Electricity markets

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Outline

ELECTRICITY MARKET DESIGN

- The electricity value chain
- Electricty market microstructure (balancing mechanism)
- Tools for power generation, typical supply curve in electricity markets
- Key drivers of European electricity prices

MODELLING ELETRICITY PRICES

- Main features of power prices
- Overview of spot and forward models
- A structural model for electricity prices (Aïd et al., 2011)
- Factorial models for energy prices (e.g. Kiesel et al., 2008)

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ELECTRICITY MARKET DESIGN

Some generalities, the electricity value chain

- Electricity market microstructure
- Power production tools and drivers of European electricity prices



Main electricity features

Electricity is a local commodity due to the non-storability and transport constraints

- Electricity is a local commodity :
 - Electricity is non-storable

Present best way to store large volumes of power: hydro-reservoir.

A too long excess of demand compared to supply may lead to dramatic blackouts (example in July 30th, 2012: India, 670 millions people).

⇒ Minute by minute real-time assessment of the equilibrium between demand and supply

Electricity transport satisfies specific laws (Kirchhoff's laws).

In a meshed electricity network, power will go from one point to another using all available paths, causing possible electricity flow interference.

⇒ Cross-border trading opportunities, up to transfert capacities available

A common market structure for a local commodity :

- Electricity being a local commodity, there are as many electricity markets as they are states...
- Market microstructure highly depends on national regulation.
- Nevertheless, common structure emerges driven by the necessary equilibrium between consumption and production.
- A central role of the Transport System Operator, in France : RTE and in Europe : ENTSO (European Network System Operator).



Electricity markets sequenced by maturity

Common market structure

- There is not only one market but a sequence of markets that can be ordered by time horizon.
 - The intraday market and/or balancing mechanism
 - The day-ahead market
 - The forward market



The electricity value chain

General design and the case for France : RTE is the Transport System Operator





Profile of customers in France

Figures and analysis from CRE (Rapport 2012)

- Deregulation of the electricity market in France since 2007
- New scheme : deregulated and non-integrated due to a legal separation of activities
- All customers (residential or not) can choose between regulated or market-indexed contracts
- In fact, the switching rate (nb of supplier change/total nb of customers) is less than 1.2%



Consumption spots (Mio) & volume (TWh) Consumption spots (Mio) & volume (TWh)

Offres aux tarifs réglementés

Offres de marché fournisseurs historiques

Offres de marché fournisseurs alternatifs

The electricity market actors

Four kinds of actors in competition

- Generators (Producers) / Retailers (Suppliers) / Consumers / Traders
- Producers have naturally a long position, consumers a short position.
- Traders take directional positions to exploit market opportunities.





Participants and money flows in the electricity industry



European power exchanges



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Coexistence of different markets

- Two kinds of transactions on electricity markets
 - Bilateral contracts traded Over The Counter (OTC)
 Standard or specially designed products (power profile, maturity, ...) : forwards, options
 - Trades on energy exchanges (trading markets) Standard products with a standardized bid/offer procedure and clearing mechanism "Financial" markets: futures, options

"Physical" markets: Day-1 (spot or day-ahead), intraday (D day)

Focus first on the intraday market on the D day

- Ensure the security of the system ⇒ Balancing Mechanism
- For this "physical" market :

First difficulty : Volume alea possible on the D day

Second difficulty : Network constraints

Intraday market : the role of the transport system operator RTE in France

The actions on the market are made to satisfy a global equilibrium :

Demand = Supply

Consumption = Production +/- Exchanges (market purchases and sales)

 Σ withdrawals = Σ injections

Declarations on Day-1 (16h30): production and consumption plans declared to RTE

Adjustment mechanism in real time (balancing), with delivery on the transmission system managed by the transport system operator (RTE)



Intraday market : operating reserves and adjustment mechanism

- On day D-1, the global Supply/Demand equilibrium takes in account both the market and OTC transactions.
- In real time (~ seconds to some hours), possible perturbation of this equilibrium :
 - Loss of some power generating units
 - Errors in the consumption or production forecast
 - Network congestions
- ➡ Since electricity is non-storable, the system has to be balanced.
- The transport system operator RTE has two means : operating reserves and balancing mechanism.
- An operating reserve is a generation that can be mobilised with a short-term notification :
 - Primary reserve : response time < 30 s, automatic device ~500 MW in France</p>
 - Secondary reserve : response time < 3 min, automatic ~600M W in France</p>
 - Tertiary reserve : response time < 15 min, manual ~1500 MW in France</p>
- Beyond, adjustment mechanism : balance responsible entities submit bids and offers to increase or decrease their production or consumption and RTE selects offers based on economic precedence (~30 min)
- A posteriori, each balance responsible entity receives the bill of her imbalances.

Focus on the French market : adjustment mechanism

- Bullish trend \(\Leftarrow Short system (lack of power))
- Bearish trend \(\Long system (supply glut))



Focus on the French market : adjustment mechanism and imbalance settlement

Adjustment mechanism : allow to compensate the power need/glut of the system

- Bullish trend ⇒ Offers for increasing the power injected (producers) in the system and erasing offers (cut-off injunctions, EJP) for reducing the demand
- Bearish trend ⇒ Offers for decreasing the power injected in the system (producers)
- Offers ranked by merit order (RTE)

Imbalance settlement ("Règlement des écarts")

- RTE establishes, a posteriori, the bill to be paid or received by any actor, for the differentials observed on its perimeter (injections/withdrawals).
- Formula based on the spot price and power generation costs
- Incentive to a vertuous behavior for both producers and consumers
- Example for EDF: Case of a Bull trend (lack of power)
 If EDF is long (P>C) ⇒ EDF receives the Spot
 If EDF is short (P<C) ⇒ EDF needs to pay max(Spot, P_u*)

 $*P_u$ = upper weighted average cost of power generation issued from the adjustment mechanism



Electricity markets sequenced by maturity

Common market structure

- Beside this balancing mechanism :
 - an infra-day market for energy delivery for the hours of the day
 - a day-ahead market for energy delivery for the hours of the next day



The day-ahead market : example of Epex Spot

- Day-ahead market : market for energy delivery for the hours of the next day
- Spot price fixing (Epex Spot: at 12h) by crossing the supply and demand curves



Epex Spot Auction on the 27/10/2011 for Friday 28/10/2011 19h-20h



The day-ahead market : example of Epex Spot

- This method for fixing is applied for the 24 hour products of the next day.
- Average of some of these hourly prices are called Blocks
- In particular, Baseload and Peakload prices (in France, Peakload 🗇 8h-20h)



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The day-ahead market : example of Epex Spot

France	📳 Germany/Austria (Phelix)	rissix) 🛛 📕 MC	С	03/09	3/09/2011 - 09/09/2011		
						03/09/2	D11 🔹 🛄
France Day Base	Sat, 09/03	Sun, 09/04	Mon, 09/05	Tue, 09/06	Wed, 09/07	Thu, 09/08	Fri, 09/0
Prices (€/MWh)	48.320	43.192	50.644	45.872	44.820	50.414	57.97
Volumes (MWh)	146,529.0	146,485.0	189,179.0	191,767.0	183,754.0	189,819.0	196,516.
France Day Peak							
Prices (€/MWh)	53.971	46.591	59.864	51.203	56.563	57.951	66.11
Volumes (MWh)	66,152.0	75,296.0	99,643.0	100,446.0	102,213.0	95,089.0	108,294.
BLOCKPRICES							
	Sat, 09/03	Sun, 09/04	Mon, 09/05	Tue, 09/06	Wed, 09/07	Thu, 09/08	Fri, 09/0
Middle-Night (01-04)	43.127	40.702	32.842	29.395	12.981	33.111	43.57
Early Morning (05-08)	31.495	25 801	40 469	41 728	30 850	39 744	48 54

Epex Spot table for France on the 08/09/2011

	Sat, 09/03	Sun, 09/04	Mon, 09/05	Tue, 09/06	Wed, 09/07	Thu, 09/08	Fri, 09/09
Middle-Night (01-04)	43.127	40.702	32.842	29.395	12.981	33.111	43.576
Early Morning (05-08)	31.495	25.801	40.469	41.728	30.850	39.744	48.546
Late Morning (09-12)	56.206	47.510	69.738	56.028	56.694	59.951	71.981
Early Afternoon (13-16)	52.978	45.598	59.021	47.260	57.064	58.040	68.367
> Rush Hour (17-20)	52.728	46.665	50.833	50.322	55.931	55.863	58.004
Off-Peak 2 (21-24)	53.388	52.877	50.961	50.496	55.401	55.775	57.363
Baseload (01-24)							57.973
Peakload (09-20)	53.971	46.591	59.864	51.203	56.563	57.951	66.117
> Night (01-06)	35.875	34.836	28.664	29.327	13.023	31.400	41.728
Off-Peak 1 (01-08)	37.311	33.251	36.656	35.561	21.915	36.428	46.061
Business (09-16)	54.592	46.554	64.379	51.644	56.879	58.996	70.174
Offpeak (01-08 & 21-24)	42.670	39.793	41.424	40.540	33.077	42.877	49.828
Morning (07-10)	47.215	36.616	65.096	55.860	51.985	55.126	64.072
 → High Noon (11-14)	57.754	49.100	66.797	51.291	58.336	60.839	73.746
Afternoon (15-18)	50.018	43.565	51.992	46.799	53.989	54.294	60.873
Evening (19-24)	54.081	51.745	51.322	51.526	56.717	56.750	57.703

The forward market

- The forward market presents a nested contract structure.
- Two basic characteristics of forward products : maturity and granularity.
- The availability of forward products evolves dynamically.



Availability : example with 3 MAH, 2 QAH and 1 YAH available

Time



The forward market

- The forward market corresponds to the market for products with granularities and maturities greater than one day.
- Example of EEX : are available at the same time the following forward products :
 - 6 calendars
 - 11 quarters
 - 9 months
 - 4 weeks
 - 2 weekends
 - 8 days
- In three flavours : Baseload (each hour), Peakload (8h-20h Monday to Friday) and Offpeak
- Thus, 106 contracts are available... to be compared to the 525684 hours in the next six years...

Market horizon : Last delivery date covered by the futures products quoted Market depth : Available volumes of tradable products Completeness : Ability to find products on for any market horizons and any granularity



The forward market

Some facts on electricity forward prices

- Forward prices result from an anticipation of the actors of the future supply/demand equilibrium.
- Illustration on the German Baseload forward curve dynamics :
 - Very differentiated behaviour between spot, month and yearly contracts
 - Slow motion of yearly contracts; may exhibit contango or backwardation (report or deport)
 - Seasonal pattern of monthly contracts





The forward market

Some facts on electricity forward prices

- No-arbitrage condition is respected between futures with different but recovering granularities.
- Some practice : At date 15/12/2013, we observed following quotations in €/MWh for power Base futures products :

1MAH	2MAH	3MAH	4MAH	5MAH	6MAH	1QAH	2QAH	3QAH	1YAH
61	60	56	49	47	46			46	51

- a) Give the prices of the 1QAH and 2QAH products, missing in the table above.
- b) In the same conditions, deduce the price of the October 2014 Base product.
- a) 1QAH * 90 = 1MAH * 31 + 2MAH * 28 + 3MAH * 31 ⇒ 1QAH price = 58.9 €/MWh
 2QAH * 91 = 4MAH * 30 + 5MAH * 31 + 6MAH * 30 ⇒ 2QAH price = 47.3 €/MWh
- b) October 2014 price = 4QAH price
 1YAH * 365 = 1QAH * 90 + 2QAH * 91 + 3QAH * 92 + 4QAH * 92 ⇒ 4QAH price = 51.8 €/MWh



The spot and forward markets

Some facts on products traded

- Futures are more traded than spot : less volatile, used for hedging (electricity non-storability)
- Trades are mostly made OTC (forward contracts) than through energy exchanges.



Volumes traded in 2010 in France (CRE, 2010)



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Tools for power generation

Electricity is a secondary energy source that can be produced with different technologies.

- Each production tool has its own structure of costs :
 - Fixed / variable costs
 - Link to fuel and CO₂ market prices
- Each production has its specific characteristics :
 - Life of service
 - Efficiency : heat rate, number of hours at nominal capacity
 - Environmental impact

	Coal	СССТ	Nuclear	Hydro	Wind
Construction period	4 y	2-3 y	6-10 y	8-10 y	2-5 y
CAPEX (invest.)	++	+	+++	+++	++
Service life	50 y	25 y	50-60 y	> 100 y	20 y
OPEX (O&M)	++	+	+++	+++	+++
Efficiency	38-42%	57-58%	35%	> 90%	25%
Fuel cost	++	+++	+	-	-
CO ₂ -NOX emissions	+++	++	-	-	-

Tools for power generation

Zoom on the CCGT technology

In recent years, fast development of CCGT (Cycle Combined Gas Turbine) in Europe

- More efficient and flexible technology: quick answer (~1/2 hour) to demand
- Replace old and polluting fuel-fired and coal-fired power plants
- With current gas prices, most of first generation gas-fired power plants are no more profitable





Stack curve in electricity markets

- The supply curve is build by ranking the different means of power generation by merit order.
- The spot price corresponds then to the price matching supply and demand.
- Usually, only the proportional production cost is modelled in this curve (no CAPEX).





Power generating mix Example of the EDF Group





EDF Group generating mix in 2012



Power generating mix

Overview of European countries



European Electricity Mix (2010)



European electricity market

Interconnected but distinct market zones : still fragmented into « electric plates »



References: RTE, Epex, Belpex, IPEX-GME, OMIE, EDF-T, APX

European electricity average prices and average available commercial capacity in 2012



Electricity market prices

Specific price drivers in each country, depending on several criteria





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Electricity market : main challenges

- Key factors are changing the world :
 - An increase in the urban population: 50% of people living in cities, 70% by 2050
 - Resource scarcity
 - The need to "decarbonise" energy
 - A plural and multi-polar world (new emerging powers: China, Brazil, India, etc.)
 - An ever-more sprawling, decentralised world (urban systems, local energies, smart grids, etc.)
- Today, the energy markets are facing a difficult equation :
 - Reduction of available resources : oil, gas, ...
 - Ageing production assets : nuclear power plants, ...
 - Environmental issues
 - An increasing demand
- Precise forecasting are risky. Too many factors can influence the prices...
- Tomorrow, the energy world will be even more uncertain and volatile.



MODELLING ELECTRICITY PRICES

Main features of power spot prices

- Overview of spot and forward models
- A structural model for electricity prices (Aïd et al., 2011)
- Factorial models for energy prices (e.g. Kiesel et al., 2008)

Spot prices resulting from the supply/demand equilibrium

- Market spot prices result from the supply/demand equilibrium.
- Factors/aleas underlying to supply and demand Fundamentals factors of spot prices





Qualitative features of electricity spot prices

What are the main features of electricity spot prices ?

- Correlation to temperature, cloud cover, sunset (lightning)
- ✓ Multi-scale seasonality linked to economic activity, heating, ...
 - Annual : seasons/months within the year
 - Weekly : days within the week
 - Daily : hours within the day
- Calendar effects (vacation, public holiday)
- ✓ High volatility (higher than on other markets) and seasonal volatility
- ✓ Positive correlation between price level and price volatility
- ✓ Mean-reversion (≠ divergence)
 - Fluctuations around levels representative of the S/D equilibrium
- ✓ Spikes
- ✓ Negative prices !
- \checkmark Opposite dependence of volatility to the supply level
- ✓ Correlation to commodity prices (fuel, CO₂, ...)



Dependence of demand to temperature

- Temperature effect on demand : illustration of the heating and air-conditioning gradients
- Threshold temperatures for summer and winter are identified.
- The temperature has also an effect on market supply : less depth for market purchase in winter.


Seasonality of electricity spot prices

- Weekly and daily seasonality are consequence of the seasonality of demand : high noon (11h-14h) and rush hour (17h-20h) peaks.
- Electricity prices show an hourly shape \Rightarrow **shaping coefficients** are an usual way to reproduce it.





Shaping coefficients for electricity spot prices

- 168 shaping coefficients used to reproduce the historical daily and weekly seasonality.
- For France, which graph corresponds to which month among September, October, November?



Shaping coefficients for electricity spot prices

Herebelow, an example of three sets of weekly shaping coefficients for France.



Mean reverting behavior of electricity spot prices

Electricity prices are governed by an effect of mean-reversion to trends.

- Short-term trend : supply/demand equilibrium, adjustment
- Long-term trend : investments/decommissioning, LT economic cycles





Electricity prices spikes

A spike (and not a jump !) is a fast upward movement followed by a quick return to the same level.

- The price distribution is positively skewed : spikes are mostly positive
- Can be multiplied by 100 in few hours
- Happen mainly in winter : annual seasonality of spikes (as for the volatility)

Why do spikes happen ?

- ✓ due to the non-storability of electricity : cannot absorb demand/supply desequilibrium
- rapid demand upward moves and/or sudden power plant decrease in availability (outages)
- ✓ a seasonal and relatively inelastic demand with respect to prices



Electricity prices spikes due to a very high demand

• On the 8th February, 2012, in France, exceptionally high demand due to the winter cold spell

- At 19h, historic record of 102GW demand (last one was in December 2010, with 97 GW)
- DaH price 10h-11h = 1983€/MWh ⇒ DaH Base price = 368€/MWh



Prices spikes due to a supply/demand desequilibrium

On Monday 19th October, 2009, production « default » in France (Epex Spot) caused by :

- An upward in demand forecast (+ 3000 MWh between Friday and Sunday)
- A very high late morning (9h-12h) demand
- A downward in supply (- 4100 MWh) due to a shutdown of nuclear and hydro power plants



Negative electricity spot prices

When does a negative price appear ?

- ✓ In case of a production based on non-flexible mean of power generation
 - Production units that cannot be turned off and on very fast without a very high cost
 - Renewables depending on external factors (wind, solar)
- This phenomenon is increased by the priority given to renewables for injection on the network.
- In case of a lower demand : in summer, by night, on 25th December, etc.



Negative electricity spot prices

What does a negative price mean ?

- This is not a theoretical concept and so, buyers can receive both money and power !
- Producers accept negative prices to maintain a minimal production rather than pay stopping and restarting costs : this decision create a supply surplus, that can only be solved by paying buyers their production.



Negative electricity spot prices

- First negative prices appear in Germany from 2007, in France from 2010 (Epex Spot)
- This situation happened for example the 25th December, 2012, in France :
 - Upward in wind production (+60% in comparison to the day before)
 - 9GW in consumption (public holiday)
 - Saturation of contractual flows from Germany to France (wind over-production in Germany)
- Illustration in Germany, with the effect of wind energy (~30 GW installed capacity in 2012)
 - The stronger the wind, the lower the electricity price !
 - More often negative prices from a certain level of wind production





Allemagne, données 2012 (Blog http://www.manicore.com)

MODELLING ELECTRICITY PRICES

Main features of power forward prices

- Overview of spot and forward models
- A structural model for electricity prices (Aïd et al., 2011)
- Factorial models for energy prices

Forward prices : effect of term structure

- The volatility decreases with products granularity.
 - Week/monthly products being more volatile than calendar products
 - Any new information (shock) has an important impact at short-term but its effect is diluted on longer time horizons.





Seasonality of forward prices volatility

As for spot prices, the volatility of monthly products tends be seasonal.





Forward prices : effect of maturity

The volatility increases when the quotation date becomes closer to the delivery date.

- Samuelson effect : exponential increase when becoming closer to the maturity
- Linked to the relative effect of available information between short and medium term : possibility or not to adjust production to meet demand





Seasonality of forward prices

The forward level depends on the delivery date as well : seasonality.

- Linked to the way the market anticipates the known cyclic fluctuations of demand
- Economic activity, weather (heating and air-conditioning)





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Using electricity and commodity price models

Why do we need price models?

- 1. Take investments decisions over the long term
 - Based on ecomonic and structural arguments, used for 10-20 years horizon
- Valuate energy derivatives
 - Physical assets and financial contracts in a context of a competitive market
- Assess the risk and hedge energy portfolios
- Propose commercial offers : competitive sales prices but covering the production costs 4.
- Models for both spot and forward prices are required.



Supply \uparrow and demand aleas

Electricity prices models

Typology of prices model and their applications



Electricity prices models

How can we represent the electricity prices features in our models ?

Seasonality

- Usually represented as a determinisitic component of the price signal
- Objective : build an initial market forward curve presenting a seasonal shape
- Discrete method using shaping coefficients or continuous-time method (sine-cosine functions)

Volatility

- Log-normality of prices is mostly often assumed (e.g. financial models)
- Volatility function can be assumed time-dependent
- Mean-reversion : log-prices can be modelled by mean-reverting processes (Ornstein-Uhlenbeck)
- Negative prices : log-normal models become irrelevant...
- Spikes : model with jumps, Lévy-driven stochastic processes, leptokurtic distribution for prices return
- Correlation to commodity prices : multi-dimensional models, structural models
- Correlation to temperature : prices return driven in addition by a stochastic process modelling the temperature level



MODELLING ELECTRICITY PRICES

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Framework : Equilibrium model for spot prices

Main ideas

- Marginal fuel = most convenient fuel to produce electricity among the different available fuels
- The power spot price is given by the marginal fuel cost
- Correction allowing to take into accound price spikes
- Forward prices deduced by the no-arbitrage condition :

Forward price = Expected spot price $F(t,T) = \mathbb{E}\left[S_T | \mathcal{F}_t\right]$

- In this model, the electricty spot price is assumed to be determined by :
 - ✓ The demand
 - ✓ The different technologies for power generation
 - ✓ Their capacities
 - ✓ Some scarcity factor

A structural model for electricity prices (Aïd et al., 2011) Simple case with 2 fuels

- Assume first that there are only 2 technologies for electricity production
- The variables used in this model are :

D_t	Demand (in MW)
C_{t}^{1}, C_{t}^{2}	Capacities (in MW)
S^1_t, S^2_t	Fuel prices
h_{1}, h_{2}	Heat rates with $h_i S_t^i$ in \in /MWh

h_iSⁱ_t corresponds to the price of the quantity of fuel number i necessary to produce 1 MWh
 After a (random but measurable) permutation, production costs can be ordered among fuels :
 h₁S¹_t ≤ h₂S²_t

Then, the electricity spot price can be given by the marginal fuel cost :

$$S_t = h_1 S_t^1 \mathbb{1}_{\{D_t \le C_t^1\}} + h_2 S_t^2 \mathbb{1}_{\{C_t^1 \le D_t\}}$$



A structural model for electricity prices (Aïd et al., 2011) General case with n fuels

More generally, for a set of n technologies for electricity production :

 $\begin{array}{|c|c|c|c|c|} D_t & \text{Demand (in MW)} \\ n & \text{Fuels available } i = 1, \dots, n \\ C_t^i & \text{Capacity for fuel } i \text{ (in MW)} \\ S_t^i & \text{Price of fuel } i \\ h_i & \text{Heat rate associated to fuel } i \text{ with } h_i S_t^i \text{ in } \in /\text{MWh} \end{array}$

The producer order the fuels from the cheapest to the most expensive (random permutation) :

$$h_1 S_t^1 \le h_2 S_t^2 \le \ldots \le h_n S_t^n$$

Then, the spot price given by the marginal fuel cost can be written as :

$$S_t = \sum_{i=1}^n h_i S_t^i \mathbb{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^{i} C_t^k\right\}}$$



A structural model for electricity prices (Aïd et al., 2011) Backtesting the model

- Here, model with two fuels: coal and oil.
- Spikes are not modelled by this first model.



Spot price (in €/MWh)



Improving the model to model spikes

- Model based on the fundamental of electricity prices :
 - Consistency between power prices and fuel prices, power prices and demand
 - All variables are observable (but only electricity and fuels are tradable).
- But, the marginal fuel cost is not exactly the market spot price !
 - Technical constraints
 - Strategic behavior, very high starting costs of peak-load generation plants
 - Effect of margin capacity = capacity limit demand

Improvement of the model :

- Include the modelization of price spikes
- ... by multiplying the marginal fuel cost by a factor, allowing the power price to deviate from the marginal fuel price when the demand becomes closer to the capacity limit
- This factor Scarcity of production capacity (non storability of electricity)



The model, improved to reproduce price spikes

- The marginal fuel cost modelled as previously : $MC_t := \sum_{i=1}^{i} h_i S_t^i \mathbb{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^{i} C_t^k\right\}}$
- Take into account the maximal available power capacity: \boldsymbol{n}

$$\bar{C}_t = \sum_{k=1}^n C_t^k$$

- Price spikes occur when the power system is under stress, corresponding to a small margin capacity of the system $x_t := \overline{C}_t - D_t$
- Scarcity factor estimation \Rightarrow Observe variable $y_t := \frac{S_t}{MC_t}$

as function of the margin capacity

as function of the margin capacity Decreasing relation between $y_t := \frac{S_t}{MC_t}$ and $x_t := C_t - D_t$

High deviation f spot prices w.r.t. the marginal fuel cost





The model, improved to reproduce price spikes

- Use a quantile estimation to fit the relation between $y_t := \frac{S_t}{MC_t}$ and $x_t := \bar{C}_t D_t$
- Estimated relation : $y_t = \frac{\gamma}{x_t^{\nu}} \quad \gamma, \nu > 0$



A structural model for electricity prices (Aïd et al., 2011) Backtesting the improved model

Final modelization for electricity spot prices :

$$S_{t} = g\left(\sum_{k=1}^{n} C_{t}^{k} - D_{t}\right) \times \left(\sum_{i=1}^{n} h_{i} S_{t}^{i} \mathbb{1}_{\left\{\sum_{k=1}^{i-1} C_{t}^{k} \le D_{t} \le \sum_{k=1}^{i} C_{t}^{k}\right\}}\right)$$

with the scarcity function : $g(x) := \min\left(\frac{\gamma}{x^{\nu}}, M\right) \mathbb{1}_{\{x \ge 0\}} + M \mathbb{1}_{\{x \le 0\}}$

Parameter M is used for matching the high cap on electricity spot prices defined on the considered power exchange (cf. technical maximal price of 3000 €/MWh).



Pricing energy derivatives in this model

- In this model, forward prices can be retrieved by the no-arbitrage condition.
- Unitary forward prices are equal to :

$$F(t,T) = \sum_{i=1}^{n} h_i G_i^T(t, C_t, D_t) F^i(t,T)$$

in which G^{T} is a conditional expectation of the scarcity function.

- This equation shows that in this model, an electricity forward is represented as a basket of fuels forwards with stochastic weights driven by electricity demand and production capacities.
- Under some assumptions, we can get quasi-analytical formulas for futures and options' prices.
- Typical assumptions assumed in Aïd et al. :
 - Fuel spot prices are independent from electricity demand and production capacities
 - Diffusion models (geometric Brownian motion) for fuels spreads $Y_t^i = h_i S_t^i h_{i-1} S_t^{i-1}$
 - Diffusion models (deterministic seasonality + Ornstein-Uhlenbeck process) for demand and capacities



MODELLING ELECTRICITY PRICES

- Main features of power forward prices
- Overview of spot and forward models
- A structural model for electricity prices (Aïd et al., 2011)
- Factorial models for energy prices (e.g. Kiesel et al., 2008)



Diffusion models for the forward curve

Factorial models are close to HJM models used for the yield curve (interest rates).

Level-Slope-Curvature approaches

- Based on historical futures prices...
- Principal Component Analysis (PCA) aims at determining factors (that is principal components) in order to explain as much of the total variation of the data as possible
- Allows to order the factors by major contribution
- A PCA shows that 95% of the forward curve dynamics can be modeled by 3 factors :
- 1. Level factor (or Shifting effect) : upwards or downwards movement of the forward curve
- 2. Slope factor (or Twisting effect) : tilting of the curve (curve extremities inversion)
- **3.** Curvature factor (or Bending effect) : curve distortion



Twisting





Some basics on factorial models

Basic decomposition of the (unitary) forward price in a factorial model :

$$F(t,T) = F(t_0,T) Y(t,T)$$

al forward curve Stochastic part (diffusion term)

Initia

Initial forward curve

- Deterministic seasonality
- Typically an hourly curve ٠

 $(F(t_0,T))_{T>t_0}$



Diffusion calibration

- Depending on a limited nb of stoch. factors
- Typically, restriction to 2 or 3 factors
- Corresponds to fitting volatility functions
- Maturity-dependent volatility ٠

Notations : t_0 = start of the diffusion, t = future observation date, T = start of delivery



Case of a constant volatility...

- One Gaussian factor : Geometric Brownian motion ⇒ This is the Black model.
- Log-normal dynamics of the prices :

$$\frac{dF(t,T)}{F(t,T)} = \sigma dW_t \Rightarrow F(t,T) = F(t_0,T) \exp\{-\frac{1}{2}\sigma^2(t-t_0) + \sigma \left(W_t - W_{t_0}\right)\}$$

Initial price

Diffusion term

- In this case, the diffusion does not depend on the maturity.
- For the log-forward curve, the deformation consists here in a translation.

$$\ln F(t,T) = \ln F(t_0,T) - \frac{1}{2}\sigma^2(t-t_0) + \sigma^2(t-t_0)\varepsilon, \varepsilon \sim \mathcal{N}(0,1)$$





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Case of an exponentially weighted volatility

Model by Clewlow and Strickland : mean-reverting model with one Gaussian factor

$$\frac{dF(t,T)}{F(t,T)} = \sigma e^{-a(T-t)} dW_t \quad \begin{cases} \ln F(t,T) &= \ln F(t_0,T) - \frac{1}{2}e^{-2a(T-t)} \operatorname{var} (X_t) + e^{-a(T-t)} X_t \\ X_t &:= \int_{t_0}^t \sigma e^{-a(t-s)} dW_s \sim \mathcal{N} \left(0, \frac{\sigma^2}{2a} \left(1 - e^{-2a(t-t_0)} \right) \right) \end{cases}$$

Model the dependence of the volatility functions w.r.t. the maturity : the deviation of the forward curve w.r.t. the initial forward curve decreases as the maturity increases
Ample Mean-reverting



Model by Kiesel et al. (2008) : allows to better represent the mooves of the forward curve

- 2 correlated Gaussian factors
- Short term : exponentially weighted volatility
- Long term : constant volatility



Illustration with a one factor model

- Diffusion of the forward curve with a single and constant volatility
 - The initial forward curve incorporates the seasonality effect.
 - The forward curve is shifted by a same diffusion factor, whatever the maturity.
- Example with $t_0 = 01/12$, t = 15/12, forward maturities from 01/01/2011 to 01/03/2011




Forward curve modelling by factorial models

Illustration with a two factors model

- Diffusion of the forward curve with a constant LT volatility & weighted ST volatility
 - The impact of the long term volatility is constant, whatever the maturity.
 - The short term volatility is weighted by a mean-reverting coefficient. It has a decreasing impact : high at short term, zero at long term.

Example with $t_0 = 01/12$, t = 15/12, forward maturities from 01/01/2011 to 01/03/2011



Parameters: sigma_S = 100%, a = 10, sigma_L = 10%, rho = 0

