Stochastic dynamic optimization for crude oil procurement of refineries

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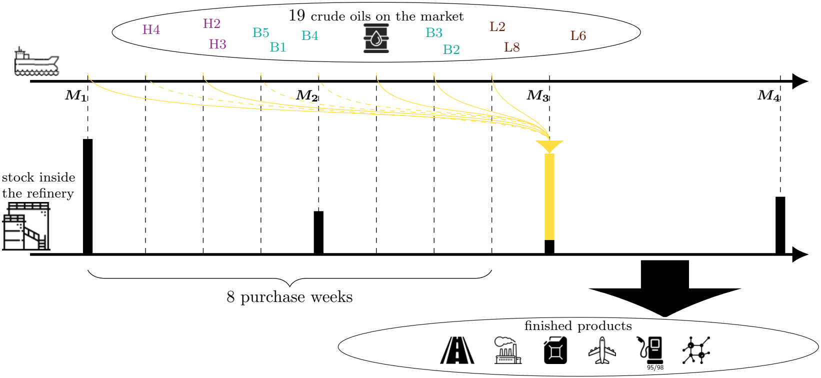
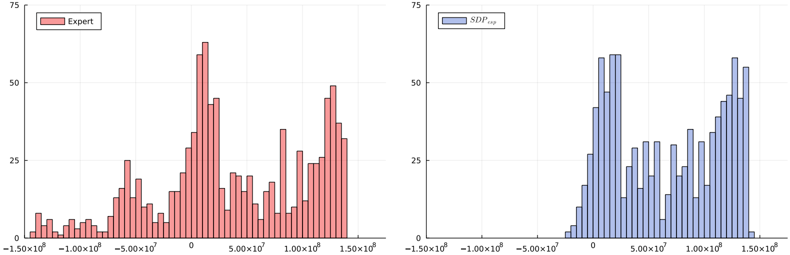
Crude oil procurement overview



How do we optimally manage the crude purchases while taking into account delivery times and uncertainties?

Comparing policies under uncertainty: average and spread

Histograms of the operating margins for two policies (higher is better)



Outline of the presentation

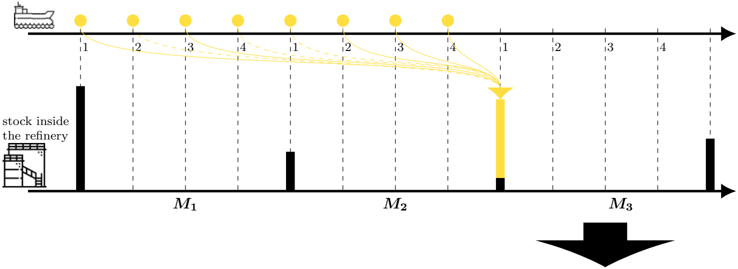
1. **Part I: Monthly crude oil procurement problem**

2. **Part II: Time-blocks decomposition and the multi-months procurement problem**

Outline of the presentation

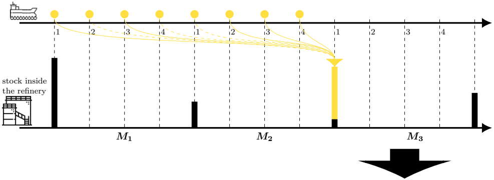
Outline of the presentation

Purchase and delivery timeline



- One single delivery/consumption month M_3
- Crude purchases over the two months M_1 and M_2 preceding M_3

Purchase and delivery timeline



$$\begin{tikzcd}
(M_1,1) \arrow[r] \arrow[d] & (M_1,2) \arrow[r] \arrow[d] & (M_1,3) \arrow[r] \arrow[d] & (M_1,4) \arrow[r] \arrow[d] & (M_2,1) \arrow[r] \arrow[d] & (M_2,2) \arrow[r] \arrow[d] & (M_2,3) \arrow[r] \arrow[d] & (M_2,4) \arrow[r] \arrow[d] & (M_3,1) \arrow[d] \\
1 \arrow[r] & 2 \arrow[r] & 3 \arrow[r] & 4 \arrow[r] & 5 \arrow[r] & 6 \arrow[r] & 7 \arrow[r] & 8 \arrow[r] & 9
\end{tikzcd}$$

For ease of use we denote the timespan of the problem

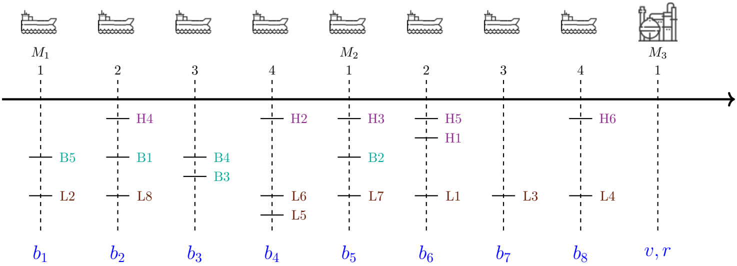
$$T = (1, 2, 3, 4, 5, 6, 7, 8)$$

with $\underline{t} = 1 = (M_1, 1)$ $\bar{t} = 8 = (M_2, 4)$

$t^+ =$ the successor of t $\bar{t}^+ = 9 = (M_3, 1)$

We identify decision variables

Each week, a set of crudes is available for purchase

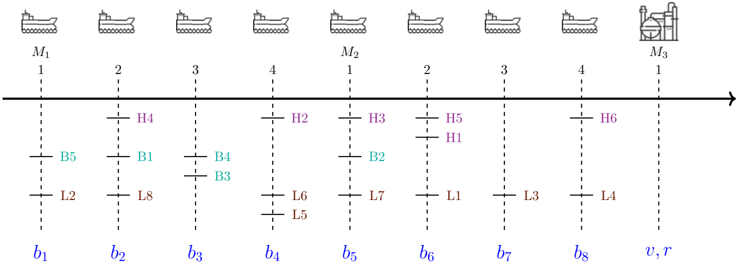


We identify 3 types of decision

- Cargos $\{b_t\}_{t \in T}$ represent the quantities of crude purchased
- Volumes v represent the crude oil consumed
- Settings r of the refinery are applied during the month M_3

We identify decision variables

Each week, a set of crudes is available for purchase

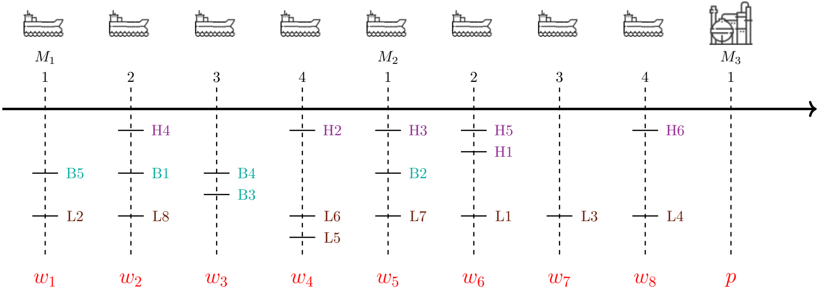


Crude oil is available in fixed quantities (full tanker)

$$b_1 = (0, \ldots, 0, b_1^{B5}, 0, \ldots, 0, b_1^{L2}, 0, \ldots, 0) \in \mathcal{B}_1 \subset \mathbb{R}_+^{19}$$

$$\left.\begin{array}{lcl} b_1^{B5} & = & 0 \text{ or } 1.5 \text{ million barrels} \\ b_1^{L2} & = & 0 \text{ or } 2.3 \text{ million barrels} \end{array}\right\} \quad \rightarrow \quad |\mathcal{B}_1| = 4$$

We identify sources of uncertainty



We model two sources of uncertainty

- Prices $\{w_t\}_{t \in T}$ of all crudes at the beginning of each week t
- Price p of all products at the beginning of the month M_3

Checkpoint

We have

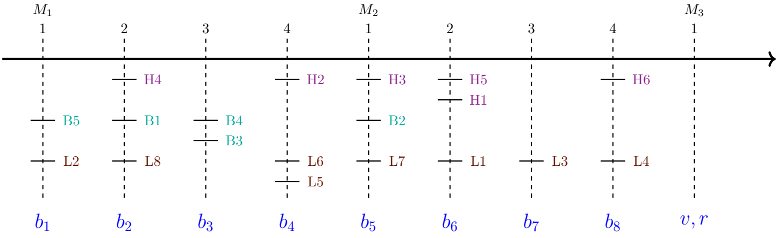
- specified a time structure
- identified decision variables
- identified sources of uncertainty

We will

- explicit a coupling constraint on crude purchases
- propose an economic function
- write multistage stochastic optimization problems

Not all crudes can be processed together

- Due to the limited treatment capacity of the refinery
- Due to chemical properties making crudes incompatible



$H4 + H2 + L8 =$ ✗

$H4 + B3 + L3 =$ ✓

- Purchase exactly 3 shipments
- No more than one cargo of heavy crude
- No more than one cargo of light crude
- No constraint on balanced crude

Coupling constraint on the purchases

The compatible crude combinations are described by a set $\mathcal{D}$

$$\overbrace{\sum_{\mathrm{t}\in\mathrm{T}} b_{\mathrm{t}} \in \mathcal{D}}^{\text{compatible combinations}} \qquad \subset \mathbb{R}_{+}^{19} \qquad\qquad |\mathcal{D}| = 520$$

where, for example

$$\sum_{\mathrm{t}\in\mathrm{T}} b_{\mathrm{t}} = (\underbrace{1.5}_{H4}, 0, \ldots, 0, \underbrace{1.4}_{B3}, 0, \ldots, 0, \underbrace{2}_{L3}) \times 10^{6} \text{ barrels}$$

Cost function

$$\overbrace{\sum_{\mathrm{t}\in\mathrm{T}} \underbrace{b_{\mathrm{t}} \cdot w_{\mathrm{t}}}_{\substack{\text{purchase costs}\\ \text{in week t}}} + \underbrace{\Psi\left(\sum_{\mathrm{t}\in\mathrm{T}} b_{\mathrm{t}}, v, r, p\right)}_{\substack{\text{refinery cost function}\\ \text{(operation costs}\\ -\ \text{incomes from selling products)}}}}^{\text{cost function}}$$

Formulation of a multistage stochastic optimization problem

$$\min_{\substack{\{\boldsymbol{b}_t\}_{t \in T} \\ \boldsymbol{v}, \boldsymbol{r}}} \mathbb{E}\Big[\sum_{t \in T} \boldsymbol{b}_t \cdot \boldsymbol{w}_t + \Psi\big(\sum_{t \in T} \boldsymbol{b}_t, \boldsymbol{v}, \boldsymbol{r}, \boldsymbol{p} \big) \Big]$$

$$\begin{aligned}
s.t \quad & \sum_{t \in T} \boldsymbol{b}_t \in \mathcal{D} && \text{coupling constraint} \\
& \boldsymbol{b}_t \in \mathcal{B}_t \ , \ \ \forall t \in T && \text{cargos availability} \\
& \boldsymbol{v} \in \mathcal{V} \ , \ \ \boldsymbol{r} \in \mathcal{R} && \text{management of the refinery} \\
& \sigma(\boldsymbol{b}_t) \subset \sigma(\boldsymbol{w}_{\underline{t}}, \cdots, \boldsymbol{w}_t) \ , \ \ \forall t \in T && \\
& \underbrace{\sigma(\boldsymbol{v}, \boldsymbol{r})}_{\sigma-\text{algebra}} \subset \sigma(\boldsymbol{w}_{\underline{t}}, \cdots, \boldsymbol{w}_{\bar{t}}, \boldsymbol{p}) &&
\end{aligned}$$

Nonanticipativity constraints

The last two constraints are nonanticipativity constraints: they represent, in mathematical terms, that decisions taken at time t only depend on past uncertainties

$$\begin{aligned} w_1 \rightsquigarrow b_1 \rightsquigarrow w_2 \rightsquigarrow b_2 \rightsquigarrow w_3 \rightsquigarrow b_3 \rightsquigarrow w_4 \rightsquigarrow b_4 \\ \rightsquigarrow w_5 \rightsquigarrow b_5 \rightsquigarrow w_6 \rightsquigarrow b_6 \rightsquigarrow w_7 \rightsquigarrow b_7 \rightsquigarrow w_8 \rightsquigarrow b_8 \\ \rightsquigarrow p \rightsquigarrow (v, r) \end{aligned}$$

- The purchase decision b_t is taken knowing past prices

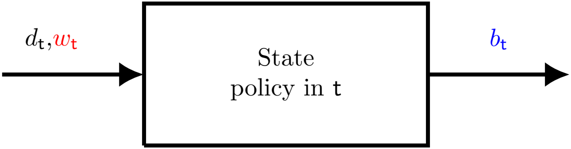
$$\sigma(\boldsymbol{b}_t) \subset \sigma(\boldsymbol{w}_{\underline{t}}, \cdots, \boldsymbol{w}_t) \quad \Leftrightarrow \quad b_t = \phi_t(\{w_{t'}\}_{t' \leq t}) \, , \ \forall t \in T$$

- Consumption and settings are decided after the product prices are revealed

$$\sigma(\boldsymbol{v}, \boldsymbol{r}) \subset \sigma(\boldsymbol{w}_{\underline{t}}, \cdots, \boldsymbol{w}_{\bar{t}}, \boldsymbol{p}) \quad \Leftrightarrow \quad (v, r) = \phi(\{w_t\}_{t \in T}, p)$$

We turn to stochastic optimal control

- We propose a state d with a dynamic
- We formulate a stochastic optimal control problem
- We then propose 5 policies:
  Expert, MPC, SDP$_{esp}$, SDP$_{CVaR}$, Succ-SDP



We introduce a buffer for the month M_3

- We propose the state variable (buffer)

$$d_{\mathrm{t}} = \sum_{\mathrm{t}'<\mathrm{t}} b_{\mathrm{t}'} \in \mathbb{R}^{\mathrm{C}} \ , \ \ \forall \mathrm{t} \in \mathrm{T} \cup \{\bar{\mathrm{t}}^{+}\}$$

with dynamics $d_{\mathrm{t}^+} = d_{\mathrm{t}} + b_{\mathrm{t}} \ , \ \ \forall \mathrm{t} \in \mathrm{T}$

- Backward recursive propagation of the target constraint

$$\mathcal{D}_{\bar{\mathrm{t}}^+} = \mathcal{D}$$

$$\mathcal{D}_{\mathrm{t}} = \{d_{\mathrm{t}} \in \mathcal{D} \mid \exists b_{\mathrm{t}} \in \mathcal{B}_{\mathrm{t}} \ , \ \ d_{\mathrm{t}} + b_{\mathrm{t}} \in \mathcal{D}_{\mathrm{t}^+}\} \ , \ \ \forall \mathrm{t} \in \mathrm{T}$$

- We reduce the size of the decision set

$$\widetilde{\mathcal{B}}_{\mathrm{t}}(d) = \{b \in \mathcal{B}_{\mathrm{t}} \mid d_{\mathrm{t}} + b \in \mathcal{D}_{\mathrm{t}^+}\} \subset \mathcal{B}_{\mathrm{t}} \ , \ \ \forall \mathrm{t} \in \mathrm{T}$$

Reformulation as a stochastic optimal control problem

$$\min_{\substack{\{\boldsymbol{b}_{\mathsf{t}}\}_{\mathsf{t}\in\mathsf{T}} \\ \boldsymbol{v},\boldsymbol{r}}} \mathbb{E}\Big[\sum_{\mathsf{t}\in\mathsf{T}} \boldsymbol{b}_{\mathsf{t}} \cdot \boldsymbol{w}_{\mathsf{t}} + \Psi(\boldsymbol{d}_{\bar{\mathsf{t}}^+}, \boldsymbol{v}, \boldsymbol{r}, \boldsymbol{p})\Big]$$

$$\begin{aligned}
s.t \quad & \boldsymbol{d}_{\bar{\mathsf{t}}^+} \in \mathcal{D}_{\bar{\mathsf{t}}^+} && \text{target constraint} \\
& \boldsymbol{d}_{\mathsf{t}} \in \mathcal{D}_{\mathsf{t}} \;, \;\; \forall \mathsf{t} \in \mathsf{T} && \text{state variable} \\
& \boldsymbol{d}_{\mathsf{t}^+} = \boldsymbol{d}_{\mathsf{t}} + \boldsymbol{b}_{\mathsf{t}} \;, \;\; \forall \mathsf{t} \in \mathsf{T} && \text{state dynamic} \\
& \boldsymbol{b}_{\mathsf{t}} \in \widetilde{\mathcal{B}}_{\mathsf{t}}(\boldsymbol{d}_{\mathsf{t}}) \;, \;\; \forall \mathsf{t} \in \mathsf{T} \\
& \boldsymbol{v} \in \mathcal{V} \;, \;\; \boldsymbol{r} \in \mathcal{R} \\
& \sigma(\boldsymbol{b}_{\mathsf{t}}) \subset \sigma(\boldsymbol{w}_{\underline{\mathsf{t}}}, \cdots, \boldsymbol{w}_{\mathsf{t}}) \;, \;\; \forall \mathsf{t} \in \mathsf{T} \\
& \sigma(\boldsymbol{v}, \boldsymbol{r}) \subset \sigma(\boldsymbol{w}_{\underline{\mathsf{t}}}, \cdots, \boldsymbol{w}_{\bar{\mathsf{t}}}, \boldsymbol{p})
\end{aligned}$$

Outline of the presentation

Single scenario based methods

Expert's method (static deterministic)

- crude prices w_t are observed
- a forecast $\widetilde{p}$ of the products prices is given
- all 19 crudes are tested individually with w_t and $\widetilde{p}$
- if the best one is available, purchase it

Model Predictive Control (deterministic dynamic)

- crude prices w_t are observed
- a forecast $\widetilde{p}$ of the products prices is given
- a forecast $(\widetilde{w}_{t^+}, \dots, \widetilde{w}_{\bar{t}})$ of crude prices is given
- solve the deterministic problem over the interval $[\![t, \bar{t}]\!]$
- take the first optimal decision b_t^*

Stochastic dynamic programming (value functions)

Before the first week

- we build
    - 600 crude prices scenarios $\big\{\big(\hat{w}^s_{\underline{\mathrm{t}}}, \dots, \hat{w}^s_{\overline{\mathrm{t}}}\big)\big\}_{s \in [\![1,600]\!]}$
    - 10 product prices $\big\{\hat{p}^m\big\}_{m \in [\![1,10]\!]}$
- we recursively compute value functions

$$V_{\overline{\mathrm{t}}^+}(d) = \frac{1}{10} \sum_{m=1}^{10} \min_{v^m, r^m} \Psi(d, v^m, r^m, \hat{p}^m) \;, \;\; \forall d \in \mathcal{D}_{\overline{\mathrm{t}}^+}$$

$$V_{\mathrm{t}}(d) = \frac{1}{600} \sum_{s=1}^{600} \min_{b^s_{\mathrm{t}} \in \widetilde{\mathcal{B}}_{\mathrm{t}}(d)} \Big(b^s_{\mathrm{t}} \cdot \hat{w}^s_{\mathrm{t}} + V_{\mathrm{t}^+}(d + b^s_{\mathrm{t}}) \Big) \;, \;\; \forall d \in \mathcal{D}_{\mathrm{t}} \;, \;\; \forall \mathrm{t} \in \mathrm{T}$$

Using the sets $\{\mathcal{D}_{\mathrm{t}}\}_{\mathrm{t} \in \mathrm{T}}$ and $\{\widetilde{\mathcal{B}}_{\mathrm{t}}\}_{\mathrm{t} \in \mathrm{T}}$ reduces computation by a factor > 10 compared to using $\mathcal{D}$ and $\{\mathcal{B}_{\mathrm{t}}\}_{\mathrm{t} \in \mathrm{T}}$

Using a risk measure

We can use a risk measure other than $\mathbb{E}$ in the value functions

$$V_{\bar{\mathrm{t}}^+}^{CVaR_\alpha}(d) = CVaR_{\alpha\,\hat{\boldsymbol{p}}}\left[\min_{\boldsymbol{v},\boldsymbol{r}} \Psi(d, \boldsymbol{v}, \boldsymbol{r}, \hat{\boldsymbol{p}})\right], \quad \forall d \in \mathcal{D}_{\bar{\mathrm{t}}^+}$$

$$V_{\mathrm{t}}^{CVaR_\alpha}(d) = CVaR_{\alpha\,\hat{\boldsymbol{w}}_{\mathrm{t}}}\left[\min_{\boldsymbol{b}_{\mathrm{t}} \in \widetilde{\mathcal{B}}_{\mathrm{t}}(d)} \left(\boldsymbol{b}_{\mathrm{t}} \cdot \hat{\boldsymbol{w}}_{\mathrm{t}} + V_{\mathrm{t}^+}^{CVaR_\alpha}(d + \boldsymbol{b}_{\mathrm{t}})\right)\right], \quad \forall d \in \mathcal{D}_{\mathrm{t}}, \quad \forall \mathrm{t} \in \mathrm{T}$$

Stochastic dynamic programming (optimal policy)

- Each week t, we solve a static optimization problem after the observation of the crude prices w_t

$$\min_{b_t \in \widetilde{\mathcal{B}}_t(d_t)} \quad b_t \cdot w_t + V_{t^+}(d_t + b_t)$$

- The solution to this problem is the control given by the SDP-policy

Successive SDP (Succ-SDP)

Each week, we have time to compute new value functions

- crude prices w_t are observed
- build N crude prices scenarios $\{(\hat{w}^s_{t^+} \dots, \hat{w}^s_{\bar{t}})\}_{s \in [\![1,N]\!]}$
- a forecast $\widetilde{p}$ of the products prices is given
- recursively compute new value functions every week

$$\hat{V}_{\bar{t}^+}(d) = \min_{v,r} \Psi(d, v, r, \widetilde{p}) \; , \;\; \forall d \in \mathcal{D}_{\bar{t}^+}$$

$$\hat{V}_{t'}(d) = \frac{1}{N} \sum_{s=1}^{N} \Bigg(\min_{b^s_{t'} \in \widetilde{\mathcal{B}}_{t'}(d)} \Big(b^s_{t'} \cdot \hat{w}^s_{t'} + \hat{V}_{t'+1}(d + b^s_{t'}) \Big) \Bigg) \; , \;\; \forall d \in \mathcal{D}_{t'} \; , \;\; \forall t' \in [\![t^+, \bar{t}]\!]$$

- the decision b^*_t is a solution of

$$\min_{b_t \in \widetilde{\mathcal{B}}_t(d_t)} \quad b_t \times w_t + \hat{V}_{t^+}(d_t + b_t)$$

Outline of the presentation

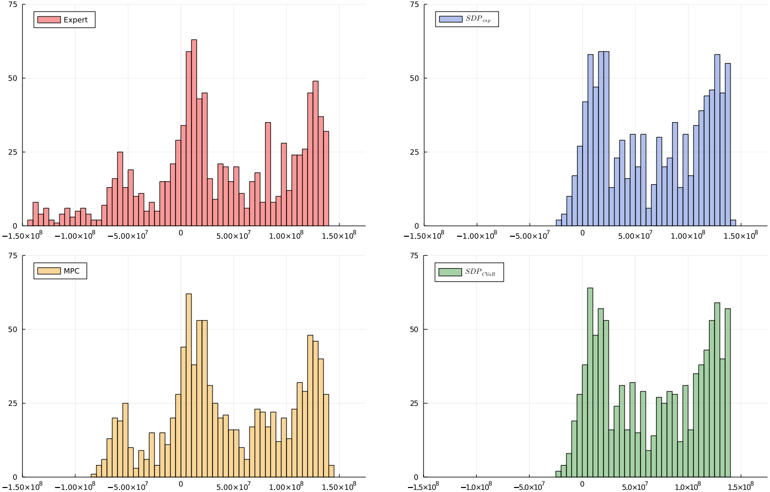
Policy assessment in an industrial setting

- Monte-Carlo simulations (1000 scenarios)
    - Succ-SDP is too heavy to be tested this way
    - assessment of 4/5 policies
    - comparison of histograms of margins

- Historical scenarios
    - replay the past for all 5 policies
    - comparison of margins and decisions

Net margins assessed over 1000 scenarios (Monte-Carlo)

Histograms of margins (higher to the right = better)



Policies are compared on historical scenarios

- We replay the scenario of December 2020

$$(w_{(O,1)}, w_{(O,2)}, w_{(O,3)}, w_{(O,4)}, w_{(N,1)}, w_{(N,2)}, w_{(N,3)}, w_{(N,4)}, p_D)$$

- We test each policy on this historical scenario

	Expert	MPC	SDP_{esp}	$\text{SDP}_{CVaR_{5\%}}$	Succ-SDP
margin ($\times 10^7$\$)	5.1	7.5	6.4	6.4	7.5
gap	−−	46%	25%	25%	46%
crude 1	H2	H4	L2	L2	H5
crude 2	L2	L2	H1	H1	L2
crude 3	B5	B1	B1	B1	B1

Policies are compared on historical scenarios

- We replay every month from October 2020 to February 2021
- We compare the cumulated performances over 5 months

	Expert	MPC	SDP_{esp}	$\text{SDP}_{CVaR_{5\%}}$	Succ-SDP
margin ($\times 10^7$\$)	5.4	26.7	10.1	10.1	27.2
gap	−−	394%	88%	88%	402%

- Only MPC and Succ-SDP yield positive margins for all months
- Only Succ-SDP outperforms Expert every month
- Succ-SDP slightly edges out MPC;
  they are the best performing policies

Conclusion of Part I

- Model for crude oil procurement under uncertainty in which
  - we purchase crude oil every week
  - we pilot the refinery every month
- Multistage stochastic optimization problem
  for a single delivery month
- We have compared 5 resolution methods
- MPC and Succ-SDP are the best performing policies
  and they use the price forecast $\widetilde{p}$

There are substantial potential gains in designing policies
based on multistage (stochastic) optimization

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1.1 Modeling of the crude oil procurement
and formulation of an optimal control problem

1.2 Resolution methods

1.3 Numerical results

2. **Part II: Time-blocks decomposition and the multi-months procurement problem**

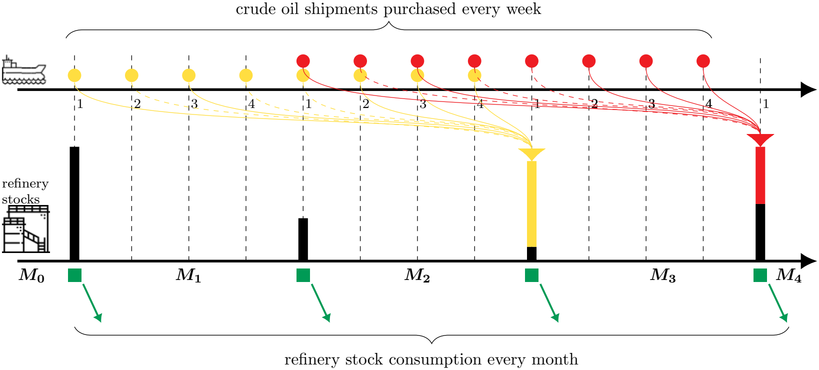
2.1 The multi-months procurement problem

2.2 Time-blocks decomposition

2.3 Two time scales optimization problem

2.4 Back to the procurement problem

We manage the refinery for any number of months



crude oil shipments purchased every week

refinery stock consumption every month

New elements for multi-months procurement

- Oil can only be purchased up to 2 months in advance

$$\begin{array}{ll} (\mathrm{m},\mathrm{w})\mathfrak{P}\mathrm{m}' & (\mathrm{m},\mathrm{w}) \text{ is a purchase week for the month } \mathrm{m}' \\ \mathfrak{P}M_3 & = \{(M_1,1),(M_1,2),\cdots,(M_2,4)\} \\ \mathfrak{P}M_4 & = \{(M_2,1),(M_2,2),\cdots,(M_3,4)\} \end{array}$$

- Purchases (and buffers) target a specific month

$b^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})}$	oil purchased in (m,w) for a delivery in $\mathrm{m}' > \mathrm{m}$
$d^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})}$	state of the $\mathrm{m}'-$buffer in (m,w)

- The refinery is operated on a monthly basis

v_{m}	crude oil consumption for the month m
r_{m}	refinery settings for the month m

- Stocks follow a monthly dynamic

s_{m}	oil in stocks at the beginning of the month m

$$\boldsymbol{s}_{\mathrm{m}^+} = \mathcal{F}_{\mathrm{m}}\left(\boldsymbol{s}_{\mathrm{m}}, \boldsymbol{d}^{\mathrm{m}}_{(\mathrm{m},\underline{\mathrm{w}})}, \boldsymbol{v}_{\mathrm{m}}\right)$$

We formulate a corresponding optimization problem

$$\min_{\substack{\{\boldsymbol{b}^{m'}_{(m,w)}\}_{((m,w),m')\in\mathfrak{P}} \\ \{\boldsymbol{v}_m,\boldsymbol{r}_m\}_{m\in M}}} \mathbb{E}\left[\sum_{(m,w)\in M\times W}\left(\sum_{m'\in(m,w)\mathfrak{P}}\Omega^{m'}_{(m,w)}\big(\boldsymbol{d}^{m'}_{(m,w)},\boldsymbol{b}^{m'}_{(m,w)},\boldsymbol{w}_{(m,w)}\big)\right)+\sum_{m\in M}\Psi_m\big(\boldsymbol{s}_m,\boldsymbol{d}^{m}_{(m,\underline{w})},\boldsymbol{v}_m,\boldsymbol{r}_m,\boldsymbol{p}_m\big)\right]$$

$$\begin{array}{lll}
\text{s.t} & \boldsymbol{b}^{m'}_{(m,w)}\in\mathcal{B}^{m'}_{(m,w)}\ ,\ \ \forall m'\in M\ ,\ \ \forall(m,w)\in\mathfrak{P}m' & \text{constraints on decisions} \\
 & \boldsymbol{v}_m\in\mathcal{V}_m\ ,\ \ \forall m\in M & \\
 & \boldsymbol{r}_m\in\mathcal{R}_m\ ,\ \ \forall m\in M & \\
 & & \\
 & \boldsymbol{d}^{m}_{(m,\underline{w})}\in\mathcal{D}^{m}\ ,\ \ \forall m\in M & \text{constraints on stocks} \\
 & \boldsymbol{s}_m\in\mathbb{S}_m\ ,\ \ \forall m\in M & \\
 & & \\
 & \boldsymbol{d}^{m}_{\min\mathfrak{P}m}=0\ ,\ \ \forall m\in M & \text{dynamics on the stocks} \\
 & \boldsymbol{d}^{m'}_{(m,w)^+}=\mathcal{F}^{m'}_{(m,w)}\big(\boldsymbol{d}^{m'}_{(m,w)},\boldsymbol{b}^{m'}_{(m,w)}\big)\ ,\ \ \forall((m,w),m')\in\overline{\mathfrak{P}} & \\
 & \boldsymbol{s}_{m^+}=\mathcal{F}_m\big(\boldsymbol{s}_m,\boldsymbol{d}^{m}_{(m,\underline{w})},\boldsymbol{v}_m\big)\ ,\ \ \forall m\in M & \\
 & & \\
 & \sigma\big(\boldsymbol{b}^{m'}_{(m,w)}\big)\subset\sigma\big(\{\boldsymbol{p}_{m''}\}_{m''\leq m},\{\boldsymbol{w}_{(m'',w'')}\}_{(m'',w'')\leq(m,w)}\big) & \text{nonanticipativity constraints} \\
 & \sigma\big(\boldsymbol{v}_m,\boldsymbol{r}_m\big)\subset\sigma\big(\{\boldsymbol{p}_{m''}\}_{m''\leq m},\{\boldsymbol{w}_{(m'',w'')}\}_{(m'',w'')\leq(m,\underline{w})}\big) &
\end{array}$$

Checkpoint

Can we leverage the month/week repetitive structure ?

that is, decompose the problem by monthly blocks to

- solve the problem by dynamic programming (DP) at the monthly scale
- without having to do DP at the weekly scale

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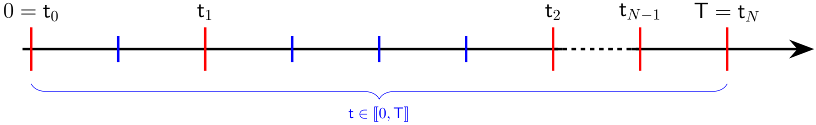
   - 2.3 Two time scales optimization problem
   - 2.4 Back to the procurement problem

Dynamic programming equations on subset of instants

We consider a subset of N instants in $[\![0, \mathsf{T}]\!]$

$$0 = \mathsf{t}_0 < \mathsf{t}_1 < \cdots < \mathsf{t}_N = \mathsf{T}$$



We will build

- reduced Bellman operators $\{\widetilde{\mathcal{B}}_{\mathsf{t}_{i+1}:\mathsf{t}_i}\}_{i \in [\![0, N-1]\!]}$
- reduced value functions $\{\widetilde{V}_{\mathsf{t}_i}\}_{i \in [\![0, N]\!]}$

$$\widetilde{V}_{\mathsf{t}_N} = \widetilde{\mathcal{J}}$$
$$\widetilde{V}_{\mathsf{t}_i} = \widetilde{\mathcal{B}}_{\mathsf{t}_{i+1}:\mathsf{t}_i} \widetilde{V}_{\mathsf{t}_{i+1}} \ , \ \ \forall i \in [\![0, N-1]\!]$$

We introduce histories . . .

- $(\mathbb{U}_0, \mathcal{U}_0)$,. . . , $(\mathbb{U}_{T-1}, \mathcal{U}_{T-1})$ are measurable control spaces
- $(\mathbb{W}_0, \mathcal{W}_0)$,. . . , $(\mathbb{W}_T, \mathcal{W}_T)$ are measurable noise spaces

We define histories for the full timespan

$$\mathbb{H}_0 = \mathbb{W}_0$$

$$\mathbb{H}_t = \mathbb{W}_0 \times \prod_{s=1}^{t} (\mathbb{U}_{s-1} \times \mathbb{W}_s) \ , \ \ \forall t \in [\![1, T]\!]$$

$h_t \in \mathbb{H}_t$ contains all the past information

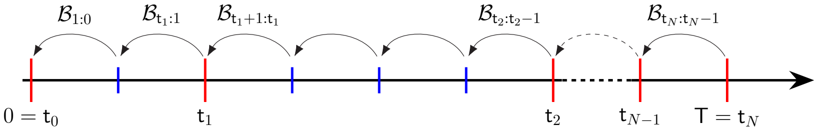
. . . with which we can define an elementary Bellman operator

We define the elementary Bellman operator $\mathcal{B}_{t+1:t}$ by

$$\big(\mathcal{B}_{\mathrm{t+1:t}}\varphi\big)(h_{\mathrm{t}}) = \inf_{u_{\mathrm{t}}\in\mathbb{U}_{\mathrm{t}}}\int_{\mathbb{W}_{\mathrm{t+1}}} \varphi(\underbrace{h_{\mathrm{t}}, u_{\mathrm{t}}, w_{\mathrm{t+1}}}_{=h_{\mathrm{t+1}}})\rho_{\mathrm{t:t+1}}(dw_{\mathrm{t+1}} \mid h_{\mathrm{t}})$$

where $\rho_{\mathrm{t:t+1}}$ is the stochastic kernel at time t (noise distribution)

$$\rho_{\mathrm{t:t+1}} : \mathbb{H}_{\mathrm{t}} \longrightarrow \Delta(\mathbb{W}_{\mathrm{t+1}})$$



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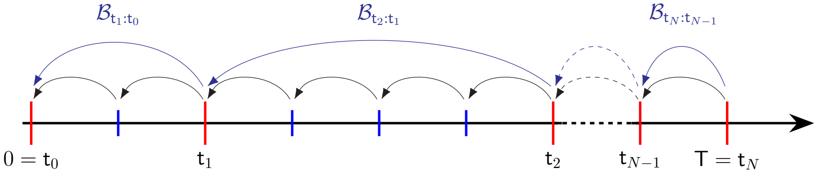
We define the elementary Bellman operator $\mathcal{B}_{t+1:t}$ by

$$\big(\mathcal{B}_{t+1:t}\varphi\big)(h_t) = \inf_{u_t \in \mathbb{U}_t} \int_{\mathbb{W}_{t+1}} \varphi\big(\underbrace{h_t, u_t, w_{t+1}}_{=h_{t+1}}\big)\rho_{t:t+1}(dw_{t+1} \mid h_t)$$

where $\rho_{t:t+1}$ is the stochastic kernel at time t (noise distribution)

$$\rho_{t:t+1} : \mathbb{H}_t \longrightarrow \Delta(\mathbb{W}_{t+1})$$



where $\mathcal{B}_{t_2:t_1} = \mathcal{B}_{t_1+1:t_1} \circ \mathcal{B}_{t_1+2:t_1+1} \circ \cdots \circ \mathcal{B}_{t_2:t_2-1}$

Assuming a state reduction . . .

$$\begin{tikzcd}
\mathbb{H}_{\mathsf{t}_i} \arrow[d, "\theta_{\mathsf{t}_i}"] & \times & \mathbb{H}_{\mathsf{t}_i+1:\mathsf{t}_{i+1}} \arrow[r, "I_d"] \arrow[d, "I_d"] & \mathbb{H}_{\mathsf{t}_{i+1}} \arrow[d, "\theta_{\mathsf{t}_{i+1}}"] \\
\mathbb{X}_{\mathsf{t}_i} & \times & \mathbb{H}_{\mathsf{t}_i+1:\mathsf{t}_{i+1}} \arrow[r, "f_{\mathsf{t}_i:\mathsf{t}_{i+1}}"] & \mathbb{X}_{\mathsf{t}_{i+1}}
\end{tikzcd}$$

We assume

- $\{(\mathbb{X}_{\mathsf{t}_i}, \mathcal{X}_{\mathsf{t}_i})\}_{i \in [\![0,N]\!]}$, measurable state sets
- $\{\theta_{\mathsf{t}_i}\}_{i \in [\![0,N]\!]}$, measurable state mappings
- $\{f_{i:i+1}\}_{i \in [\![0,N-1]\!]}$, measurable dynamics

such that

$$\theta_{\mathsf{t}_{i+1}}\big((h_{\mathsf{t}_i}, h_{\mathsf{t}_i+1:\mathsf{t}_{i+1}})\big) = f_{\mathsf{t}_i:\mathsf{t}_{i+1}}\big(\theta_{\mathsf{t}_i}(h_{\mathsf{t}_i}), h_{\mathsf{t}_i+1:\mathsf{t}_{i+1}}\big)$$

. . . that is compatible with kernels . . .

$$\begin{tikzcd}
\mathbb{H}_{\mathtt{t}_i} & \times & \mathbb{H}_{\mathtt{t}_i+1:\mathtt{t}} \arrow[r, "\rho_{\mathtt{t}:\mathtt{t}+1}"] \arrow[d, "I_d"] & \Delta(\mathbb{W}_{\mathtt{t}+1}) \\
\mathbb{X}_{\mathtt{t}_i} \arrow[from=1-1, d, "\theta_{\mathtt{t}_i}"] & \times & \mathbb{H}_{\mathtt{t}_i+1:\mathtt{t}} \arrow[ur, "\widetilde{\rho}_{\mathtt{t}:\mathtt{t}+1}"'] &
\end{tikzcd}$$

There exists a family $\{\widetilde{\rho}_{s-1:s}\}_{s\in[\![\mathtt{t}_i+1,\mathtt{t}_{i+1}]\!]}$ of reduced stochastic kernels such that

$$\widetilde{\rho}_{\mathtt{t}_i:\mathtt{t}_{i+1}} : \mathbb{H}_{\mathtt{t}_i} \to \Delta(\mathbb{W}_{\mathtt{t}_{i+1}})$$

$$\rho_{\mathtt{t}_i:\mathtt{t}_{i+1}}\big(\mathrm{d}w_{\mathtt{t}_i+1} \bigm| h_{\mathtt{t}_i}\big) = \widetilde{\rho}_{\mathtt{t}_i:\mathtt{t}_{i+1}}\big(\mathrm{d}w_{\mathtt{t}_i+1} \bigm| \theta_{\mathtt{t}_i}(h_{\mathtt{t}_i})\big)$$

$$\widetilde{\rho}_{\mathtt{t}-1:\mathtt{t}} : \mathbb{H}_{\mathtt{t}_i} \times \mathbb{H}_{\mathtt{t}_i+1:\mathtt{t}-1} \to \Delta(\mathbb{W}_{\mathtt{t}})$$

$$\rho_{\mathtt{t}-1:\mathtt{t}}\big(\mathrm{d}w_{\mathtt{t}} \bigm| h_{\mathtt{t}_i}, h_{\mathtt{t}_i+1:\mathtt{t}}\big) = \widetilde{\rho}_{\mathtt{t}-1:\mathtt{t}}\big(\mathrm{d}w_{\mathtt{t}} \bigm| \theta_{\mathtt{t}_i}(h_{\mathtt{t}_i}), h_{\mathtt{t}_i+1:\mathtt{t}-1}\big)$$

. . . then we can write a reduced Bellman operator

$$\begin{tikzcd}
\mathbb{L}^0_+(\mathbb{H}_{\mathtt{t}_{i+1}}, \mathcal{H}_{\mathtt{t}_{i+1}}) \arrow[r, "\mathcal{B}_{\mathtt{t}_{i+1}:\mathtt{t}_i}"] & \mathbb{L}^0_+(\mathbb{H}_{\mathtt{t}_i}, \mathcal{H}_{\mathtt{t}_i}) \\
\mathbb{L}^0_+(\mathbb{X}_{\mathtt{t}_{i+1}}, \mathcal{X}_{\mathtt{t}_{i+1}}) \arrow[u, "\theta^\star_{\mathtt{t}_{i+1}}"] \arrow[r, "\widetilde{\mathcal{B}}_{\mathtt{t}_{i+1}:\mathtt{t}_i}"] & \mathbb{L}^0_+(\mathbb{X}_{\mathtt{t}_i}, \mathcal{X}_{\mathtt{t}_i}) \arrow[u, "\theta^\star_{\mathtt{t}_i}"']
\end{tikzcd}$$

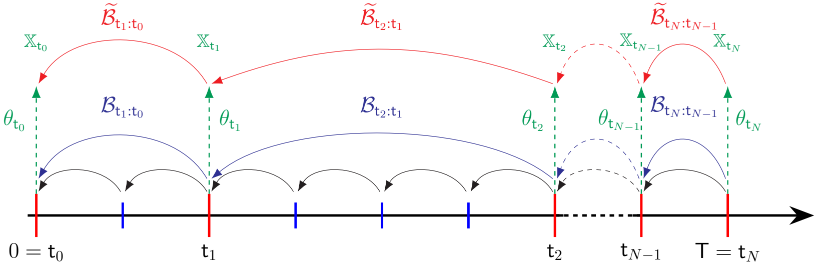
Consequently, there exists the family $\{\widetilde{\mathcal{B}}_{\mathtt{t}_{i+1}:\mathtt{t}_i}\}_{\imath\in[\![1,N-1]\!]}$ of reduced Bellman operators such that

$$\widetilde{\mathcal{B}}_{\mathtt{t}_{i+1}:\mathtt{t}_i} : \mathbb{L}^0_+(\mathbb{X}_{\mathtt{t}_{i+1}}, \mathcal{X}_{\mathtt{t}_{i+1}}) \to \mathbb{L}^0_+(\mathbb{X}_{\mathtt{t}_i}, \mathcal{X}_{\mathtt{t}_i})$$

$$\left(\widetilde{\mathcal{B}}_{\mathtt{t}_{i+1}:\mathtt{t}_i}\widetilde{\varphi}_{\mathtt{t}_{i+1}}\right) \circ \theta_{\mathtt{t}_i} = \mathcal{B}_{\mathtt{t}_{i+1}:\mathtt{t}_i}\left(\widetilde{\varphi}_{\mathtt{t}_{i+1}} \circ \theta_{\mathtt{t}_{i+1}}\right)$$

We can now write a Bellman operator across $(\mathsf{t}_i, \mathsf{t}_{i+1})$

$$
\begin{aligned}
\big(\widetilde{\mathcal{B}}_{\mathsf{t}_{i+1}:\mathsf{t}_i}\widetilde{\varphi}_{\mathsf{t}_{i+1}}\big)(x_{\mathsf{t}_i}) = \inf_{u_{\mathsf{t}_i}\in\mathbb{U}_{\mathsf{t}_i}} & \int_{\mathbb{W}_{\mathsf{t}_i+1}} \widetilde{\rho}_{\mathsf{t}_i:\mathsf{t}_i+1}\big(\mathrm{d}w_{\mathsf{t}_i+1} \,\big|\, x_{\mathsf{t}_i}\big) \\
& \inf_{u_{\mathsf{t}_i+1}\in\mathbb{U}_{\mathsf{t}_i+1}} \int_{\mathbb{W}_{\mathsf{t}_i+2}} \widetilde{\rho}_{\mathsf{t}_i+1:\mathsf{t}_i+2}\big(\mathrm{d}w_{\mathsf{t}_i+2} \,\big|\, x_{\mathsf{t}_i}, u_{\mathsf{t}_i}, w_{\mathsf{t}_i+1}\big) \quad \cdots \\
& \quad \inf_{u_{\mathsf{t}_{i+1}-1}\in\mathbb{U}_{\mathsf{t}_{i+1}-1}} \int_{\mathbb{W}_{\mathsf{t}_{i+1}}} \widetilde{\rho}_{\mathsf{t}_{i+1}-1:\mathsf{t}_{i+1}}\big(\mathrm{d}w_{\mathsf{t}_{i+1}} \,\big|\, x_{\mathsf{t}_i}, u_{\mathsf{t}_i}, w_{\mathsf{t}_i+1}, \dots, u_{\mathsf{t}_{i+1}-2}, w_{\mathsf{t}_{i+1}-1}\big) \\
& \qquad \widetilde{\varphi}_{\mathsf{t}_{i+1}}\Big(f_{\mathsf{t}_i:\mathsf{t}_{i+1}}\big(x_{\mathsf{t}_i}, u_{\mathsf{t}_i}, w_{\mathsf{t}_i+1}, \dots, u_{\mathsf{t}_{i+1}-1}, w_{\mathsf{t}_{i+1}}\big)\Big)
\end{aligned}
$$



Checkpoint

- Computing reduced Bellman operators does not produce computational gains
- In practice we can now
  - decompose the problem block-by-block
  - compute approximate value functions in the subset of instants
- We will now apply time-blocks decomposition to a two time scales problem

Outline of the presentation

1. Part I: Monthly crude oil procurement problem

1.1 Modeling of the crude oil procurement and formulation of an optimal control problem

1.2 Resolution methods

1.3 Numerical results

2. Part II: Time-blocks decomposition and the multi-months procurement problem

2.1 The multi-months procurement problem

2.2 Time-blocks decomposition

2.3 Two time scales optimization problem

2.4 Back to the procurement problem

We now consider two time scales

- A slow scale (e.g months)

$$\min \mathsf{S} = \underline{\mathsf{s}} \prec \cdots \prec \mathsf{s}^- \prec \mathsf{s} \prec \mathsf{s}^+ \prec \cdots \prec \bar{\mathsf{s}} = \max \mathsf{S}$$

where s^- is the predecessor to s

s^+ is the successor to s

- A fast scale (e.g weeks)

$$\min \mathsf{F} = \underline{\mathsf{f}} \prec \cdots \prec \mathsf{f}^- \prec \mathsf{f} \prec \mathsf{f}^+ \prec \cdots \prec \bar{\mathsf{f}} = \max \mathsf{F}$$

Two time scales setting

- Slow
  - $\{\mathbb{U}^{\mathsf{s}}_{\mathsf{s}}\}_{\mathsf{s}\in\overline{\mathsf{S}}\setminus\{\bar{\mathsf{s}}\}}$, slow scale decision measurable sets
  - $\{\mathbb{W}^{\mathsf{s}}_{\mathsf{s}}\}_{\mathsf{s}\in\mathsf{S}}$, slow scale uncertainty measurable sets
- Fast
  - $\{\mathbb{U}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}\}_{(\mathsf{s},\mathsf{f})\in\mathsf{S}\times(\mathsf{F}\setminus\{\bar{\mathsf{f}}\})}$, fast scale decision measurable sets
  - $\{\mathbb{W}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}\}_{(\mathsf{s},\mathsf{f})\in\mathsf{S}\times(\mathsf{F}\setminus\{\underline{\mathsf{f}}\})}$, fast scale uncertainty measurable sets
- States
  - $\{\mathbb{X}^{\mathsf{s}}_{\mathsf{s}}\}_{\mathsf{s}\in\overline{\mathsf{S}}}$, slow time scale state sets
    $\hookrightarrow$ with the dynamic $\mathcal{F}^{\mathsf{s}}_{\mathsf{s}} : \mathbb{X}^{\mathsf{s}}_{\mathsf{s}}\times\mathbb{U}^{\mathsf{s}}_{\mathsf{s}}\times\mathbb{W}^{\mathsf{s}}_{\mathsf{s}^+} \to \mathbb{X}^{\mathsf{sf}}_{(\mathsf{s}^+,\underline{\mathsf{f}})}$
  - $\{\mathbb{X}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}\}_{(\mathsf{s},\mathsf{f})\in\mathsf{S}\times(\mathsf{F}\setminus\{\bar{\mathsf{f}}\})}$, fast time scale state sets
    $\hookrightarrow$ with the dynamic $\mathcal{F}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})} : \mathbb{X}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}\times\mathbb{U}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}\times\mathbb{W}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})^+} \to \mathbb{X}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})^+}$

We introduce criterion and kernels

We consider the criterion

$$j(h_{(\bar{\mathsf{s}},\bar{\mathsf{f}})}) = \sum_{\mathsf{s}\in\mathsf{S}} \underbrace{\Lambda_{\mathsf{s}^-}\Big(\overbrace{x^{\mathsf{s}}_{\mathsf{s}^-}, u_{\mathsf{s}^-}, w^{\mathsf{s}}_{\mathsf{s}}}^{\text{slow scale variables}}, \overbrace{\{x^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}, u^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}, w^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})^+}\}_{\mathsf{f}\in\mathsf{F}\setminus\{\bar{\mathsf{f}}\}}}^{\text{fast scale variables in } [\mathsf{s}^-,\mathsf{s}[}\Big)}_{\text{cost for the period } [\mathsf{s}^-,\mathsf{s}[} + \underbrace{\Lambda_{\bar{\mathsf{s}}}\big(x^{\mathsf{s}}_{\bar{\mathsf{s}}}\big)}_{\text{final cost}}$$

We consider 2 types of stochastic kernels

that ensure block-wise independence:

- Constant slow scale kernels

$$\rho^{\mathsf{s}}_{\mathsf{s}:\mathsf{s}^+} \in \Delta(\mathbb{W}^{\mathsf{s}}_{\mathsf{s}^+})$$

- Fast scale stochastic kernels

$$\rho^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f}):(\mathsf{s},\mathsf{f})^+} : \mathbb{W}^{\mathsf{s}}_{\mathsf{s}} \times \underbrace{\prod_{\mathsf{f}'=\underline{\mathsf{f}}^+}^{\mathsf{f}} \mathbb{W}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f}')}}_{\text{interval } [\mathsf{s}^-,\mathsf{s}[} \longrightarrow \Delta(\mathbb{W}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})^+})$$

We represent both time scales on a unified timeline

$$\begin{tikzcd}
\underline{s}^- \arrow[rr] \arrow[d] \arrow[rrrrrrr, bend left, dotted, no head, "\overline{\mathsf{S}}"] & & \underline{s} \arrow[rr] \arrow[d] \arrow[rrrrr, bend left, dotted, no head, "\mathsf{S}"] & & \underline{s}^+ \arrow[r, "\cdots" description] \arrow[d] & \bar{s}^- \arrow[rr] \arrow[d] & & \bar{s} \arrow[d] \\
(\underline{s}^-, \bar{f}) \arrow[r] \arrow[rrrrrrr, bend right, dotted, no head, "\overline{\mathsf{S}\times\mathsf{F}}"'] & (\underline{s}, \underline{f}) \arrow[r, dashed] \arrow[r, bend right, dotted, no head, "\{\underline{s}\}\times\mathsf{F}"'] \arrow[rrrrrr, bend right, dotted, no head, "\mathsf{S}\times\mathsf{F}"'] & (\underline{s}, \bar{f}) \arrow[r] & (\underline{s}^+, \underline{f}) \arrow[r, dashed] \arrow[r, bend right, dotted, no head, "\{\underline{s}^+\}\times\mathsf{F}"'] & (\underline{s}^+, \bar{f}) \arrow[r, dashed] & (\bar{s}^-, \bar{f}) \arrow[r] & (\bar{s}, \underline{f}) \arrow[r, dashed] \arrow[r, bend right, dotted, no head, "\{\bar{s}\}\times\mathsf{F}"'] & (\bar{s}, \bar{f})
\end{tikzcd}$$

where we denote the successor of (s, f) by

$$(\mathsf{s}, \mathsf{f})^+ = \begin{cases} (\mathsf{s}, \mathsf{f}^+) & \text{if } \mathsf{f} \neq \bar{\mathsf{f}} \\ (\mathsf{s}^+, \underline{\mathsf{f}}) & \text{if } \mathsf{f} = \bar{\mathsf{f}} \end{cases}$$

We represent both time scales on a unified timeline

$$\mathbb{X}_{(s,f)} = \begin{cases} \mathbb{X}^{s}_{s} & \text{if } f = \bar{f} \\ \mathbb{X}^{sf}_{(s,f)} & \text{if } f \neq \bar{f} \end{cases}, \quad \forall (s,f) \in \overline{S \times F}$$

$$\mathbb{U}_{(s,f)} = \begin{cases} \mathbb{U}^{s}_{s} & \text{if } f = \bar{f} \\ \mathbb{U}^{sf}_{(s,f)} & \text{if } f \neq \bar{f} \end{cases}, \quad \forall (s,f) \in \overline{S \times F} \setminus \{(\bar{s},\bar{f})\}$$

$$\mathbb{W}_{(s,f)} = \begin{cases} \mathbb{W}^{s}_{s} & \text{if } f = \underline{f} \\ \mathbb{W}^{sf}_{(s,f)} & \text{if } f \neq \underline{f} \end{cases}, \quad \forall (s,f) \in S \times F$$

$$\mathbb{W}_{(\underline{s}^{-},\bar{f})} = \mathbb{X}_{(\underline{s}^{-},\bar{f})}$$

$$\mathcal{F}_{(s,f)} = \begin{cases} \mathcal{F}^{s}_{s} & \text{if } f = \bar{f} \\ \mathcal{F}^{sf}_{(s,f)} & \text{if } f \neq \bar{f} \end{cases}, \quad \forall (s,f) \in \overline{S \times F} \setminus \{(\bar{s},\bar{f})\}$$

We can write a dynamic programming equation at the slow time scale

We perform a time-block decomposition
on the subset of instants $\Big\{(\mathsf{s},\bar{\mathsf{f}})\Big\}_{\mathsf{s}\in\mathsf{S}\cup\{\underline{\mathsf{s}}^-\}} \subset \overline{\mathsf{S}\times\mathsf{F}}$

$$
\begin{aligned}
V_{\mathsf{s}}(x^{\mathsf{s}}_{\mathsf{s}}) = \inf_{u_{\mathsf{s}}\in\mathbb{U}^{\mathsf{s}}_{\mathsf{s}}} \int_{\mathbb{W}^{\mathsf{s}}_{\mathsf{s}^+}} & \rho^{\mathsf{s}}_{\mathsf{s}:\mathsf{s}^+}(\,\mathrm{d}w^{\mathsf{s}}_{\mathsf{s}^+}) \\
& \inf_{u^{\mathsf{sf}}_{(\mathsf{s}^+,\underline{\mathsf{f}})}\in\mathbb{U}^{\mathsf{sf}}_{(\mathsf{s}^+,\underline{\mathsf{f}})}} \int_{\mathbb{W}^{\mathsf{sf}}_{(\mathsf{s}^+,\underline{\mathsf{f}}^+)}} \rho^{\mathsf{sf}}_{(\mathsf{s}^+,\underline{\mathsf{f}}):(\mathsf{s}^+,\underline{\mathsf{f}}^+)}(\,\mathrm{d}w^{\mathsf{sf}}_{(\mathsf{s}^+,\underline{\mathsf{f}}^+)} \mid w^{\mathsf{s}}_{\mathsf{s}^+}) \;\cdots \\
& \quad \inf_{u^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}}^-)}\in\mathbb{U}^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}}^-)}} \int_{\mathbb{W}^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}})}} \rho^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}}^-):(\mathsf{s}^+,\bar{\mathsf{f}})}(\,\mathrm{d}w^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}})} \mid w^{\mathsf{s}}_{\mathsf{s}^+}, w^{\mathsf{sf}}_{(\mathsf{s}^+,\underline{\mathsf{f}}^+)}, \cdots, w^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}}^-)}) \\
& \quad\quad \Big(\Lambda_{\mathsf{s}}(x^{\mathsf{s}}_{\mathsf{s}}, u_{\mathsf{s}}, w^{\mathsf{s}}_{\mathsf{s}^+}, \ldots, u^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}}^-)}, w^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}})}) \\
& \quad\quad\quad + V_{\mathsf{s}^+}\big(\mathcal{F}_{\mathsf{s}:\mathsf{s}^+}(x^{\mathsf{s}}_{\mathsf{s}}, u_{\mathsf{s}}, w^{\mathsf{s}}_{\mathsf{s}^+}, \ldots, u^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}}^-)}, w^{\mathsf{sf}}_{(\mathsf{s}^+,\bar{\mathsf{f}})})\big)\Big)
\end{aligned}
$$

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and the multi-months procurement problem**

2.1 The multi-months procurement problem

2.2 Time-blocks decomposition

2.3 Two time scales optimization problem

2.4 Back to the procurement problem

We recall the multi-months procurement problem

$$\min_{\substack{\{\boldsymbol{b}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})}\}_{((\mathrm{m},\mathrm{w}),\mathrm{m}')\in\mathfrak{P}} \\ \{\boldsymbol{v}_{\mathrm{m}},\boldsymbol{r}_{\mathrm{m}}\}_{\mathrm{m}\in\mathrm{M}}}} \mathbb{E}\Bigg[\sum_{(\mathrm{m},\mathrm{w})\in\mathrm{M}\times\mathrm{W}}\Bigg(\sum_{\mathrm{m}'\in(\mathrm{m},\mathrm{w})\mathfrak{P}}\Omega^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})}\big(\boldsymbol{d}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})},\boldsymbol{b}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})},\boldsymbol{w}_{(\mathrm{m},\mathrm{w})}\big)\Bigg)+\sum_{\mathrm{m}\in\mathrm{M}}\Psi_{\mathrm{m}}\big(\boldsymbol{s}_{\mathrm{m}},\boldsymbol{d}^{\mathrm{m}}_{(\mathrm{m},\underline{\mathrm{w}})},\boldsymbol{v}_{\mathrm{m}},\boldsymbol{r}_{\mathrm{m}},\boldsymbol{p}_{\mathrm{m}}\big)\Bigg]$$

s.t

$$\begin{aligned}
&\boldsymbol{b}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})}\in\mathcal{B}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})}\ , \quad \forall\mathrm{m}'\in\mathrm{M}\ , \quad \forall(\mathrm{m},\mathrm{w})\in\mathfrak{P}\mathrm{m}'\\
&\boldsymbol{v}_{\mathrm{m}}\in\mathcal{V}_{\mathrm{m}}\ , \quad \forall\mathrm{m}\in\mathrm{M}\\
&\boldsymbol{r}_{\mathrm{m}}\in\mathcal{R}_{\mathrm{m}}\ , \quad \forall\mathrm{m}\in\mathrm{M}
\end{aligned}$$

constraints on decisions

$$\begin{aligned}
&\boldsymbol{d}^{\mathrm{m}}_{(\mathrm{m},\underline{\mathrm{w}})}\in\mathcal{D}^{\mathrm{m}}\ , \quad \forall\mathrm{m}\in\mathrm{M}\\
&\boldsymbol{s}_{\mathrm{m}}\in\mathbb{S}_{\mathrm{m}}\ , \quad \forall\mathrm{m}\in\mathrm{M}
\end{aligned}$$

constraints on stocks

$$\begin{aligned}
&\boldsymbol{d}^{\mathrm{m}}_{\min\mathfrak{P}\mathrm{m}}=0\ , \quad \forall\mathrm{m}\in\mathrm{M}\\
&\boldsymbol{d}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})^+}=\mathcal{F}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})}\big(\boldsymbol{d}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})},\boldsymbol{b}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})}\big)\ , \quad \forall((\mathrm{m},\mathrm{w}),\mathrm{m}')\in\overline{\mathfrak{P}}\\
&\boldsymbol{s}_{\mathrm{m}^+}=\mathcal{F}_{\mathrm{m}}\big(\boldsymbol{s}_{\mathrm{m}},\boldsymbol{d}^{\mathrm{m}}_{(\mathrm{m},\underline{\mathrm{w}})},\boldsymbol{v}_{\mathrm{m}}\big)\ , \quad \forall\mathrm{m}\in\mathrm{M}
\end{aligned}$$

dynamics on the stocks

$$\begin{aligned}
&\sigma\big(\boldsymbol{b}^{\mathrm{m}'}_{(\mathrm{m},\mathrm{w})}\big)\subset\sigma\big(\{\boldsymbol{p}_{\mathrm{m}''}\}_{\mathrm{m}''\leq\mathrm{m}},\{\boldsymbol{w}_{(\mathrm{m}'',\mathrm{w}'')}\}_{(\mathrm{m}'',\mathrm{w}'')\leq(\mathrm{m},\mathrm{w})}\big)\\
&\sigma\big(\boldsymbol{v}_{\mathrm{m}},\boldsymbol{r}_{\mathrm{m}}\big)\subset\sigma\big(\{\boldsymbol{p}_{\mathrm{m}''}\}_{\mathrm{m}''\leq\mathrm{m}},\{\boldsymbol{w}_{(\mathrm{m}'',\mathrm{w}'')}\}_{(\mathrm{m}'',\mathrm{w}'')\leq(\mathrm{m},\underline{\mathrm{w}})}\big)
\end{aligned}$$

nonanticipativity constraints

Translation of the two time scales to the procurement

Notations	Crude oil procurement
S	set of months during which we manage the refinery;
F	set of weeks in each month;
$\mathbb{U}^{s}_{s}$	set of crude oil consumptions during the month s^{+}
$\mathbb{W}^{s}_{s^{+}}$	set of product prices for the month s^{+}
$\mathbb{U}^{sf}_{(s,f)}$	set of crude shipments purchased in week (s, f)
$\mathbb{W}^{sf}_{(s,f)^{+}}$	set of crude oil prices in week (s, f)
$\mathcal{F}^{sf}_{(s,f)}$	dynamic accumulation of shipments purchased in (s, f)
$\mathcal{F}^{s}_{s}$	dynamics of the stocks inside the refinery between s and s^{+}
Λ_{s}	operational costs during the month s (crude oil purchases during s - earnings from production)

The general procurement problem fits a two time scales problem

$$\inf \mathbb{E}\Big[\sum_{\mathsf{s}\in\mathsf{S}} \Lambda_{\mathsf{s}}\big(X^{\mathsf{s}}_{\mathsf{s}^-}, U^{\mathsf{s}}_{\mathsf{s}^-}, W_{\mathsf{s}}, \{X^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}, U^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}, W^{\mathsf{f}}_{(\mathsf{s},\mathsf{f})^+}\}_{\mathsf{f}\in\mathsf{F}\setminus\{\bar{\mathsf{f}}\}}\big) + \Lambda_{\bar{\mathsf{s}}}\big(X^{\mathsf{s}}_{\bar{\mathsf{s}}}\big)\Big]$$

$$s.t.\ U^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})} \in \mathbb{U}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}\ ,\ \ \forall (\mathsf{s},\mathsf{f}) \in \overline{\mathsf{S}\times\mathsf{F}}$$

$$U^{\mathsf{s}}_{\mathsf{s}} \in \mathbb{U}^{\mathsf{s}}_{\mathsf{s}}\ ,\ \ \forall \mathsf{s} \in \overline{\mathsf{S}}$$

$$X^{\mathsf{s}}_{\mathsf{s}} \in \mathbb{X}^{\mathsf{s}}_{\mathsf{s}}\ ,\ \ \forall \mathsf{s} \in \overline{\mathsf{S}}$$

$$X^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})} \in \mathbb{X}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}\ ,\ \ \forall (\mathsf{s},\mathsf{f}) \in \overline{\mathsf{S}\times\mathsf{F}}$$

$$X^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})^+} = \mathcal{F}^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}\big(X^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}, U^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}, W^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})^+}\big)$$

$$X^{\mathsf{s}}_{\mathsf{s}^+} = \mathcal{F}^{\mathsf{s}}_{\mathsf{s}}\big(X^{\mathsf{s}}_{\mathsf{s}}, U^{\mathsf{s}}_{\mathsf{s}}, W^{\mathsf{s}}_{\mathsf{s}^+}\big)\ ,\ \ \forall \mathsf{s} \in \mathsf{S}\setminus\{\bar{\mathsf{s}}\}$$

$$\sigma\big(U^{\mathsf{sf}}_{(\mathsf{s},\mathsf{f})}\big) \subset \sigma\big(\{W^{\mathsf{s}}_{\mathsf{s}'}\}_{\mathsf{s}'\prec\mathsf{s}}, \{W^{\mathsf{sf}}_{(\mathsf{s}',\mathsf{f}')}\}_{(\mathsf{s}',\mathsf{f}')\prec(\mathsf{s},\mathsf{f})}\big)$$

$$\sigma(U^{\mathsf{s}}_{\mathsf{s}}) \subset \sigma(\{W^{\mathsf{s}}_{\mathsf{s}}\}_{\mathsf{s}'\prec\mathsf{s}}, \{W^{\mathsf{sf}}_{(\mathsf{s}',\mathsf{f}')}\}_{(\mathsf{s}',\mathsf{f}')\prec(\mathsf{s},\underline{\mathsf{f}})})$$

Conclusion and outlooks on the multi-months procurement problem

- Given a value function V_{s} and a state $x^{\mathsf{s}}_{\mathsf{s}-}$,
  computing $V_{\mathsf{s}-}(x^{\mathsf{s}}_{\mathsf{s}-}) \approx$ solving a monthly problem
- Two time scales decomposition assumes
  month-wise independence of the noises
  - $\hookrightarrow$ crude oil prices inside a month are time-dependent
- Various methods can be used to solve the problem
  inside each month
  - $\hookrightarrow$ adapt policies from Part I to approximate value functions

The end

- We built a model for the crude oil procurement that models uncertainties and delivery delays

- Multi-scenarios-based policies showed promising results on the monthly procurement problem

- We developed a framework to decompose two time scales problems at the slow scale, without independent fast scale noises

- Next: Adapt the policies from Part I to the multi-months problem