Energy derivatives – Optimisation & Pricing

February 2015

Arnaud de Latour, arnaud.de-latour@edf.fr

Outline

- **Energy derivatives modelling**
  - A typical energy portolio
  - Market modelling of some energy derivatives :

    Swaps, power plants, storage assets, load-curve contracts, …

- **Some pricing methods will be detailed for…**
  - Spread options
  - Swing options

ENERGY DERIVATIVES

- **A typical energy portfolio**
- Forwards, Swaps and Spreads
- Power plant modelling, Spread options pricing
- Gas storage modelling
- Swing options pricing
- Load curve contract modelling

Energy derivatives

Risk context and objectives

- Energy/electric utilities are exposed to a whole set of risk factors :
  - Energy market risk : electricity prices, fuel prices (coal, crude, gas, …), emission prices, for both forward and spot prices
  - Financial risk : currency, interest rates, counterpart risk
  - Volume risk : industrial/technical risk (outages, inflows), physical risk (network), commercial risk (consumption and market shares), weather risk (temperature, cloud cover, …)
  - Regulatory risk (politics and regulation), economic risk
  - Climatic risk, natural disasters

- The challenges in this context are the following :
  - Modelling realistic prices and their dependencies
  - Pricing forwards, derivatives components of the portfolio
  - Assessing risks and hedging structured products (computing Greeks) and more globally the whole portfolio
  - Designing models for a risk management system for a utility exposed to those risk factors…

Energy derivatives

A typical energy portfolio

- Like any other commodity market, energy markets have their options on quoted futures.

  But, the most challenging problems comes from the pricing, hedging and structuring of exotic tradable products linked to physical assets : can be called real derivatives.

- Physical assets are modelled (and valuate) by using the real option theory.
  - When traditional deterministic methods fail to accurately capture the economic value of physical assets in a competitive energy market (cf. extrinsic value)
  - The real option valuation framework borrows the idea from classical financial option pricing theory and views a real asset or investment project as an option on the underlying cash flows.

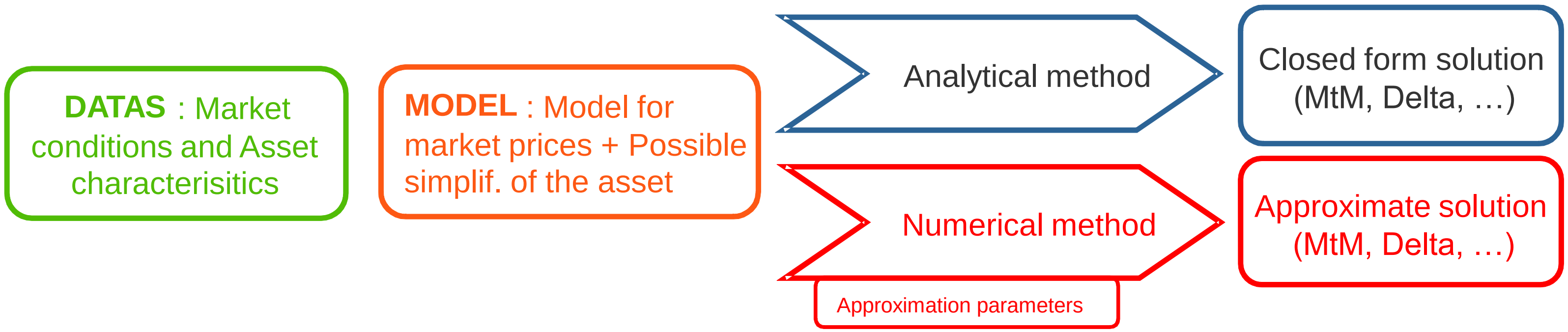
- Energy derivatives can be classified being “physical” of “financial” and regarding their complexity : more or less difficult task for pricing (assets market-dependent, “strategy”-dependent, etc.)

PHYSICAL				FINANCIAL	
Standard	**Exotic**		**Structured (Assets)**	**Standard**	**Exotic**
	Price-based	***Volumetric***			
Future	Asian option	Swing option	Take-or-Pay contract	Future	Asian option
Forward	American option	Load serving contract	Power plant	Forward	American option
Swap	Swaption	Interruptible contract	Thermal asset	Swap	Swaption
Spread	Spread option		Hydraulic asset	Spread	Spread option
European option	Tolling agreement		Storage facilities	European option	
	Transmission contract				

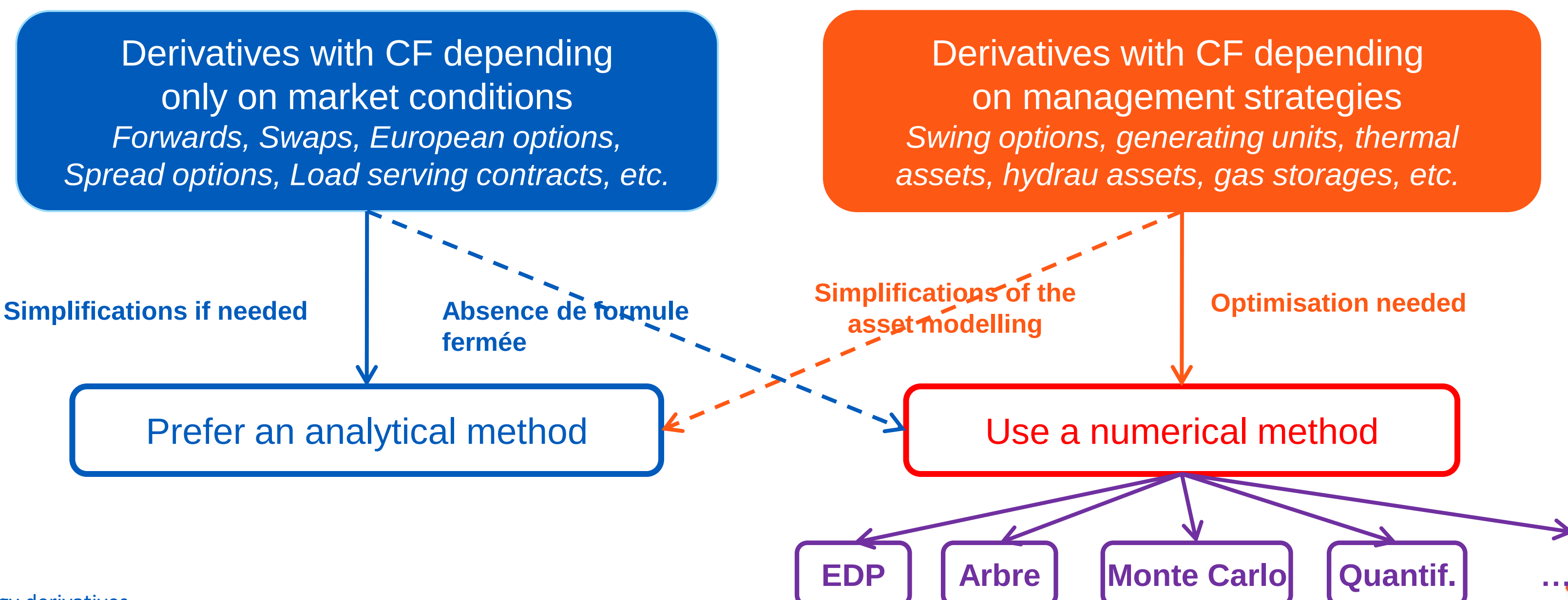
Energy derivatives

Analytical or numerical methods for pricing derivatives?

- For pricing a derivatives, one can have the choice between an analytical method or a numerical one.



- One has to make a trade-off for choosing an appropriate method for depending on the derivatives.



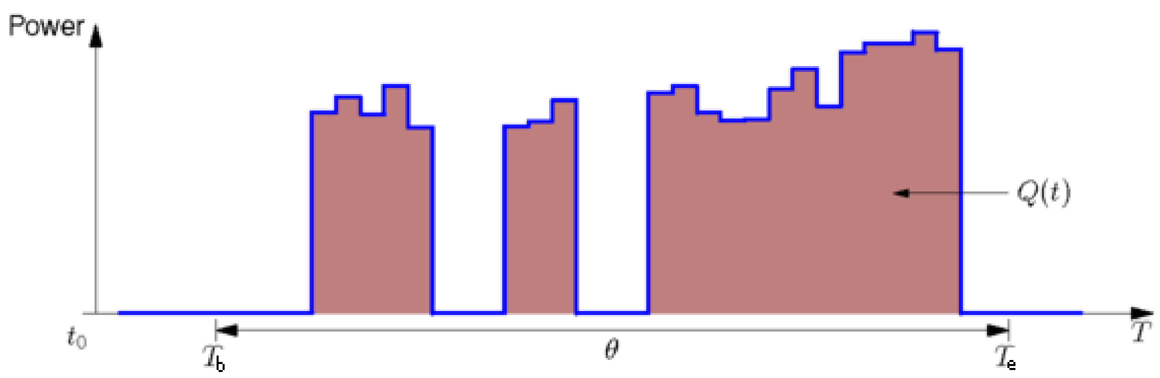
ENERGY DERIVATIVES

- A typical energy portfolio
- **Forwards, Swaps and Spreads**
- Power plant modelling, Spread options pricing
- Gas storage modelling
- Swing options pricing
- Load curve contract modelling

Forwards in energy markets

Forwards can have a non standard delivery profile.

- Futures and forward contracts in energy markets
  - Firm agreement for purchase or sale of an underlying commodity over a certain future period in time, to a price fixed in advance, when the contract is made
  - ⇨ No cash flow at the transaction date
  - Main characterisitics are the delivery period and the power delivered (e.g. Base or Peak)
  - A forward has a given "profile" :



- Buying an energy forward ⇔ Firm energy purchase contract over $[T_b, T_e]$
- Selling an energy forward ⇔ Firm energy sale contract over $[T_b, T_e]$

Forwards and Swaps in energy markets

Swaps are a natural generalization of forward products

- What are the cashflows generated by a forward contract with delivery on *[T_b, T_e]* and profile *Q* ? Deduce the fair fixed for this forward at time *t*.

	Transaction date t	Delivery period $[T_b, T_e]$
Buying a forward contract $F(t, T_b, T_e)$	0 (No cash)	For any $u \in [T_b, T_e] : Q_u \left(S_u - F(t, T_b, T_e)\right)$ = Proportion of energy to the Spot − Forward price

Forwards and Swaps in energy markets

Swaps are a natural generalization of forward products

- What are the cashflows generated by a forward contract with delivery on *$[T_b, T_e]$* and profile *Q* ? Deduce the fair fixed for this forward at time *t*.

	Transaction date t	Delivery period $[T_b, T_e]$
Buying a forward contract $F(t, T_b, T_e)$	0 (No cash)	For any $u \in [T_b, T_e] : Q_u \left(S_u - F(t, T_b, T_e)\right)$ $=$ Proportion of energy to the Spot $-$ Forward price

$$F(t, T_b, T_e) = \left(\sum_{u \in [T_b, T_e]} Q_u \mathbb{E}_t [S_u] \right) / \left(\sum_{u \in [T_b, T_e]} Q_u \right)$$

- Swaps contracts are a natural generalization of forward products (forward ⇔ one-period swap), similar to swaps in financial markets
  - Known as Contract for Differences (CfD) or Fixed for Floating forward contracts
  - Swaps are flexible, OTC, easily customizable transactions
  - Vanilla Swap (Contract for Differences) :

    Payment of a fixed predetermined price for a set of payments (financial) or deliveries (physical) at a sequence of dates in the future
  - Various possibilities exist for the fixed and floating payments/deliveries…

Swaps in energy markets

Some practice…

- In July 2013, a local gas distribution company wants to secure the base load supply of gas for the coming winter (October 2013 to March 2014) by a forward agreement.
- The winter demand is 1 MWh/day. The forward prices observed on the market are the following :

Gas Monthly Forward prices (in €/MWh)					
2013-10	**2013-11**	**2013-12**	**2014-01**	**2014-02**	**2014-03**
26.5	28	30.5	33	31.5	30.5

- Question : What is the fixed price (in €/MWh) that the distribution company should agree to pay for satisfying the winter gas supply ?

Swaps in energy markets

Some practice…

- In July 2013, a local gas distribution company wants to secure the base load supply of gas for the coming winter (October 2013 to March 2014) by a forward agreement.
- The winter demand is 1 MWh/day. The forward prices observed on the market are the following :

Gas Monthly Forward prices (in €/MWh)					
2013-10	**2013-11**	**2013-12**	**2014-01**	**2014-02**	**2014-03**
26.5	28	30.5	33	31.5	30.5

- Question : What is the fixed price (in €/MWh) that the distribution company should agree to pay for satisfying the winter gas supply ?

- Swap price in €/MWh = $\sum_{\text{month } m} (\text{Volume}_m \text{ Price}_m) / (\sum \text{Volume}_{\text{month } m})$

  Swap price = (31 * 26.5 + 30 * 28 + … + 31 * 30.5) / (31 + 30 + … + 31)

  Swap price = 29.9 €/MWh

Spreads in energy markets

Spreads are one of the most useful instrument in the “energy world”.

- Spread : price differential between two commodities viewed as 2 points of the energy price system
- Spreads can be used to describe power plants, refineries, storage facilities, etc.

- There are four classes of Spreads :
  1. Intra-commodity spread

     Ex. Spread on oils with different qualities
  2. Inter-commodity spread : price differential between two different but related commodities
     - Between two operational inputs or two operational outputs or between inputs and outputs (processing spread)
     - Crack Spread : gasoline or heating oil (refined products) versus crude oil (input)
     - Spark Spread : electricity versus a primary fuel (gas, oil, fuel oil, uranium)
     - Dark Spread : electricity versus coal
     - Clean XY Spread : X versus Y compensated for the price of CO2
  3. Time or calendar spread

     Ex. Summer 2014 versus Winter 2014
  4. Geographic spread : difference between prices of a same product in two different locations

Spark and Dark spreads…

Practical application

- We consider a gas-fired power plant with a 50% efficiency and emission costs of 0.20 ton CO_2/MWh and a coal-fired power plant with a 35% efficiency and emission costs of 0.30 ton CO_2/MWh.
- We assume following market conditions :

Prices	
Electricity	50 €/MWh
Gas	30 €/MWh
Coal	15 €/MWh
CO_2	6 €/ton

a) Compute the Clean Spark and Clean Dark spreads. What do you deduce ?

b) In these gas and coal market conditions, what is the threshold CO_2 price conducing to switch from coal to gas power generation ?

Spark and Dark spreads…

Practical application

- We consider a gas-fired power plant with a 50% efficiency and emission costs of 0.20 ton CO_2/MWh and a coal-fired power plant with a 35% efficiency and emission costs of 0.30 ton CO_2/MWh.
- We assume following market conditions :

Prices	
Electricity	50 €/MWh
Gas	30 €/MWh
Coal	15 €/MWh
CO_2	6 €/ton

a) Compute the Clean Spark and Clean Dark spreads. What do you deduce ?

CSS = 50 – (30 + 0.20*6) / 0.5 = - 12.4 €/MWh

CDS = 50 – (15 + 0.30*6) / 0.35 = 2 €/MWh

The gas-fired power plant is not profitable. We should better use coal-fired power plants.

b) In these gas and coal market conditions, what is the threshold CO_2 price conducing to switch from coal to gas power generation ?

Spark and Dark spreads…

Practical application

- We consider a gas-fired power plant with a 50% efficiency and emission costs of 0.20 ton CO_2/MWh and a coal-fired power plant with a 35% efficiency and emission costs of 0.30 ton CO_2/MWh.
- We assume following market conditions :

Prices	
Electricity	50 €/MWh
Gas	30 €/MWh
Coal	15 €/MWh
CO_2	6 €/ton

a) Compute the Clean Spark and Clean Dark spreads. What do you deduce ?

CSS = 50 – (30 + 0.20*6) / 0.5 = - 12.4 €/MWh

CDS = 50 – (15 + 0.30*6) / 0.35 = 2 €/MWh

The gas-fired power plant is not profitable. We should better use coal-fired power plants.

b) In these gas and coal market conditions, what is the threshold CO_2 price conducing to switch from coal to gas power generation ?

CO_2 price p* such that : 50 – (30 + 0.20 **p***) / 0.5 = 50 – (15 + 0.30 **p***) / 0.35

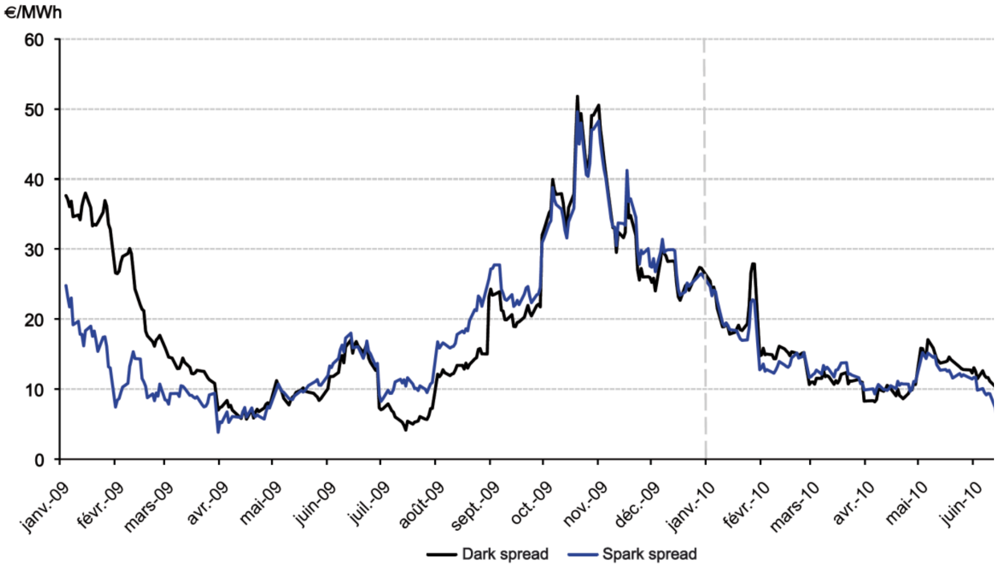
⇨ p* = 37.5 €/ton

Spark and Dark spreads…

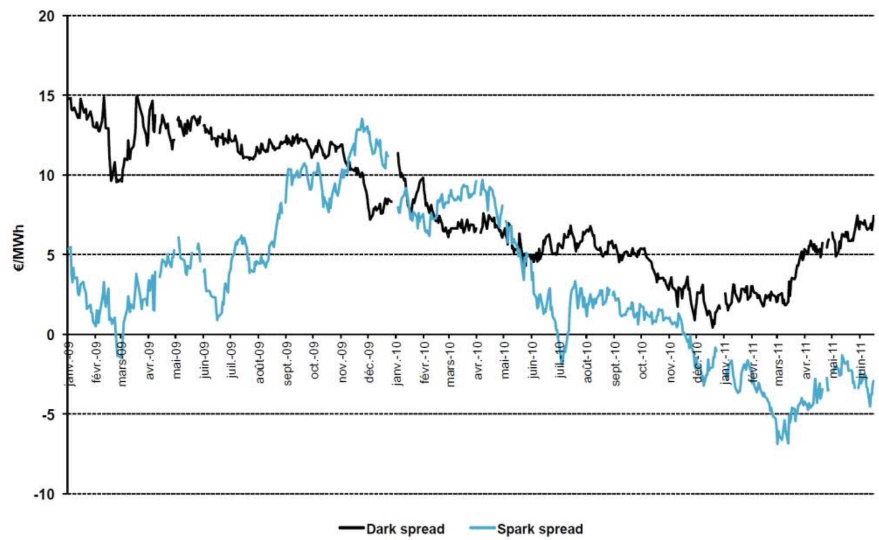
Illustrate the theoretical benefits of using gas-fired or coal-fired plants for power generation

- Spark and Dark spreads strongly correlated ⇨ No-arbitrage condition between primary fuels
- Decoupling of these two Spreads ⇨ Loss of competitiveness of one of the two means of production

- Question : What do you deduce from plots below ?



French Clean Dark and Clean Spark Spreads based on MAH forward prices (CRE, 2010)



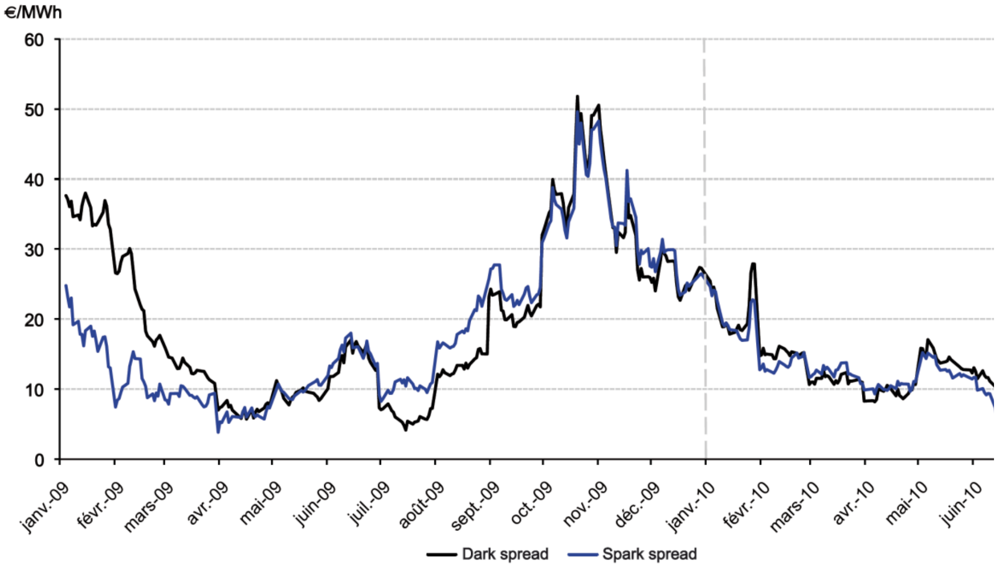
German Clean Dark and Clean Spark Spreads based on YAH forward prices (CRE, 2011)

Spark and Dark spreads…

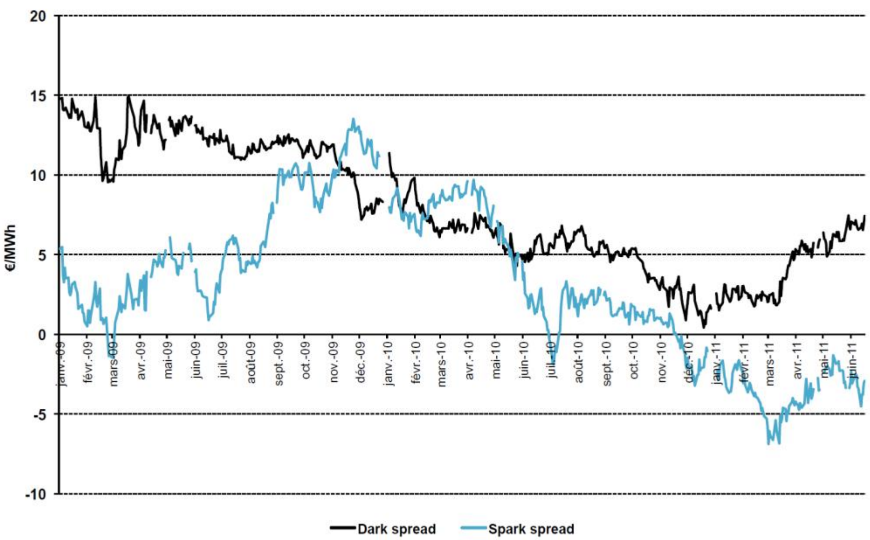
Illustrate the theoretical benefits of using gas-fired or coal-fired plants for power generation

- Spark and Dark spreads strongly correlated ⇨ No-arbitrage condition between primary fuels
- Decoupling of these two Spreads ⇨ Loss of competitiveness of one of the two means of production

- Question : What do you deduce from plots below ?
  - Beginning 2009 in France : coal-fired plants more rentable
  - Germany from June 2011 : anticipation of future use of coal-fired thermal power plant
  - Negative Spark Spreads due to an higher level of gas prices



French Clean Dark and Clean Spark Spreads based on MAH forward prices (CRE, 2010)



German Clean Dark and Clean Spark Spreads based on YAH forward prices (CRE, 2011)

Transmission contracts

Valuation as commodity derivatives

- A transmission contract corresponds to the right to tranfer a commodity from one point of the network to another point.

- Question : How would you model such kind of contract?

Transmission contracts

Valuation as commodity derivatives

- A transmission contract corresponds to the right to tranfer a commodity from one point of the network to another point.

- Question : How would you model such kind of contract?

- This contract can be modelled as a geographic Spread option : option on the Spread A-B.

$$\mathrm{MtM} = \mathbb{E}\left[\sum_{t\leq T}\left(F_t^A - F_t^B - K\right)^+\right]$$

ENERGY DERIVATIVES

- A typical energy portfolio
- Forwards, Swaps and Spreads
- **Power plant modelling, Spread options pricing**
- Gas storage modelling
- Swing options pricing
- Load curve contract modelling

Power plants

Power generating units can be viewed as derivatives.

- First approximation of a power plant market :

  Pricing as a *Strip* of calls on the spread between the electricity price and its fuel price

$$\mathrm{MtM} = \mathbb{E}\left[\sum_{t \leq T} \left(F_t^{elec} - hF_t^{fuel}\right)^+\right]$$

in which *[0, T]* is the period of time considered and *h* is the *heat rate* of the power plant

- Depending on the fuel, this leads to so-called spark spread or dark spread options.
- If log-normal dynamics are assumed for commodity prices, Margrabe closed formula (1978) for exchange options applies and provides value and Greeks.

- Spread options can be used to model a large class of real derivatives :
  - Other inter-commodity spread options : tolling agreements, raffineries, etc.
  - Calendar spread options : storage modelling, water reservoir
  - Spread options on futures prices and spot prices are both importants

Power plants

More complex pricing methods are required to model more realistic power plants.

- However, this first approximation is not totally satisfactory… Why ?

- Taking into account the CO_2 emission costs and additional operation costs :

  Pricing as a *Strip* of Clean fuel spreads

$$\mathrm{MtM} = \mathbb{E}\left[\sum_{t \leq T} \left(F_t^{elec} - hF_t^{fuel} - gF_t^{\mathrm{CO_2}} - c\right)^+\right]$$

  in which *g* is the proportional emission cost and *c* some operation costs (as start-up costs)

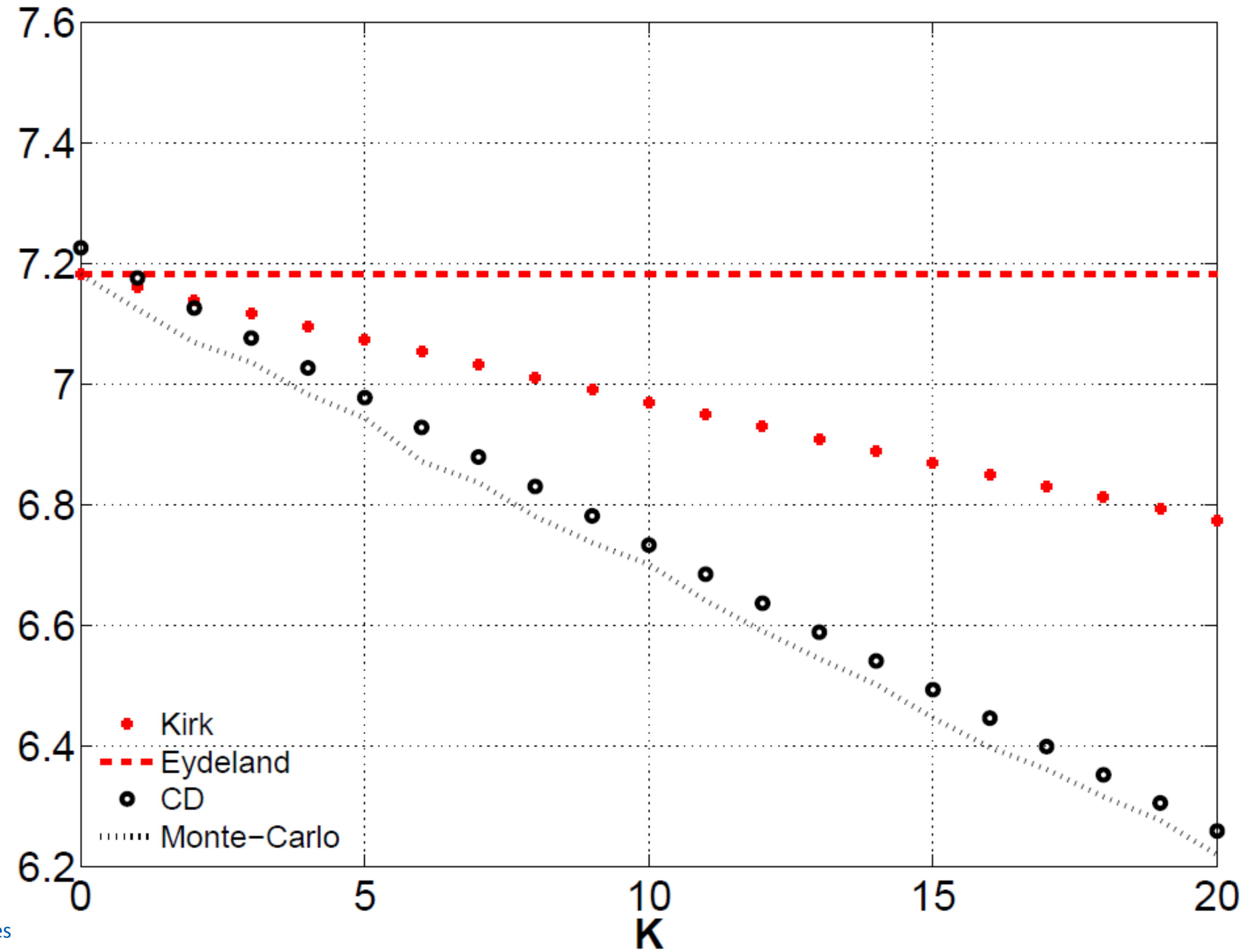
- Here, we still neglect dynamic operational constraints : ramp-up time (when they have just been started/turned off, power plants cannot be shut down/started up immediately), minimum power (arbitrage between stopping/restarting or running at Pmin), …
- Additional constraints ⇨ Lower market value

- Margrabe's formula does not apply anymore…
  - Apart from previous simple case, no closed-form solution is known.
  - A first approximation consists in assuming the equivalent fuel cost log-normal…
  - Analytical methods rely on approximations, otherwise numerical approches (Monte-Carlo, …) need to be use to solve such a stochastic control problem.

Spread options pricing

No closed formula exist for Spread options with non-zero strike.

- Spread option value decreases with substitution between fixed cost and initial input price.
- Pricing with different methods : approximations by Kirk, Eydeland and Carmona & Durrleman



Spread options pricing

Margrable formula for a simple fuel spread options

- Consider the fuel spread option with maturity *T* priced at time *t* :

$$\mathrm{MtM} = \mathbb{E}_t\left[\left(F_T^e - hF_T^f\right)^+\right]$$

- Assume log-normal correlated dynamics for the electricity and fuel prices :

$$\frac{dF_t^e}{F_t^e} = \sigma_e dW_t^e, \quad \frac{dF_t^f}{F_t^f} = \sigma_f dW_t^f \text{ with } d\left\langle W^e, W^f\right\rangle_t = \rho dt$$

- Margrabe provides the closed formula :

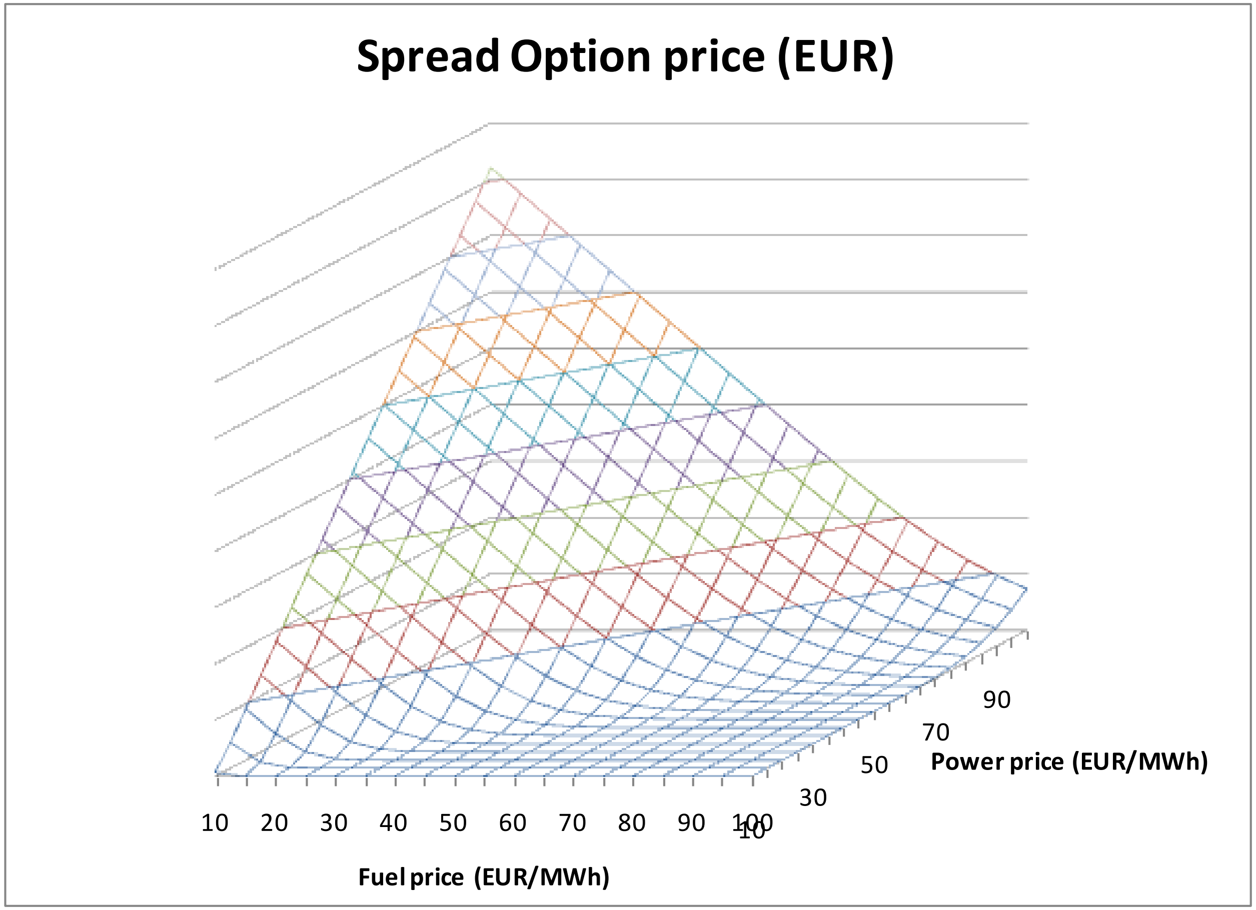
$$\mathrm{MtM} = F_t^e \mathcal{N}(d) - hF_t^f \mathcal{N}(d - \sqrt{V})$$

N is the c.d.f. of the Gaussian law and $\begin{cases} d & = \left(\ln(F_t^e/hF^f t) + V/2\right)/\sqrt{V} \\ V & = \underbrace{(\sigma_e^2 + \sigma_f^2 - 2\rho\sigma_e\sigma_f)}_{=\text{equivalent volatility}}(T-t) \end{cases}$

- Question : How does the option value vary when the elec price increases ? the fuel price increases?
- Question : What are the Deltas of this option ?

Spread options pricing

Market value of a fuel spread option with Margrabe



Greeks of spread options

Sensitivities of fuel spread options to underlying factors

- Deltas : sensitivity to the different commodities
  - Long Delta : Sensitivity of the option to the electricity price
  - Short Delta : Sensitivity of the option to the fuel price

$$\begin{cases} \Delta_e &= \dfrac{\partial \,\mathrm{MtM}}{\partial F^e} = \mathcal{N}(d) \\ \Delta_f &= \dfrac{\partial \,\mathrm{MtM}}{\partial F^f} = -h\mathcal{N}(d - \sqrt{V}) \end{cases}$$

Can be proven by using the key relation $F_t^e f_{\mathcal{N}}(d) = hF_t^f f_{\mathcal{N}}(d - \sqrt{V})$

- Vega : sensitivity to the equivalent volatility elec-fuel : what is its sign?

Greeks of spread options

Sensitivities of fuel spread options to underlying factors

- Deltas : sensitivity to the different commodities
  - Long Delta : Sensitivity of the option to the electricity price
  - Short Delta : Sensitivity of the option to the fuel price

$$\begin{cases} \Delta_e & = \frac{\partial \, \mathrm{MtM}}{\partial F^e} = \mathcal{N}(d) \\ \Delta_f & = \frac{\partial \, \mathrm{MtM}}{\partial F^f} = -h\mathcal{N}(d - \sqrt{V}) \end{cases}$$

Can be proven by using the key relation $F_t^e f_{\mathcal{N}}(d) = hF_t^f f_{\mathcal{N}}(d - \sqrt{V})$

- Vega : sensitivity to the equivalent volatility elec-fuel : what is its sign?

$$\nu = \frac{\partial \, \mathrm{MtM}}{\partial \sigma_{eq}} \geq 0$$

- Correlation Delta : sensitivity to the correlation between elec and fuel prices : what is its sign?

Greeks of spread options

Sensitivities of fuel spread options to underlying factors

- Deltas : sensitivity to the different commodities
  - Long Delta : Sensitivity of the option to the electricity price
  - Short Delta : Sensitivity of the option to the fuel price

$$\begin{cases} \Delta_e & = \dfrac{\partial \,\mathrm{MtM}}{\partial F^e} = \mathcal{N}(d) \\ \Delta_f & = \dfrac{\partial \,\mathrm{MtM}}{\partial F^f} = -h\mathcal{N}(d - \sqrt{V}) \end{cases}$$

Can be proven by using the key relation $F_t^e f_{\mathcal{N}}(d) = hF_t^f f_{\mathcal{N}}(d - \sqrt{V})$

- Vega : sensitivity to the equivalent volatility elec-fuel : what is its sign?

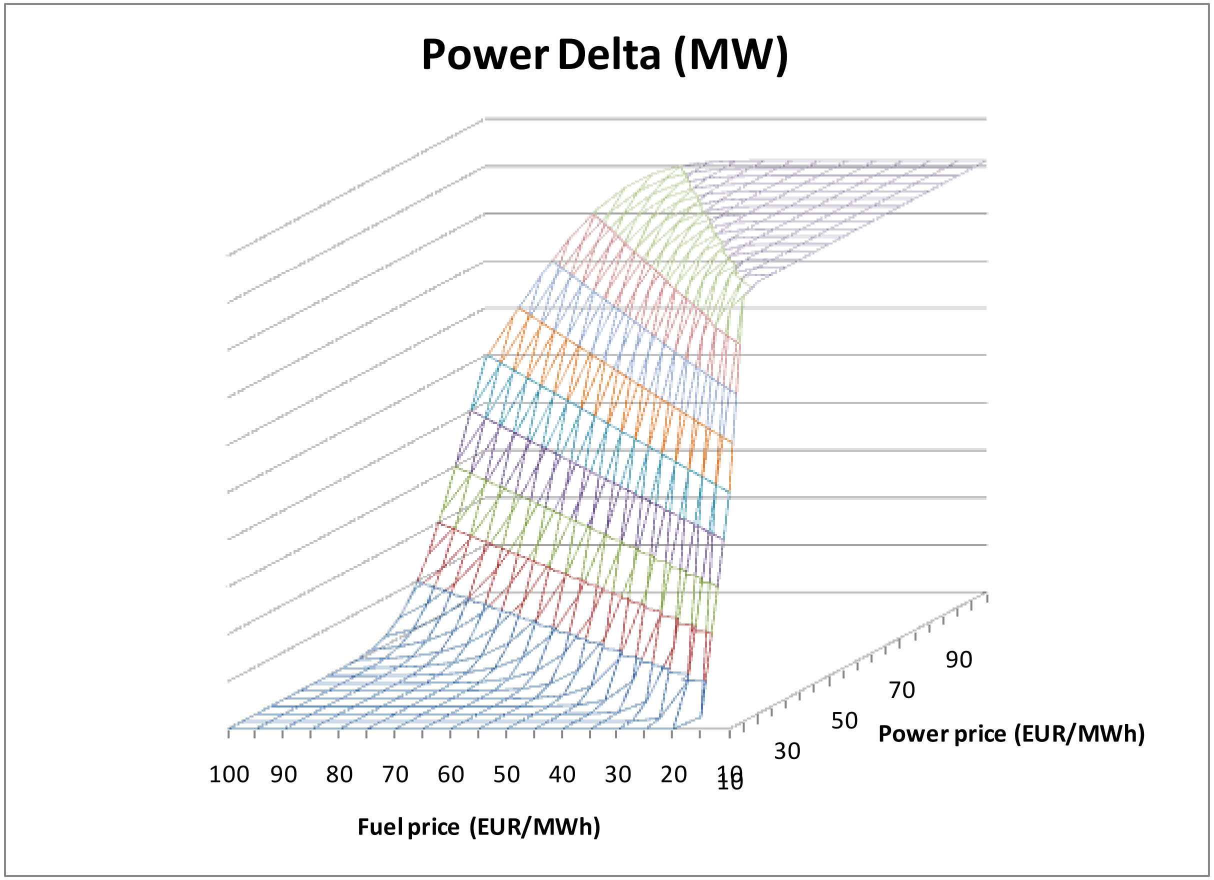
$$\nu = \frac{\partial \,\mathrm{MtM}}{\partial \sigma_{eq}} \geq 0$$

- Correlation Delta : sensitivity to the correlation between elec and fuel prices : what is its sign?

$$\pi = \frac{\partial \,\mathrm{MtM}}{\partial \rho} = -\nu \frac{\sigma_e \sigma_f}{\sigma_{eq}} \leq 0$$

Greeks of spread options

Long Delta of a fuel spread option

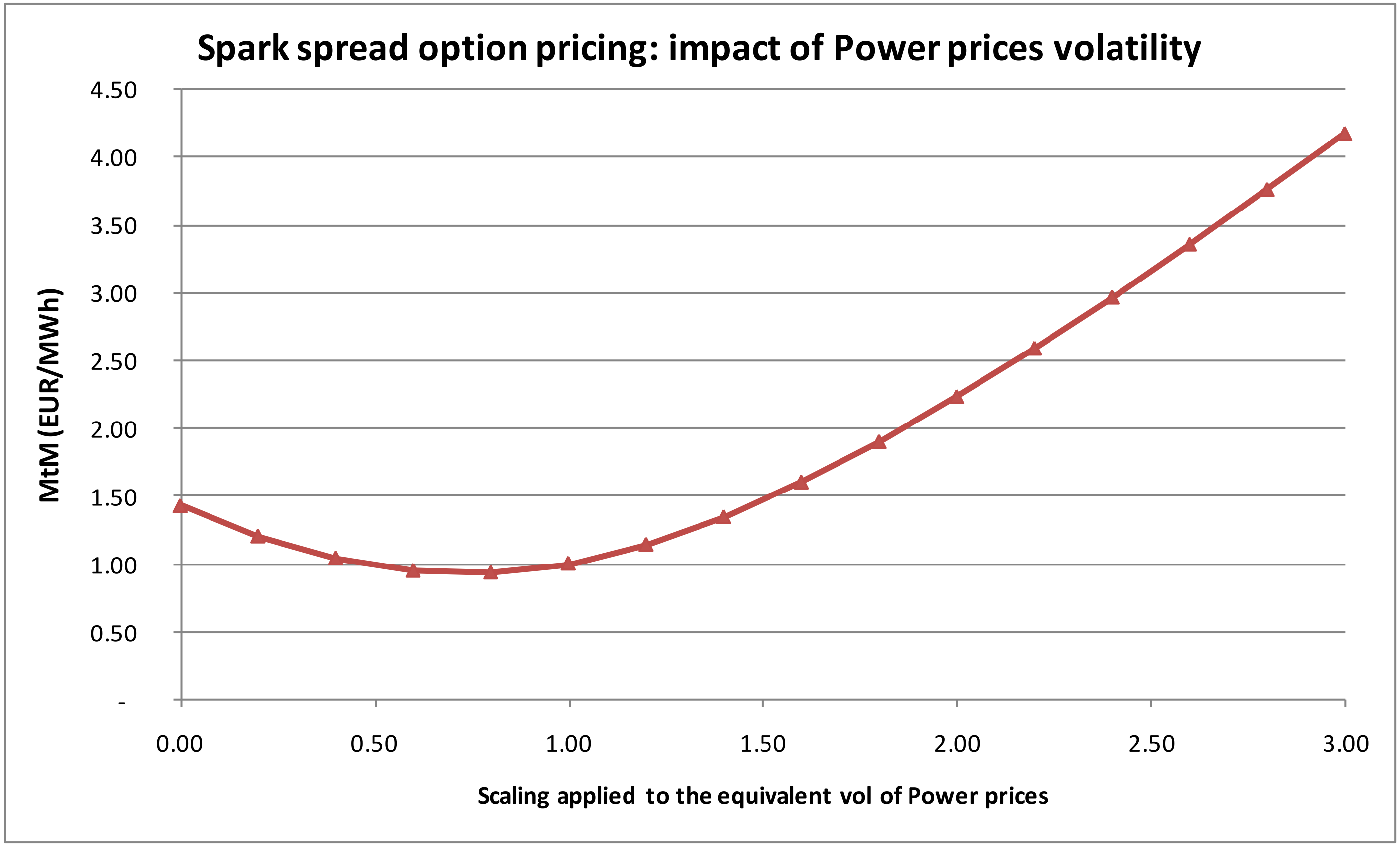


Greeks of spread options

Sensitivity to the volatility of one of the two underlyings

- How does the fuel spread option value vary when the volatility of electricity prices increases ?
- Due to (positive) correlation, it depends on the relative level of the electricity vol in comparison to the vol of fuel prices…

$$\nu_e = \frac{\partial \, \mathrm{MtM}}{\partial \sigma_e} = \nu \frac{\sigma_e - \rho \sigma_f}{\sigma_{eq}}$$



ENERGY DERIVATIVES

- A typical energy portfolio
- Forwards, Swaps and Spreads
- Power plant modelling, Spread options pricing
- **Gas storage modelling**
- Swing options pricing
- Load curve contract modelling

Storages

Storages are a major component of the gas chain

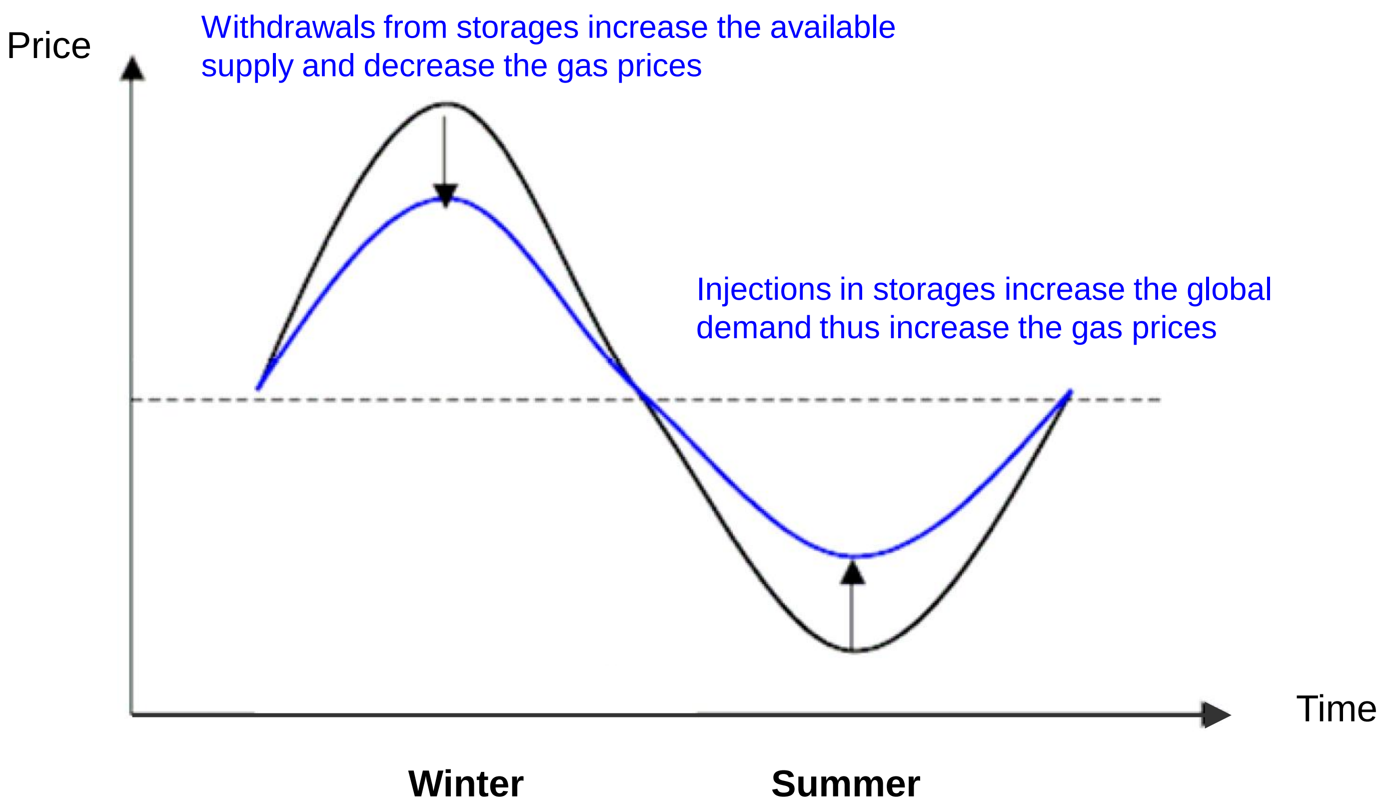
Why using storage facilities?

1. Physical and economic reasons (cost minimization)
   - Gas demand: variable (summer/winter) and inelastic
   - Inflexible gas supply: limited by the capacity of the pipeline system
   - Storages are closer to consumption areas
   - They ensure that gas will be easily accessible in response to higher demand
   - Interesting for non-producer countries with a high level of gas importation to reduce their dependance to producers
2. Regulation conditions
   - Gas supply compagnies have the obligation to own storage facilities to secure supply in periods of high demand
3. Financial reasons : arbitrage mechanism
   - Take benefit of the possible arbitrage between summer/winter or week-end/open days
   - Exploit market opportunities: inject gas in the storage while the gas is cheaper and withdraw it during periods with higher prices
   - Used for high consumption periods in gas but also for firing gas power plants (CCGT)

Gas storages

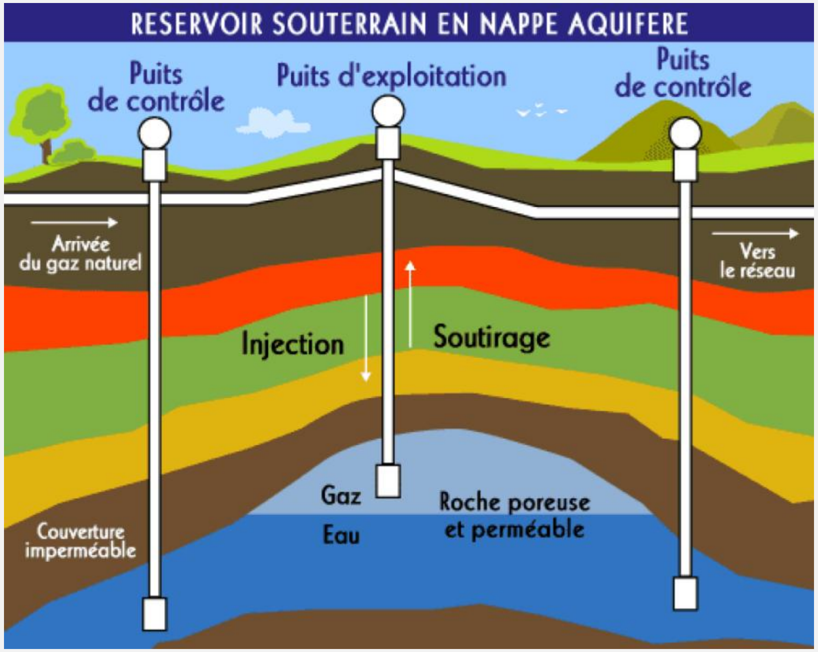
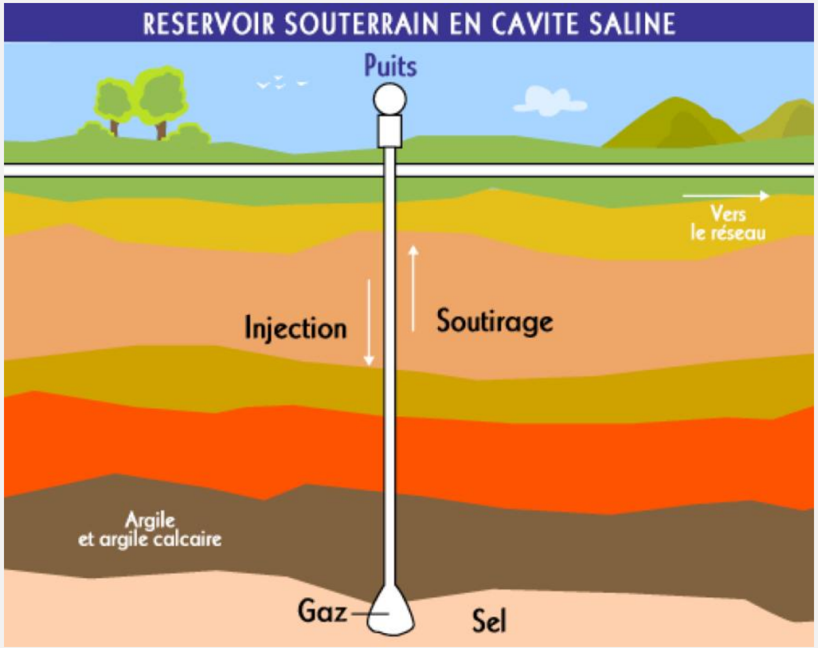
The storage has a strategic role for gas supply.

- Storage is used by supply companies to store an extra gas capacity (issued from production fields or importations).
- This buffer stock allows to hedge a part of the price risk (unexpected high demand).
- The storage has thus an effect on the summer-winter spread.



Gas storages

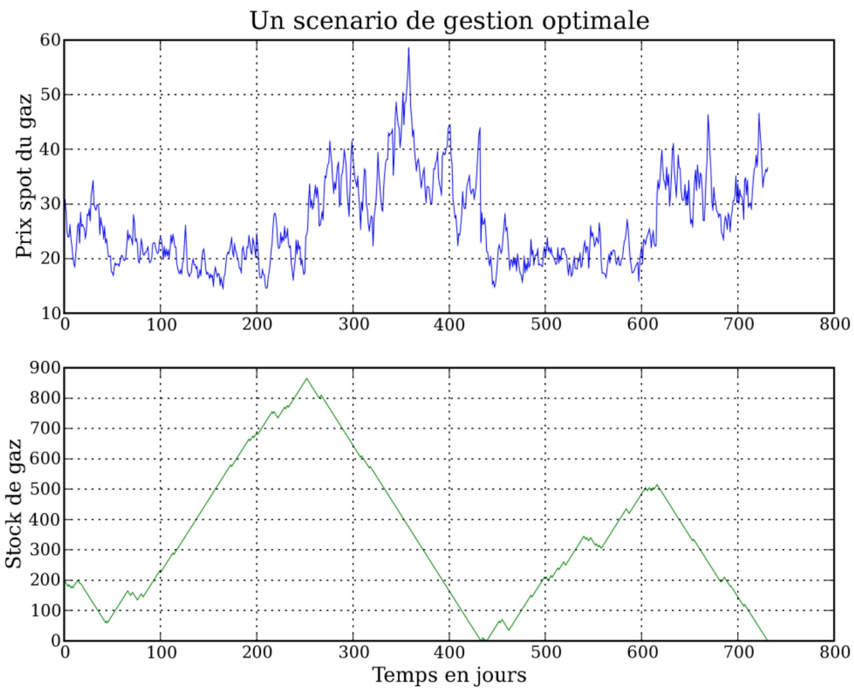
There are two main kinds of storage facilities (underground storages).

Seasonal or base load storages	Fast-cycle or peak-load storages
Aquifers and depleted fields	**Salt cavern: artificial cavity created by boring in salt rocks and dissolution**
	
Large volume capacities but low injection/withdrawal rates (~10 months to totally fill and empty)	Smaller reservoirs but high deliverability rates (around 2 weeks or less to fill and empty)
Used to satisfy the seasonal demand: gas at lower price is stored in summer and delivered in winter	Used for arbitrage on short periods in time (intraweek trading) and to respond to short-term demand

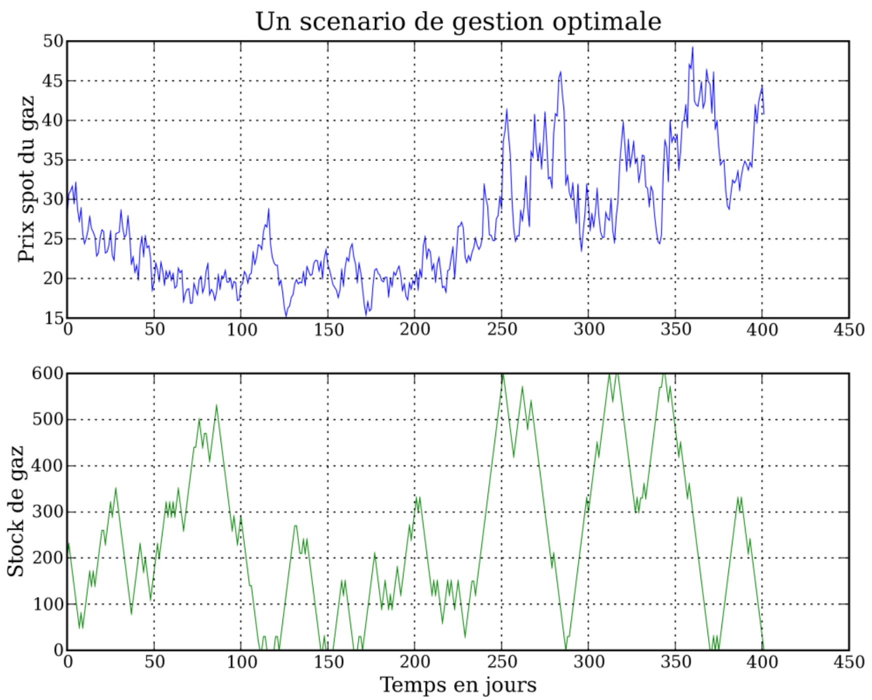
Gas storages

Optimal valuation of storages requires an optimization procedure.

- With the real option approach, the storage value is given by the maximal expected value that the storage scheduler can get by operating optimally the storage with respect to market conditions and stock constraints.
- An optimization is needed to determine the optimal gas volume to be injected/withdrawn, while respecting the contraints on the stock level.



Seasonal storage (180 days)



Fast-cycle storage (20 days)

ENERGY DERIVATIVES

- A typical energy portfolio
- Forwards, Swaps and Spreads
- Power plant modelling, Spread options pricing
- Gas storage modelling
- **Swing options pricing**
- Load curve contract modelling

Swing options in energy markets

Many physical or financial assets including an optionality can be considered as Swing options

- ***What is a Swing option ?***
  - Right to receive/purchase a given volume of energy
  - Maximal number of 'exercise' rights n_{max} before the maturity
  - The exchange price (strike) can be fixed, variable or random.
  - Can be viewed as a multiple-exercise American options
  - More complex Swing options include a variable volume : can be "swing up" or "swing down"
  - In this case, global volume constraints impose a limited total energy amount

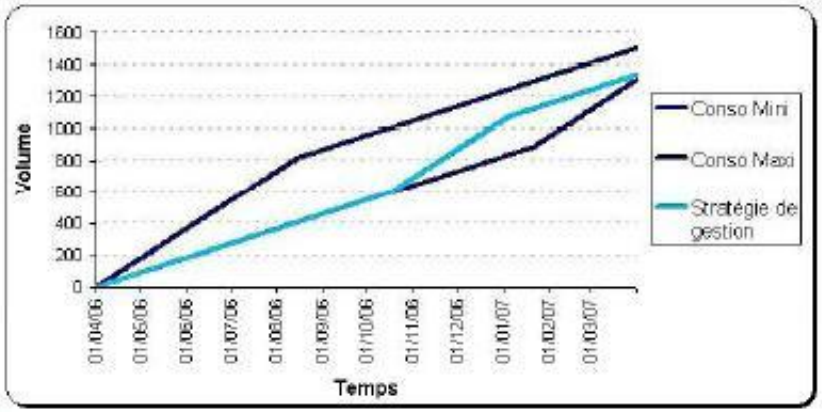
- Swing options include a large class of structured assets in energy markets…
  - Gas supply contracts including Take-or-Pay clauses
  - Options to shut down services, demand-side mgmt contracts
  - Storage facilities (but allow both injections and withdrawals)
  - Water reservoirs, hydraulic assets (but include inflows)

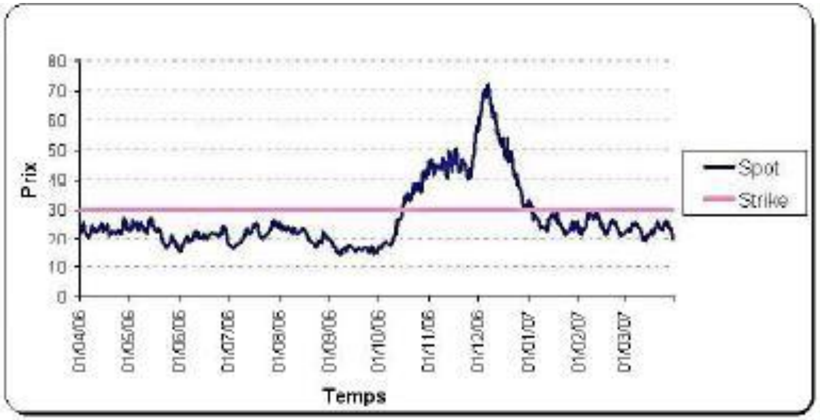
Swing options with variable volume

Typical example of a gas Swing contract

- The buyer has the right to purchase each day a gas quantity in $[q_{min}, q_{max}]$
- … at a given strike price *X*
- … but must purchase globally (over the year) a global volume in $[Q_{min}, Q_{max}]$

- NB : The optimal strategy is non necessary bang-bang, meaning that the optimal consumption can belong to $]q_{min}, q_{max}[$





Swing options pricing

The general pricing problem boils down to a stochastic control problem.

- The market value of a variable volume Swing option is :

$$\mathrm{MtM} = \sup_{(q_t)_{t\le T}} \mathbb{E}\left[\sum_{t\le T} q_t\left(S_t - \bar{X}_t\right)\right] \quad q \text{ such that : } \begin{cases} q_t \in [q_{\min}, q_{\max}] \\ Q_T = \sum_{t\le T} q_t \in [Q_{\min}, Q_{\max}] \end{cases}$$

- Main difficulties :
  - Stochastic control problem under constraints
  - Numerical method needed (optimization required)
  - Dimension of the problem : price(s) + volume
  - Variable quantity *q* : bang-bang assumption ⇨ multiple-exercise American option

- Main idea in all numerical methods : use the stochastic Dynamic Programming Principle (DDP)
  - Tree methods (binomial or trinomial) ⇨ Leading to a forest of trees
  - Monte Carlo techniques ⇨ Longstaff-Schwartz (2001) for multiple-exercise American options
  - Quantification techniques, cf. Bally and Pagès (2003)

Swing options with variable volume

Typical example of a gas Swing contract : some practice…

- Consider a gas Swing option over 180 days such that the minimal daily quantity is 10 MWh/day, the maximal daily quantity is 50 MWh/day and the global maximal quantity that can be purchased over the whole period is 6000 MWh.
- After a standard normalization, this contract is usually separated into a firm Swap contract and a normalized Swing option.

a) What is the baseload volume provided by the Swap contract ?
b) What is the remaining optional volume ?
c) How many exercise rights are provided by the normalized Swing option ?
d) Is the bang-bang assumption satisfied ?

Swing with $q \in [q_{min}, q_{max}]$ ⇔ Swap purchasing q_{min} every day + Purely Swing with $q^* \in [0, q_{max} - q_{min}]$

Swing options with variable volume

Typical example of a gas Swing contract : some practice…

- Consider a gas Swing option over 180 days such that the minimal daily quantity is 10 MWh/day, the maximal daily quantity is 50 MWh/day and the global maximal quantity that can be purchased over the whole period is 6000 MWh.
- After a standard normalization, this contract is usually separated into a firm Swap contract and a normalized Swing option.

a) What is the baseload volume provided by the Swap contract ?
b) What is the remaining optional volume ?
c) How many exercise rights are provided by the normalized Swing option ?
d) Is the bang-bang assumption satisfied ?

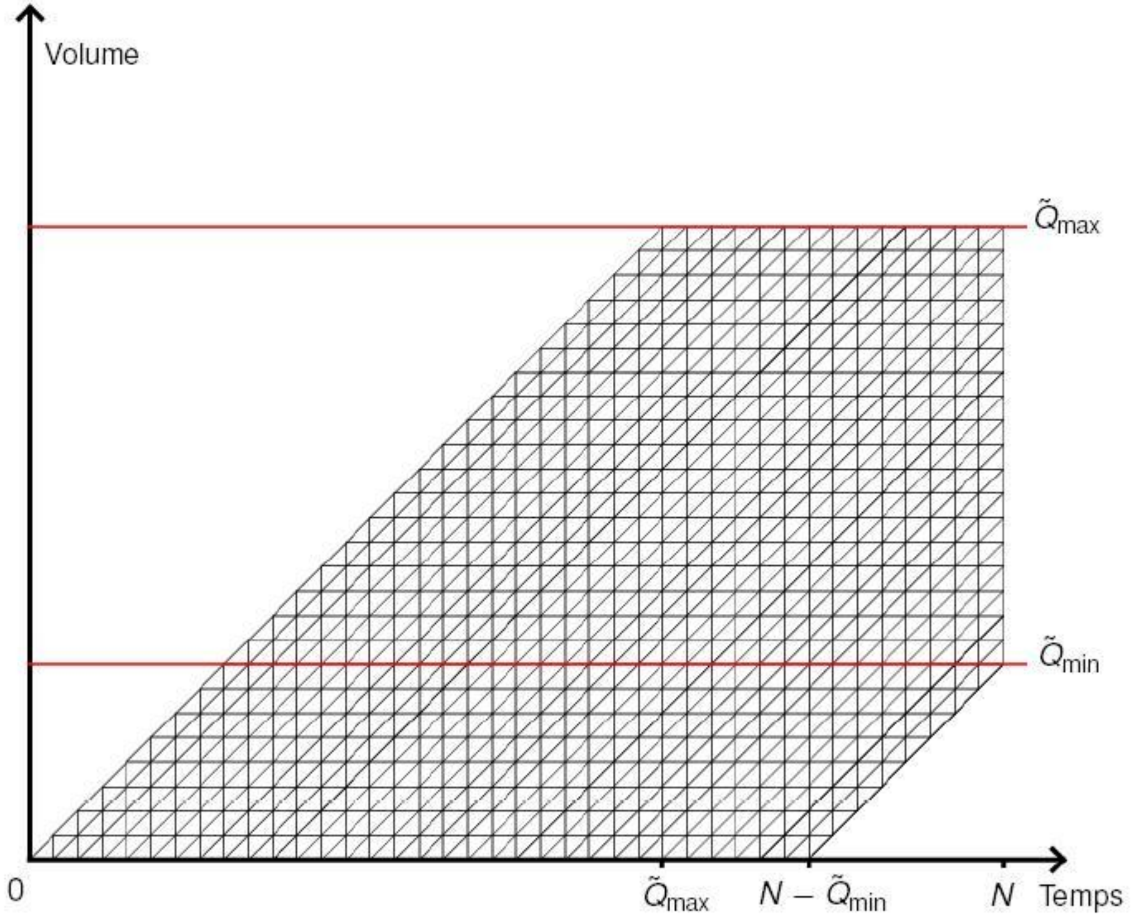
Swing with $q \in [q_{min}, q_{max}]$ ⇔ Swap purchasing q_{min} every day + Purely Swing with $q^* \in [0, q_{max} - q_{min}]$

a) Baseload volume = 10 * 180 = 1800 MWh
b) Remaining optional volume = 6000 – 1800 = 4200 MWh
c) Number of exercise rights = 4200 / (50 – 10) = 105 times
d) Yes, since there is an “integer” number of exercises in the Swing option

Swing options pricing

Reducing to multiple-exercise American options

- Swing contracts are decomposed in a firm Swap contract guaranteeing q_{min} on each date + a purely Swing part normalized to purchases belonging to [0, 1] with n_{max} possible exercises.
- The discretization of admissible volumes is equivalent to the enumeration of possible exercises rights while respecting the constraints.



Swing options pricing

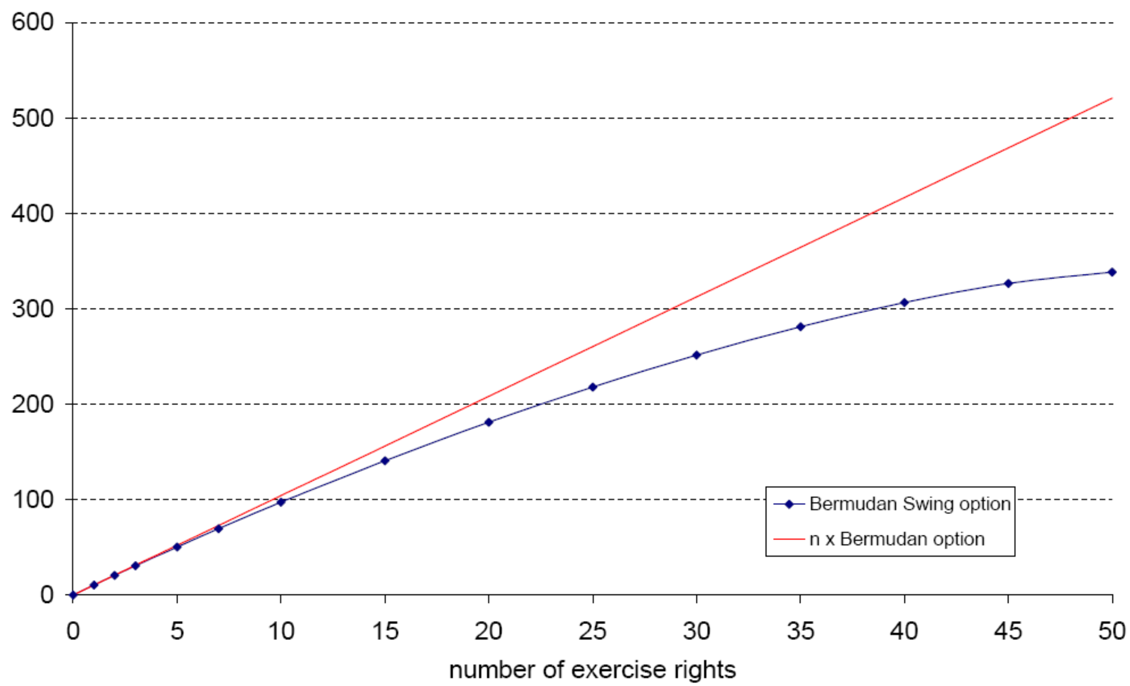
Some major properties of normalized Swing options market value

1. Case with only one exercise right : Swing option = American/Bermudan option
2. Upper bound (less constraint ⇨ bigger value)

$$\boxed{\text{Swing option} \leq n_{\max} \times \text{Identical American options}}$$

3. Lower bound (pre-determined choice of exercise dates ⇨ lower value)

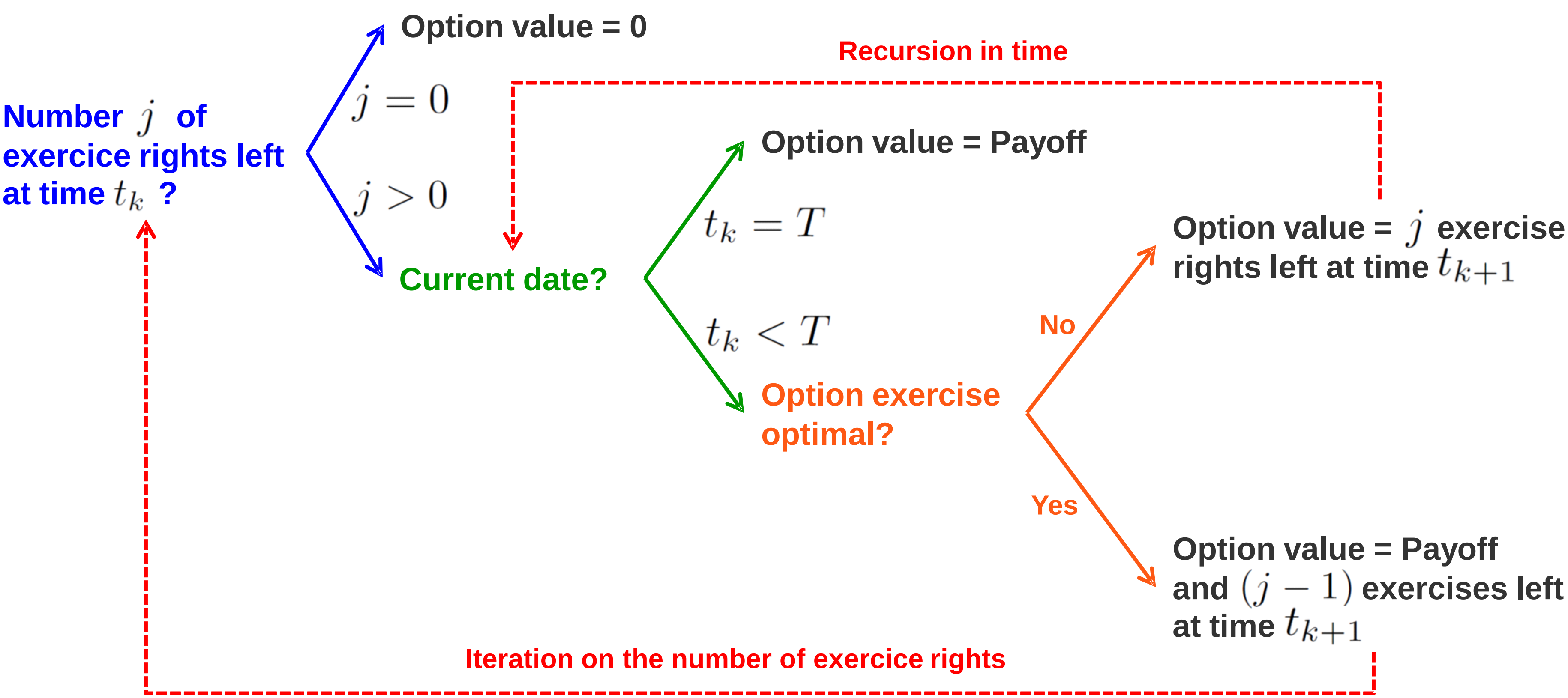
$$\boxed{\text{Swing option} \geq \sup_{\mathcal{N}:n_{\max}\text{ dates } \tau_k \leq T} \sum_{\tau_k \in \mathcal{N}} \text{European options with maturity } \tau_k}$$



Swing options pricing

Numerical method in practice

- As multiple-exercise American options, Swing options can be valuate through :
  - Backward induction in time
  - An iteration on the number of exercise rights left



Swing options pricing

Dynamic Programming Principle

- Consider the problem of pricing a Swing option with *j* exercise rights on *[t, T]* :

$$v^{(j)}(t) = \sup_{t \leq \tau_1 < \ldots < \tau_j \leq T} \mathbb{E}\left[\sum_{k=1}^{j} \left(S_{\tau_k} - \bar{X}_{\tau_k}\right)^+ | \mathcal{F}_t \right]$$

- Backward recursion on a discrete time grid

  Forward iteration on the number of exercise rights

- Mathematically, the Dynamic Programming Principle (DDP) for Swing options can be written :

$$\begin{cases} v^{(0)}(t_k) & = 0, \quad \forall t_k \in \pi \\ v^{(j)}(T) & = (S_T - \bar{X}_T)^+, \quad \forall j = 1, \ldots, n_{\max} \\ v^{(j)}(t_k) & = \max\left\{ \text{apply the } j^{th} \text{ exercise at } t_k \ ; \ \text{do not exercise at } t_k \right\} \\ & = \max\left\{ (S_{t_k} - \bar{X}_{t_k})^+ + \mathbb{E}\left[v^{(j-1)}(t_{k+1}) | \mathcal{F}_{t_k}\right] ; \mathbb{E}\left[v^{(j)}(t_{k+1}) | \mathcal{F}_{t_k}\right] \right\} \end{cases}$$

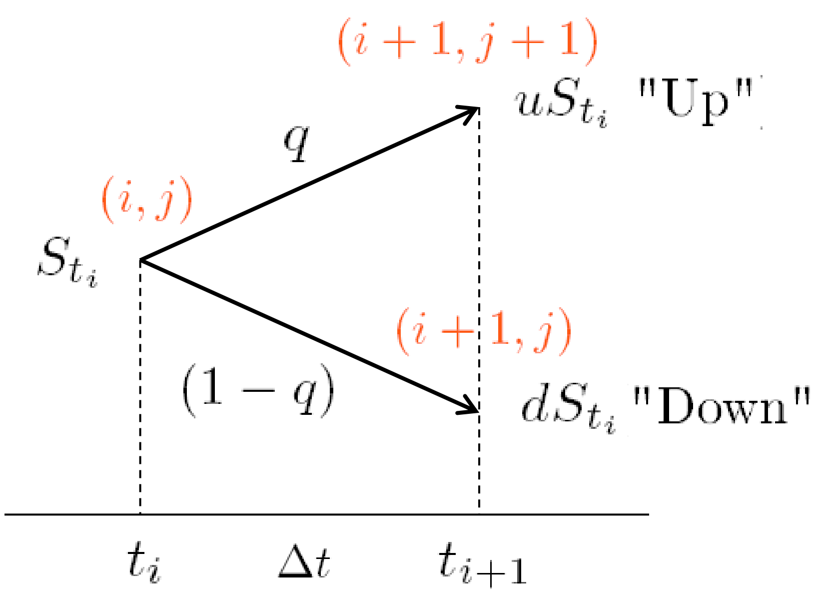
- As for American options, the main difficulty comes from the estimation of conditional expectations !

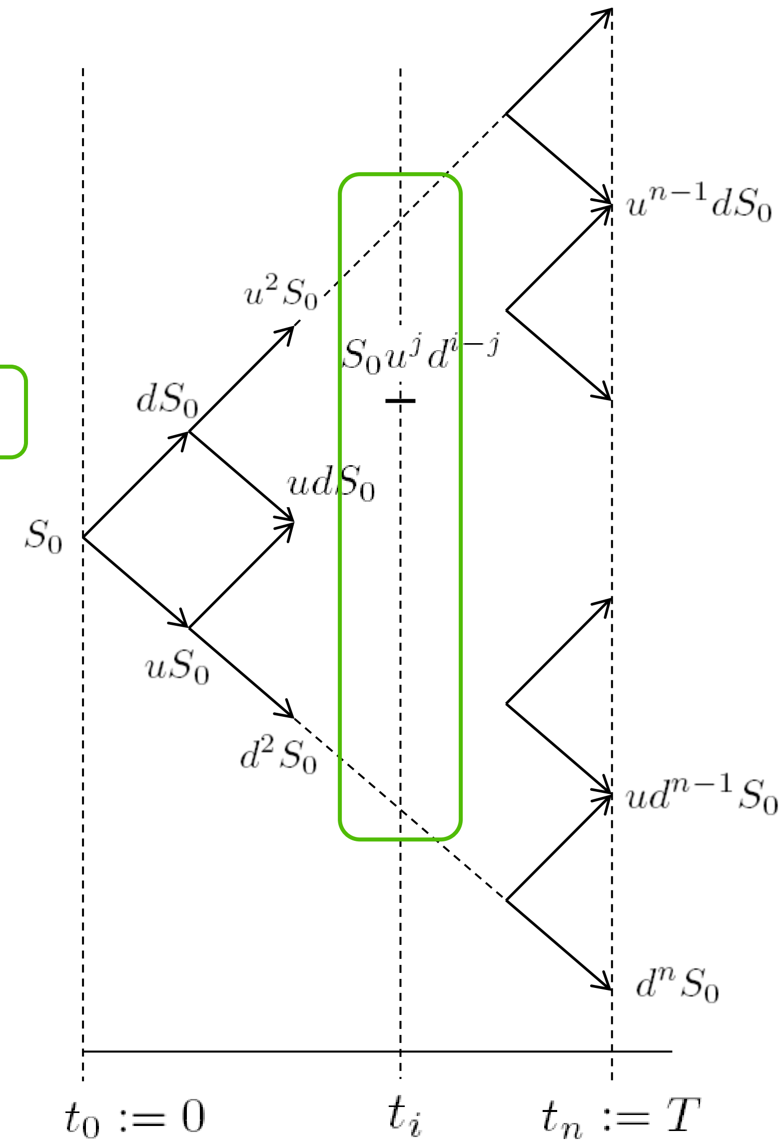
Swing options pricing

Using tree method…

- Idea : representing the price evolution on a tree
- Use of a binomial recombining tree
  - Possibility of upward and downward moves
  - At time t_i the underlying price can take (i +1) values

$$\boxed{S_0 u^i, S_0 u^{i-1} d^1, \ldots, S_0 u^j d^{i-j}, \ldots, S_0 d^i}$$

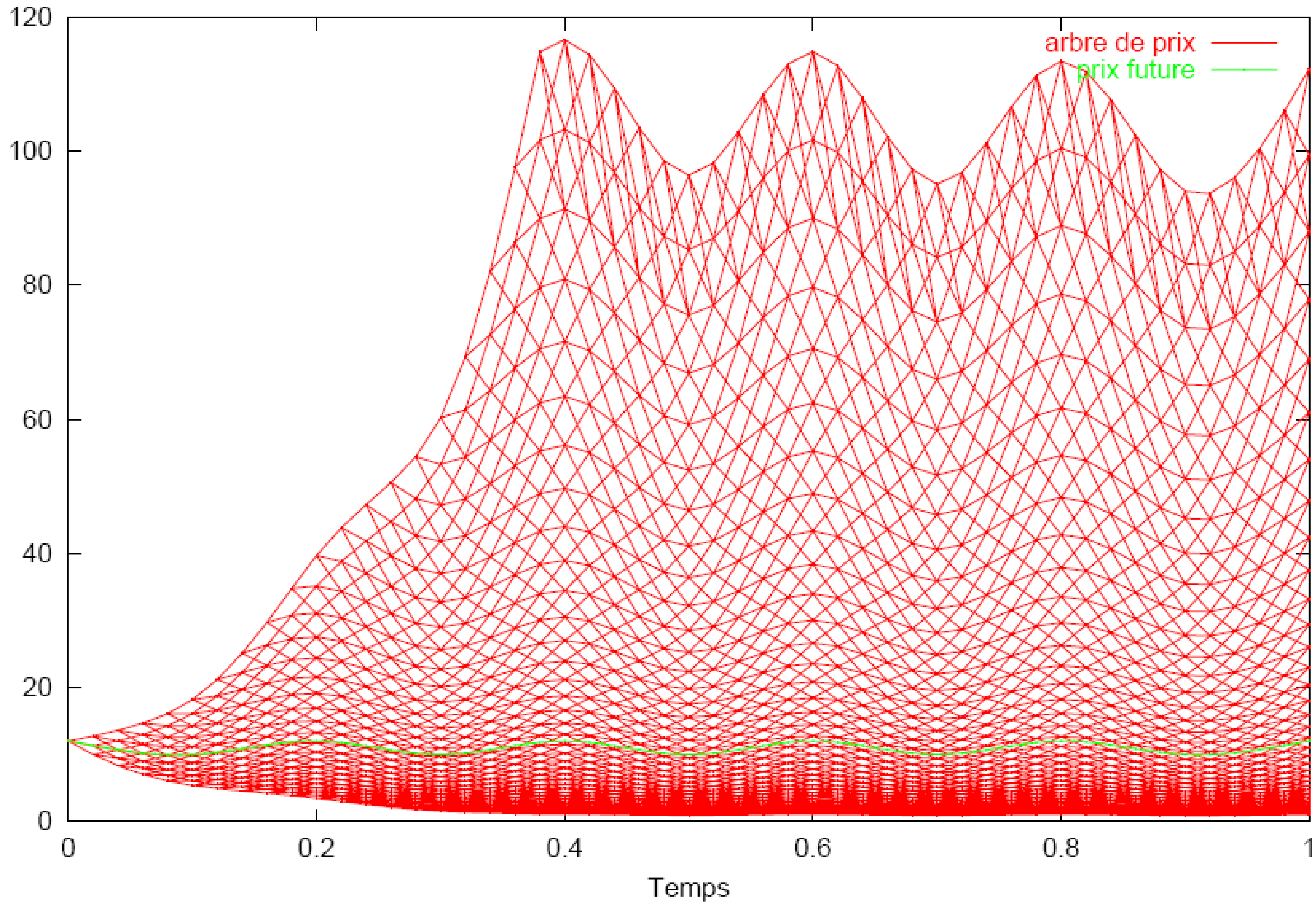




Swing options pricing

Using tree method…

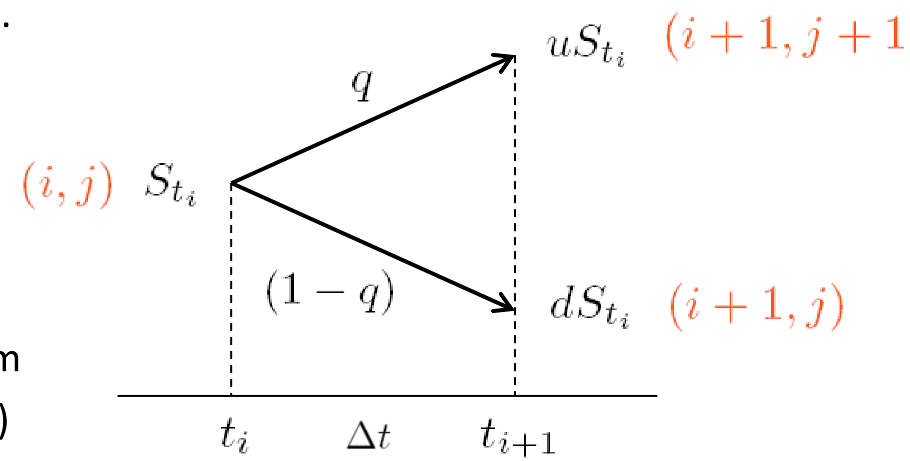
- Example of trinominal recombining tree in a one factor model for gas prices :



Swing options pricing

Using tree method…

- Assume the underlying price tree has been build.
- Backward recursion : the option price on node *(i, j)* can be expressed as function of the option prices on nodes *(i+1, j+1)* and *(i+1, j)*.



- Some recall for American options' pricing…

  DDP : The option price at a node is the maximum between the exercise value (payoff at this node) and the continuation value (expected CF if the option is not exercised).

$$\begin{aligned} v(i,j) &= \max\{\mathbb{E}_{t_i}\left[v(t_{i+1})\right]; (S_{t_i} - \bar{X}_{t_i})^+\} \\ &= \max\{q \cdot v(i+1, j+1) + (1-q) \cdot v(i+1, j); (S_{t_i} - \bar{X}_{t_i})^+\} \end{aligned}$$

- Swing options' case : it leads to a forest of trees (number of trees = number of exercise rights) !

$$v^{(0)}(i,j) = 0$$

$$\begin{aligned} v^{(k)}(i,j) &= \max\{\mathbb{E}_{t_i}\left[v^{(k)}(t_{i+1})\right]; (S_{t_i} - \bar{X}_{t_i})^+ + \mathbb{E}_{t_i}\left[v^{(k-1)}(t_{i+1})\right]\} \\ &= \max\{q \cdot v^{(k)}(i+1, j+1) + (1-q) \cdot v^{(k)}(i+1, j); \\ &\quad (S_{t_i} - \bar{X}_{t_i})^+ + q \cdot v^{(k-1)}(i+1, j+1) + (1-q) \cdot v^{(k-1)}(i+1, j)\} \end{aligned}$$

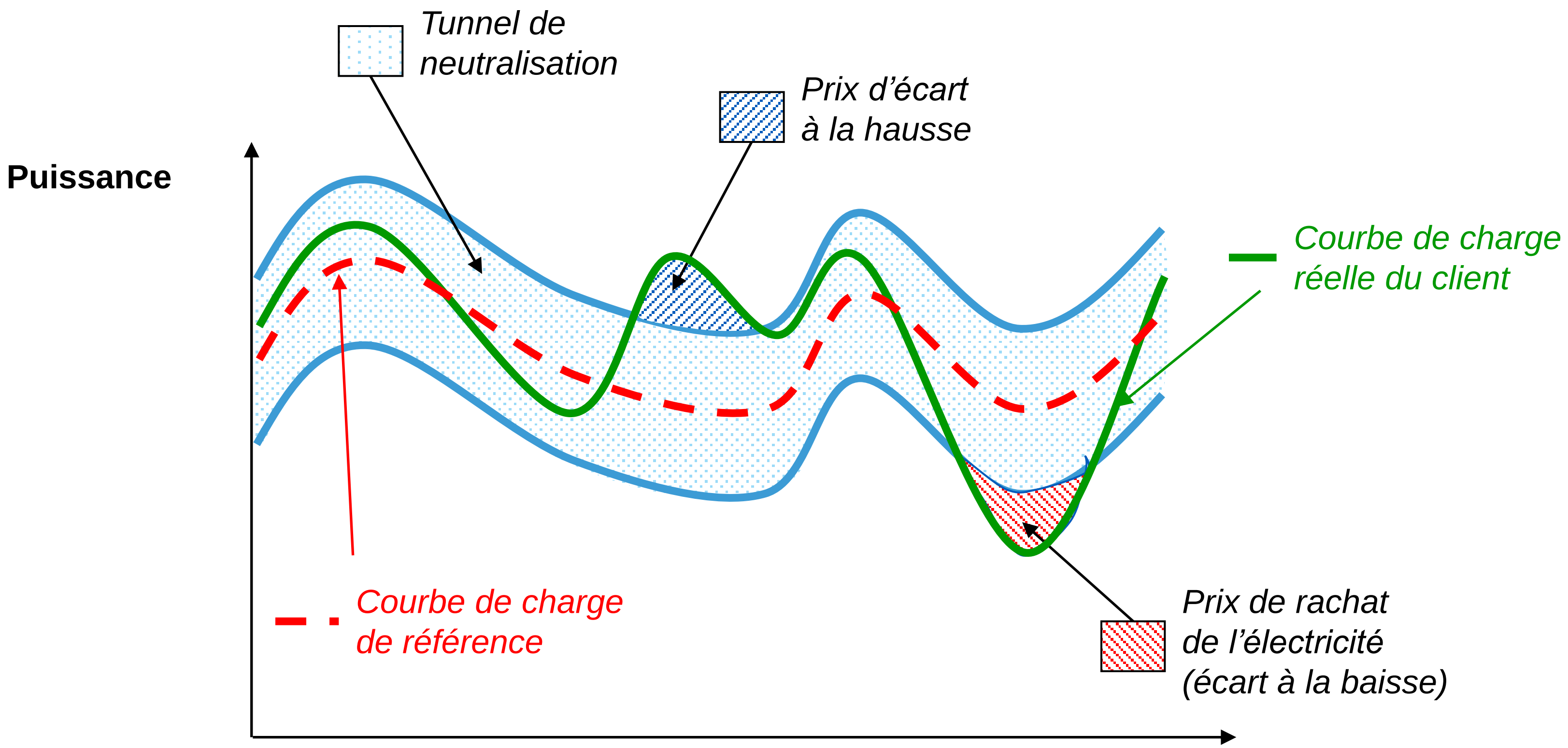
ENERGY DERIVATIVES

- A typical energy portfolio
- Forwards, Swaps and Spreads
- Power plant modelling, Spread options pricing
- Gas storage modelling
- Swing options pricing
- **Load curve contract modelling**

Load serving contract

A demand-side typical contract in energy portfolios

- The supplier is engaged to deliver some enery whatever the consumption.
- Concerns only customers, whose consumption is precisely known (load curve).
- Different prices depending on the real consumption being inside, below or above some tunnel



Cotation à la courbe de charge