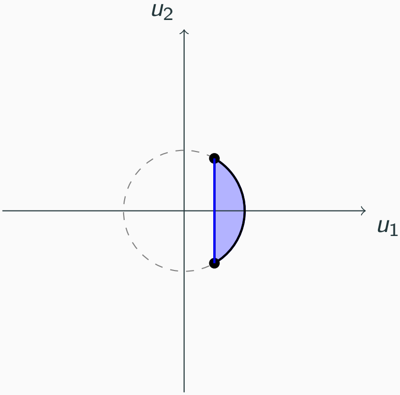
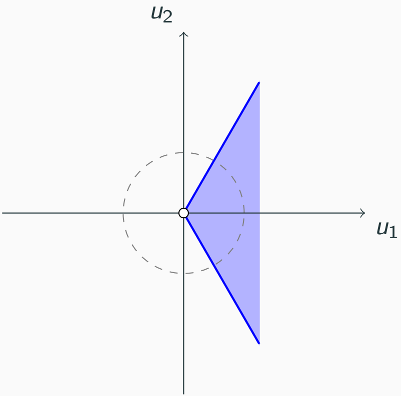
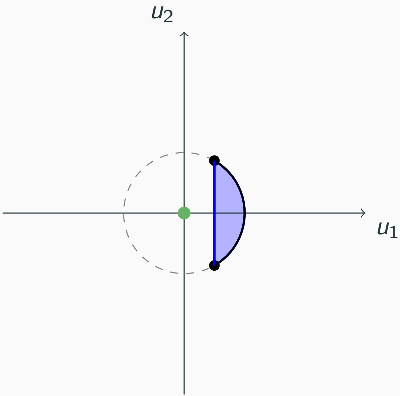


$\overline{\mathrm{co}}(n(K))$

Example with $\|\cdot\| = \ell_2$: a Capra-convex cone



K

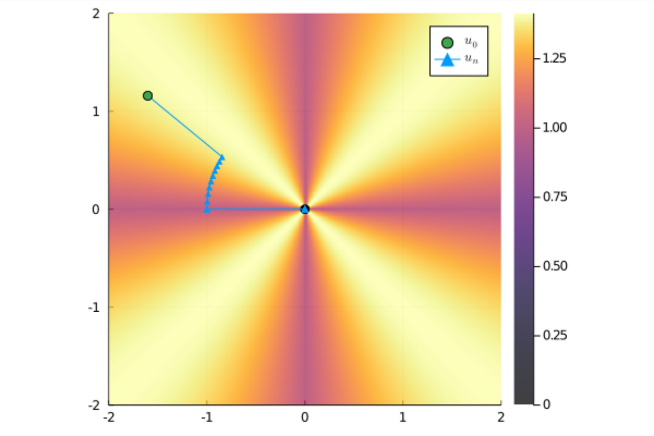


$\overline{\mathrm{co}}(n(K))$

$$K \cap \{0\} = \overline{\mathrm{co}}\big(n(K)\big) \cap \{0\}$$

Application of the mirror descent algorithm

Only a (trivial) unconstrained test so far: $\min_{u \in \mathbb{R}^2} \frac{||u||_1}{||u||_2}$



Perspectives to handle constraint sets

Turning to e.g. $\min_{u \in K} \frac{||u||_1}{||u||_2}$?

- We need to identify suitable **divergence generating functions** κ such that $\kappa + \delta_K$ **is Capra-strongly convex**
- We need to make sure that we can **compute efficiently**

$$u_{n+1} \in \arg\min_{u \in K} \big(\kappa(u) + ¢(u, \alpha_n v_n^f - v_n)\big)$$

Work in progress...

Conclusion

1.
- We have introduced a **dataset**, a **mathematical framework** and a **software** to compare microgrid controller techniques on a **publicly available** benchmark
- The EMSx benchmark is further detailed in [Le Franc et al., 2021]

2.
- We have introduced a class of **parametric multistage stochastic optimization problems** to model **day-ahead power scheduling**
- We have presented **differentiability properties** of parametric value functions and derived efficient optimization methods

3.
- We have extended **the mirror descent algorithm to OSL couplings**
- We have explicited **the Capra-subdifferential of ℓ_0**
- We have identified **Capra-convex sets** and **Capra-convex sparse optimization problems**

Perspectives

- We look forward to applications of our methods in parametric multistage stochastic optimization to several **concrete use cases in energy markets**

- We plan to study further applications of the mirror descent algorithm to solve **Capra-convex problems**

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Jean-Philippe Chancelier and Michel De Lara. Capra-convexity, convex factorization and variational formulations for the ℓ_0 pseudonorm. *Set-Valued and Variational Analysis*, pages 1–23, 2021.

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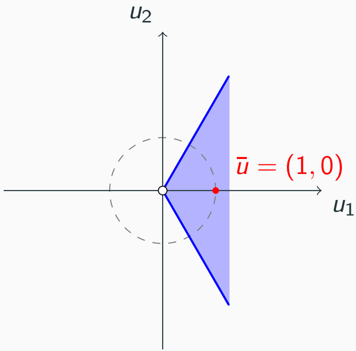
Adrien Le Franc, Pierre Carpentier, Jean-Philippe Chancelier, and Michel De Lara. Emsx: a numerical benchmark for energy management systems. *Energy Systems*, pages 1–27, 2021.

Appendix: an example where the subdifferential of the sum...

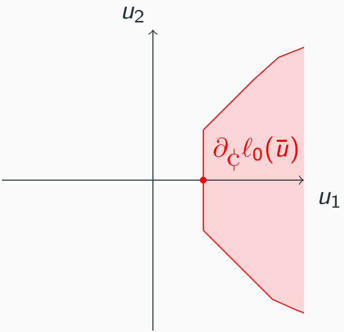
$$|||\cdot||| = \ell_2$$



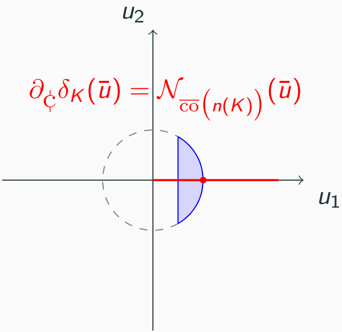
$\bar{u} \in \arg\min_K \ell_0 \implies 0 \in \partial_{\text{ç}}\big(\ell_0 + \delta_K\big)(\bar{u})$ (a property of OSL couplings)

…is not the sum of the subdifferentials

Let $v' \in \partial_{\text{Ç}} \ell_0(\bar{u})$ and $v'' \in \partial_{\text{Ç}} \delta_K(\bar{u})$



$v'_1 \geq 1$



$v''_1 \geq 0$

$0 \notin \partial_{\text{Ç}} \ell_0(\bar{u}) + \partial_{\text{Ç}} \delta_K(\bar{u})$ hence $\partial_{\text{Ç}} \ell_0(\bar{u}) + \partial_{\text{Ç}} \delta_K(\bar{u}) \subsetneq \partial_{\text{Ç}} \left(\ell_0 + \delta_K\right)(\bar{u})$