SMIP workshop

Time-multi-scale parameter identification of models describing material fatigue

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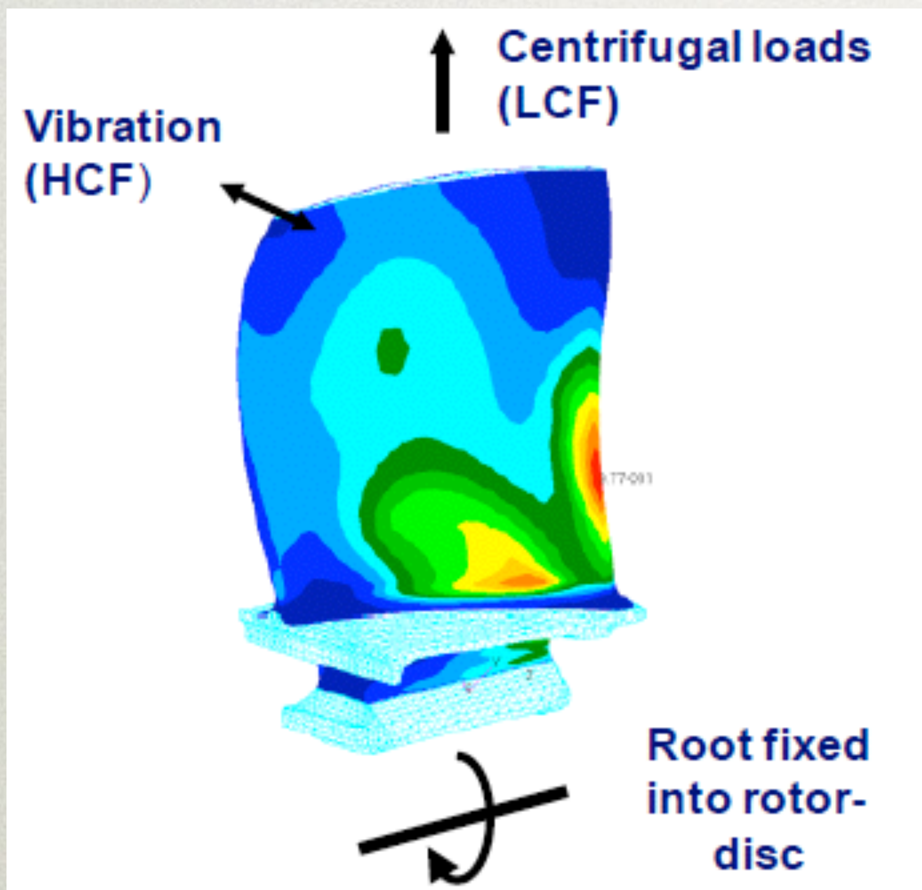
Context

- Rotor blade «Combined Cycle Fatigue» (CCF)

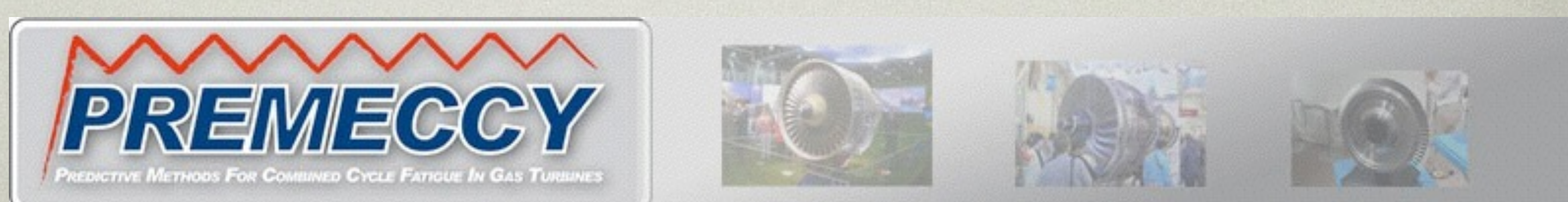


Loading on the blades:

- aerodynamic forces $\leftrightarrow$ high frequency
- centrifugal force $\leftrightarrow$ low frequency

Ratio LF/HF: $\xi \sim 10^{-4}$

EU Project (2006-2011)



PREdictive MEthods for Combined CYcle fatigue in gas turbines

Context

- Necessity of a specific method
  - complex interaction between low- and high-frequency loads, and dynamic effects
    ➡ classical cumulative laws can be inadequate
  - time-dependent simulations required
    ➡ need to efficiently describe the `slow' evolution of a structure withstanding `fast' loading cycles
  - use of a specific method to reduce the associated huge computation cost

Context

- Process: periodic time homogenization method
  [Guennouni & Aubry 1986][Guennouni 1988]
  - separation of two time scales
  - asymptotic expansion
    ➡ time-homogenized problem solved on slow time steps only
- Similarities
  with periodic space homogenization techniques
  [Bensoussan *et al.* 1978, Sanchez-Palencia 1980, ...]

Outline

- Periodic time homogenization
  - basic ingredients
  - simple case of study
  - towards industrial problems
- Time-multi-scale parameter identification
- Prospects

Basic ingredients

- Two independent time scales
  - fast time scale $\tau = \dfrac{t}{\xi}$
  - slow time scale t

$$\xi = \frac{t}{\tau} \ll 1$$

- Total differentiation rule: for $\alpha(t, \tau)$

$$\mathrm{d}_t \alpha = \partial_t \alpha + \frac{1}{\xi} \partial_\tau \alpha$$

  - time derivative w.r.t. slow time: $\partial_t \alpha = \dot{\alpha}$
  - time derivative w.r.t. fast time: $\partial_\tau \alpha = \alpha'$

Basic ingredients

- Quasi-periodicity assumption

$$\alpha(t,\tau) = \alpha\left(t, \tau + \frac{1}{F}\right) \quad \forall t$$

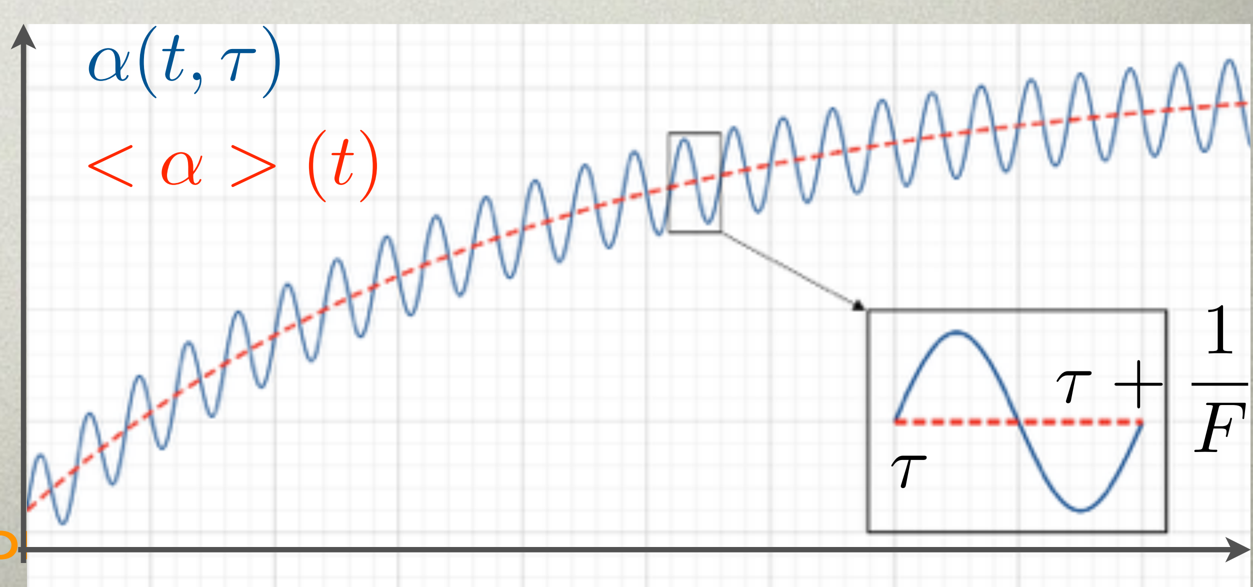
  - periodicity w.r.t. the fast period ξ / F

- Fast-time average

$$< \alpha > (t) = F \int_0^{\frac{1}{F}} \alpha(t,\tau)\,\mathrm{d}\tau$$

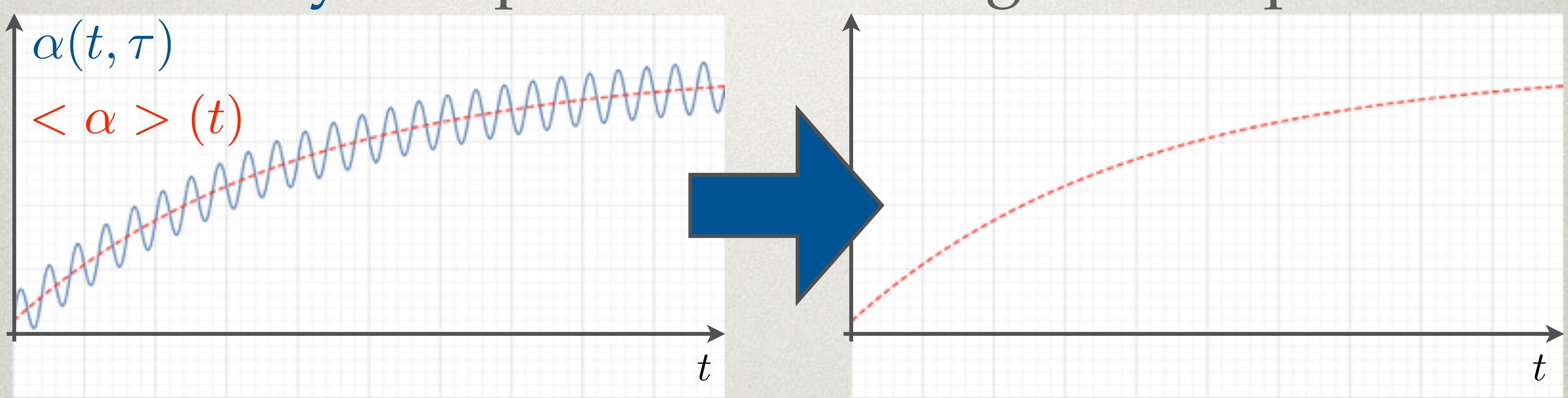
  - quasi-periodicity:

$$< \alpha' > = 0$$



Basic ingredients

- Time homogenization
  - directly compute time-homogenized quantities



  - using asymptotic expansions w.r.t. $\xi = \dfrac{t}{\tau} \ll 1$

$$\varepsilon^{\mathrm{p}}(\mathbf{x},t,\tau) = \varepsilon^{\mathrm{p}}_0(\mathbf{x},t,\tau) + \xi\varepsilon^{\mathrm{p}}_1(\mathbf{x},t,\tau) + O(\xi^2)$$

$$\vdots$$

Outline

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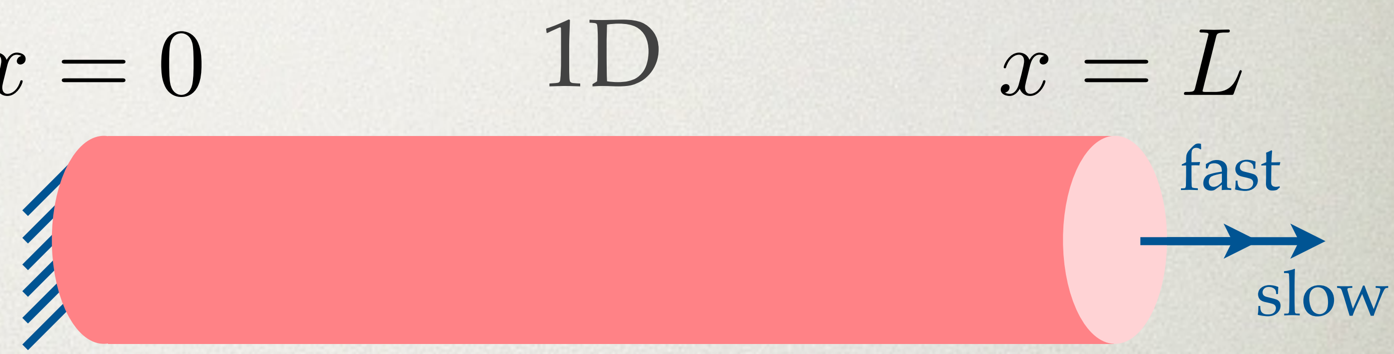
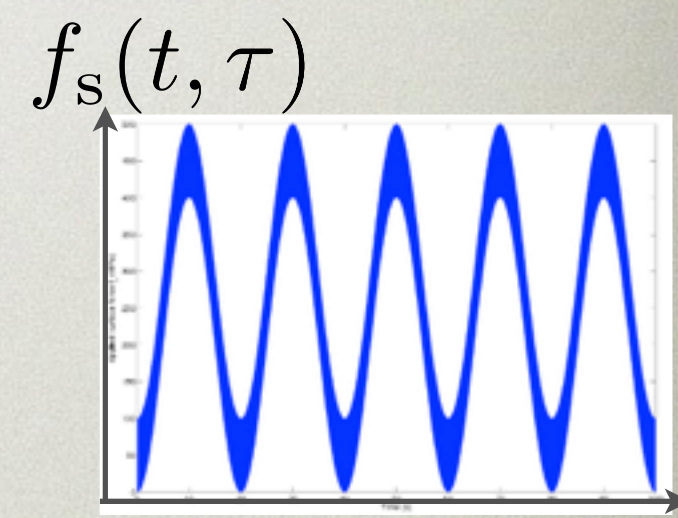
Simple case: tensile test on a bar

- Description
  - cylindrical bar
  - two-frequency tensile load
    0.129Hz / 1290Hz = 1st mode
    amplitudes ratio = 1/4





- Material: titanium alloy
  - viscoplastic flow rule with two hardenings
  - Rayleigh damping (prop. to stiffness)

Simple case: tensile test on a bar

- Reference model
  - PDEs: $\partial_x \sigma + c_K \mathrm{d}_t \partial_x \sigma = \rho \mathrm{d}_t^2 u$

    $\sigma = E\left(\partial_x u - \varepsilon^{\mathrm{p}}\right)$

$\forall x \in (0, L)$
$\forall t, \tau$

$$\mathrm{d}_t p = \left\langle \frac{|\sigma - X| - R - k}{K} \right\rangle_+^n$$
$$\mathrm{d}_t \varepsilon^{\mathrm{p}} = \mathrm{d}_t p \, \mathrm{sign}(\sigma - X)$$
$$\mathrm{d}_t X = \frac{2}{3} C \mathrm{d}_t \varepsilon^{\mathrm{p}} - \gamma_0 \mathrm{d}_t p \, X$$
$$\mathrm{d}_t R = b(Q - R) \mathrm{d}_t p$$

$\Leftrightarrow$ $\mathrm{d}_t \varepsilon^{\mathrm{p}} = a(\sigma)$

- BCs: $\sigma_{|x=L} = f_{\mathrm{s}}$

  $u_{|x=0} = 0$

  $\forall t, \tau$
- zero initial values

[Lemaître & Chaboche 1990]

ZEROTH-ORDER TIME-HOMOGENIZED PB.

- Evolution equation: $\mathrm{d}_t \varepsilon^{\mathrm{p}} = a(\sigma)$
  - asymptotic expansion:

$$\frac{1}{\xi}\varepsilon_0^{\mathrm{p}\,\prime} + (\dot{\varepsilon}_0^{\mathrm{p}} + \varepsilon_1^{\mathrm{p}\,\prime}) + \xi\,(\dot{\varepsilon}_1^{\mathrm{p}} + \varepsilon_2^{\mathrm{p}\,\prime}) + O(\xi^2)$$

$$= a(\sigma_0) + \xi \sigma_1 \mathrm{D}_\sigma a(\sigma_0) + O(\xi^2)$$

  - order -1: $\varepsilon_0^{\mathrm{p}\,\prime} = 0 \;\Rightarrow\; \varepsilon_0^{\mathrm{p}}(x,t)$ only $\;\Rightarrow\; < \varepsilon_0^{\mathrm{p}} >= \varepsilon_0^{\mathrm{p}}$

➡ viscoplasticity is a slow-evolving phenomenon

  - order 0: $\dot{\varepsilon}_0^{\mathrm{p}} + \varepsilon_1^{\mathrm{p}\,\prime} = a(\sigma_0)$ avrg. $\Rightarrow\; < \varepsilon_1^{\mathrm{p}\,\prime} >= 0$

$$\Rightarrow \boxed{\dot{\varepsilon}_0^{\mathrm{p}} = \langle a(\sigma_0) \rangle}$$

ZEROTH-ORDER TIME-HOMOGENIZED PB.

- Equilibrium equation: $\partial_x \sigma + c_K \mathrm{d}_t \partial_x \sigma = \rho \mathrm{d}_t^2 u$
    - asymptotic expansion:

$$\mathrm{d}_t^2 u = \frac{1}{\xi^2} u_0'' + \frac{1}{\xi}(2\dot{u}_0' + u_1'') + (\ddot{u}_0 + 2\dot{u}_1' + u_2'') + O(\xi)$$

    - order -2: $u_0'' = 0 \;\Rightarrow\; u_0(x,t)$ only
    - order -1: $u_1'' = 0 \;\Rightarrow\; u_1(x,t)$ only

$$\vdots$$

➡ the two time scales are not separable!

ZEROTH-ORDER TIME-HOMOGENIZED PB.

- Equilibrium equation: $\partial_x \sigma + c_K \mathrm{d}_t \partial_x \sigma = \rho \mathrm{d}_t^2 u$
  - assumption: $\boxed{\dfrac{\rho L^2 F^2}{E} = \beta \xi^2}$ with $\beta \leq O(1)$
  - equivalent physical criterion:

$$\frac{L}{\lambda_F} = \sqrt{\beta}\xi \quad \text{or} \quad \frac{L}{\lambda_{F/\xi}} = \sqrt{\beta}$$

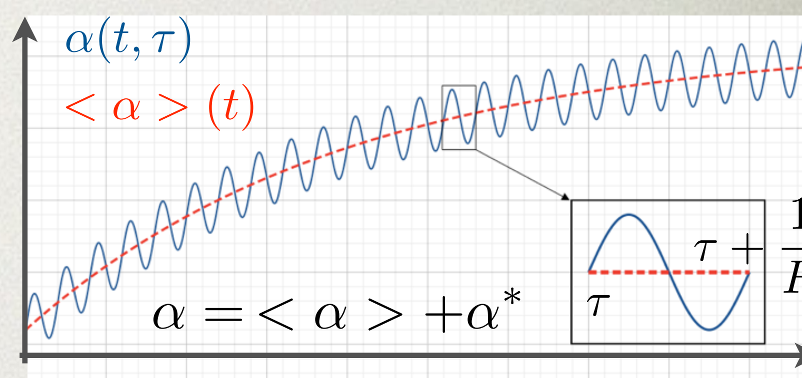
with $\lambda_F, \lambda_{F/\xi}$ wavelengths of propagating waves with frequency F and $\dfrac{F}{\xi}$ respectively

➡ $$\boxed{\rho \mathrm{d}_t^2 u = \frac{\beta E}{L^2 F^2} u_0'' + \xi \frac{\beta E}{L^2 F^2} (2\dot{u}_0' + u_1'') + O(\xi^2)}$$

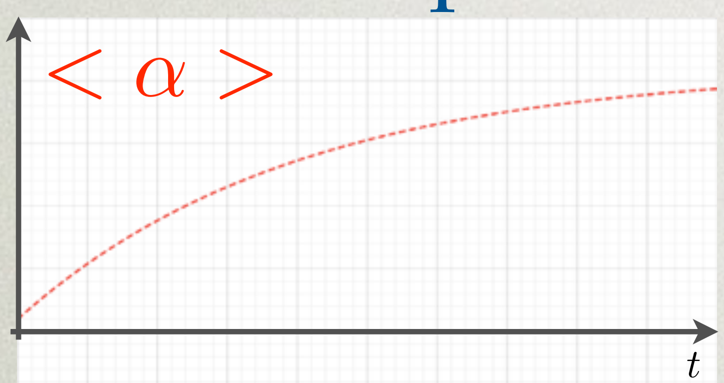
ZEROTH-ORDER TIME-HOMOGENIZED PB.

$$\dot{\varepsilon}_0^{\mathrm{p}} = \langle a(\sigma_0) \rangle$$

$$\sigma_0(x,t,\tau) = <\sigma_0>(x,t) + \sigma_0^*(x,t,\tau)$$



Slow elasto-viscoplastic q.s. pb.



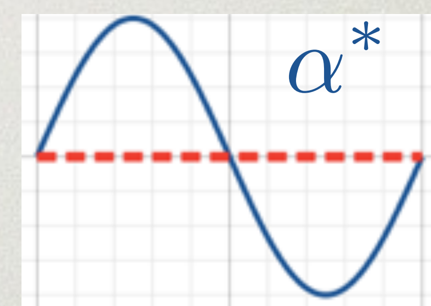
$$\partial_x <\sigma_0> = 0$$

$$<\sigma_0> = E\left(\partial_x <u_0> - \varepsilon_0^{\mathrm{p}}\right)$$

$$<u_0>_{|x=0} = 0$$

$$<\sigma_0>_{|x=L} = <f_{\mathrm{s}}>$$

Fast elastic damped dyn. pb.



$$\partial_x \sigma_0^* + \frac{\gamma}{F}\partial_x {\sigma_0^*}' = \frac{\beta E}{L^2 F^2} {u_0^*}''$$

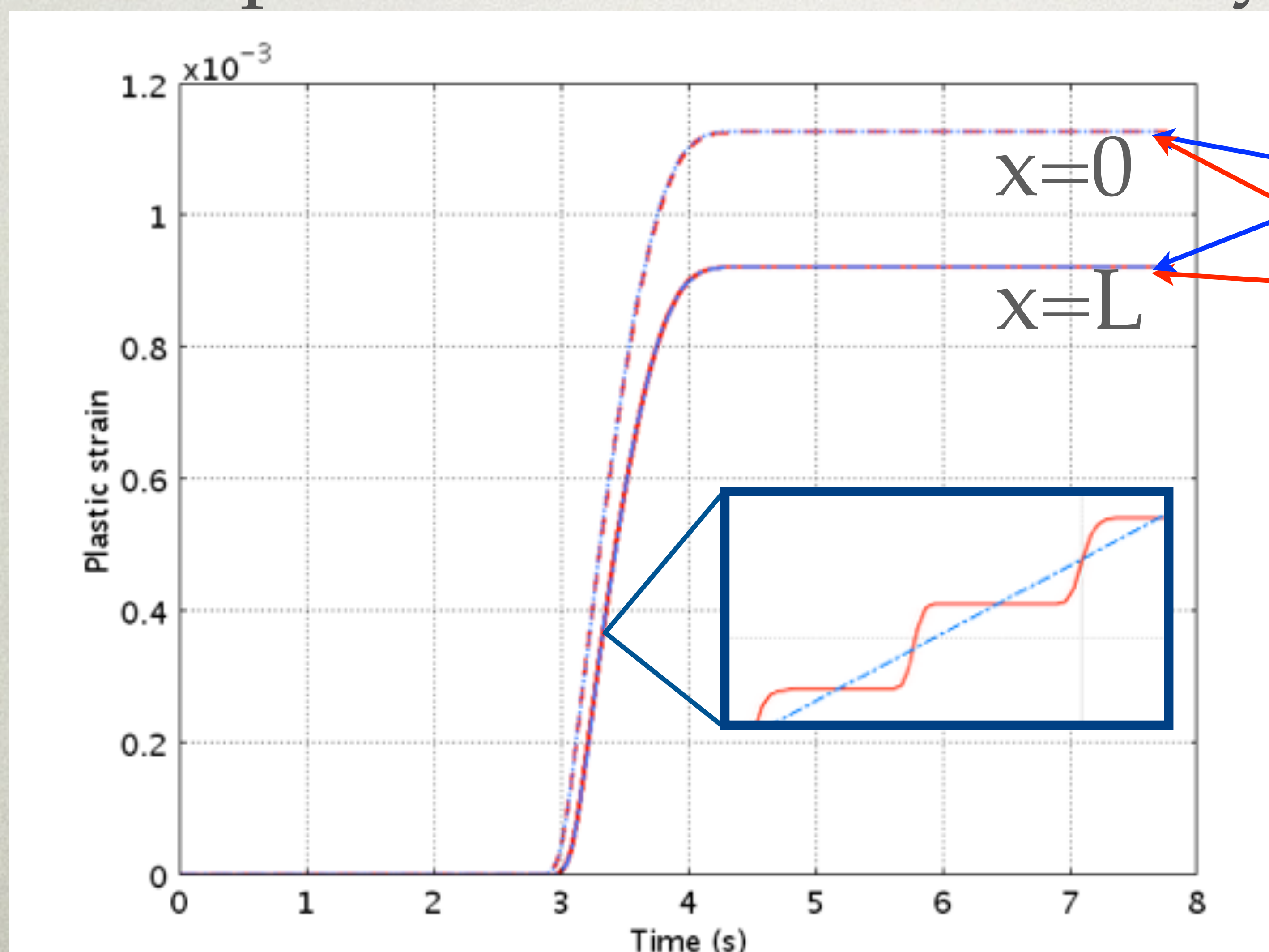
$$\sigma_0^* = E \partial_x u_0^*$$

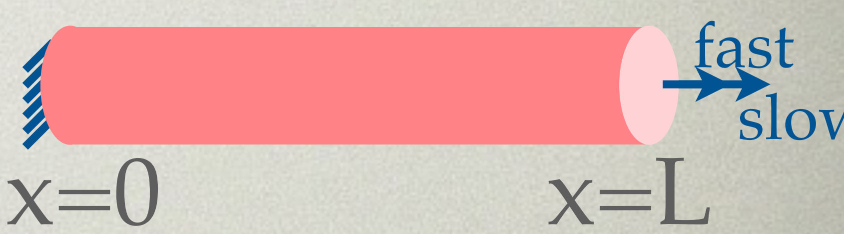
$$u^*_{0\,|x=0} = 0$$

$$\sigma^*_{0\,|x=L} = f_{\mathrm{s}}^*$$

Validation of the method

- Plastic strain: [Puel & Aubry EJCM 2012]
  - comparison for the first slow cycle of $\varepsilon_0^{\mathrm{p}}$

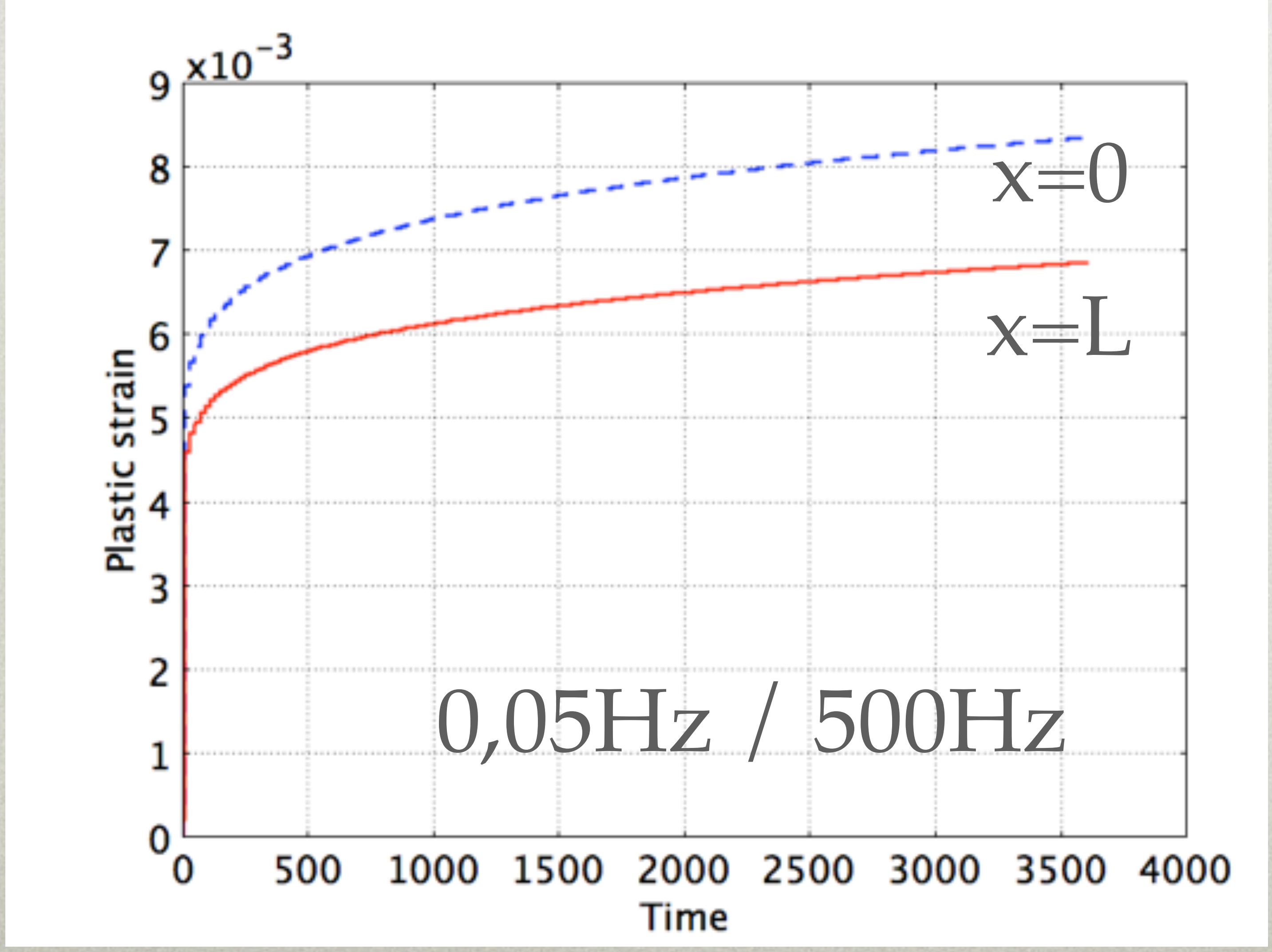




impact of inertial terms

Validation of the method

- Simulation for a 1-hour time interval
  - 180 slow cycles / 1 800 000 fast cycles



classical condition:
36 000 000 time steps

x400

time-homogenized:
90 000 time steps only

Outline

- Periodic time homogenization
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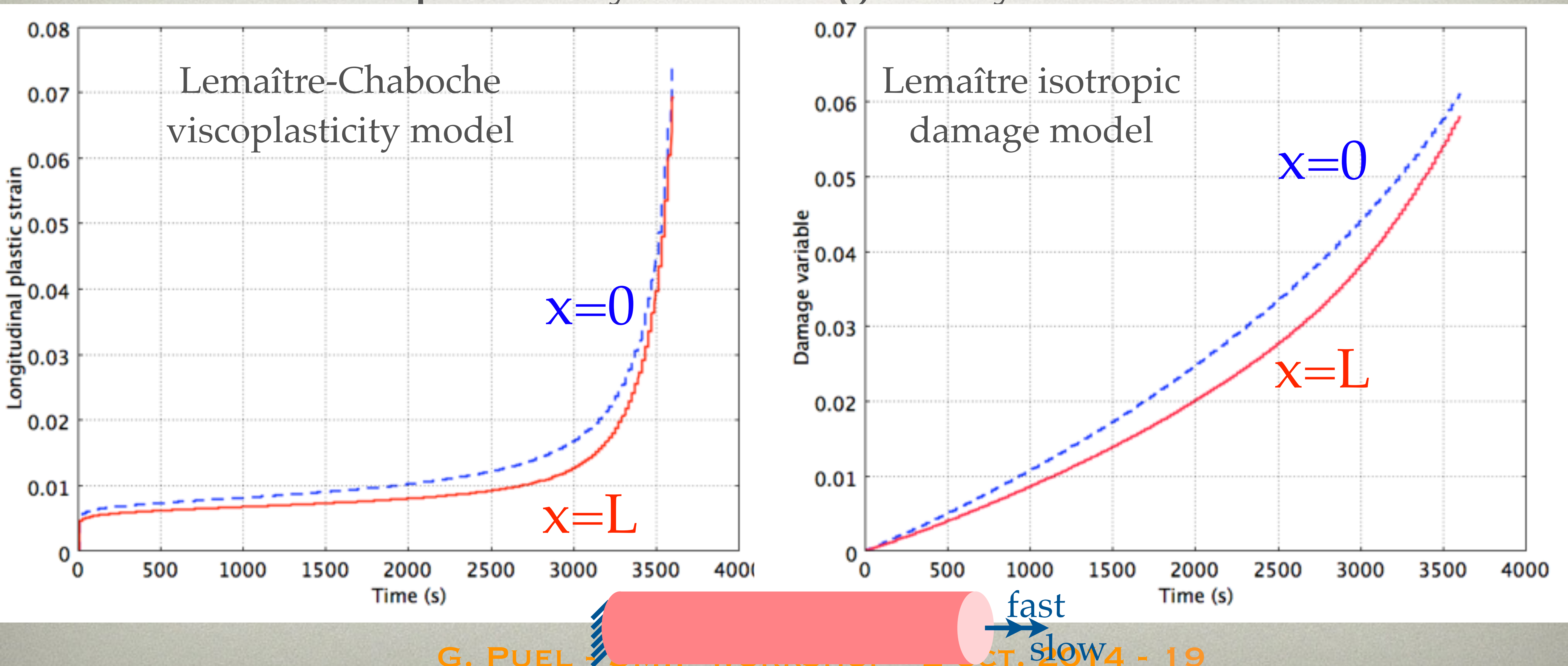
  - towards industrial problems
- Time-multi-scale parameter identification
- Prospects

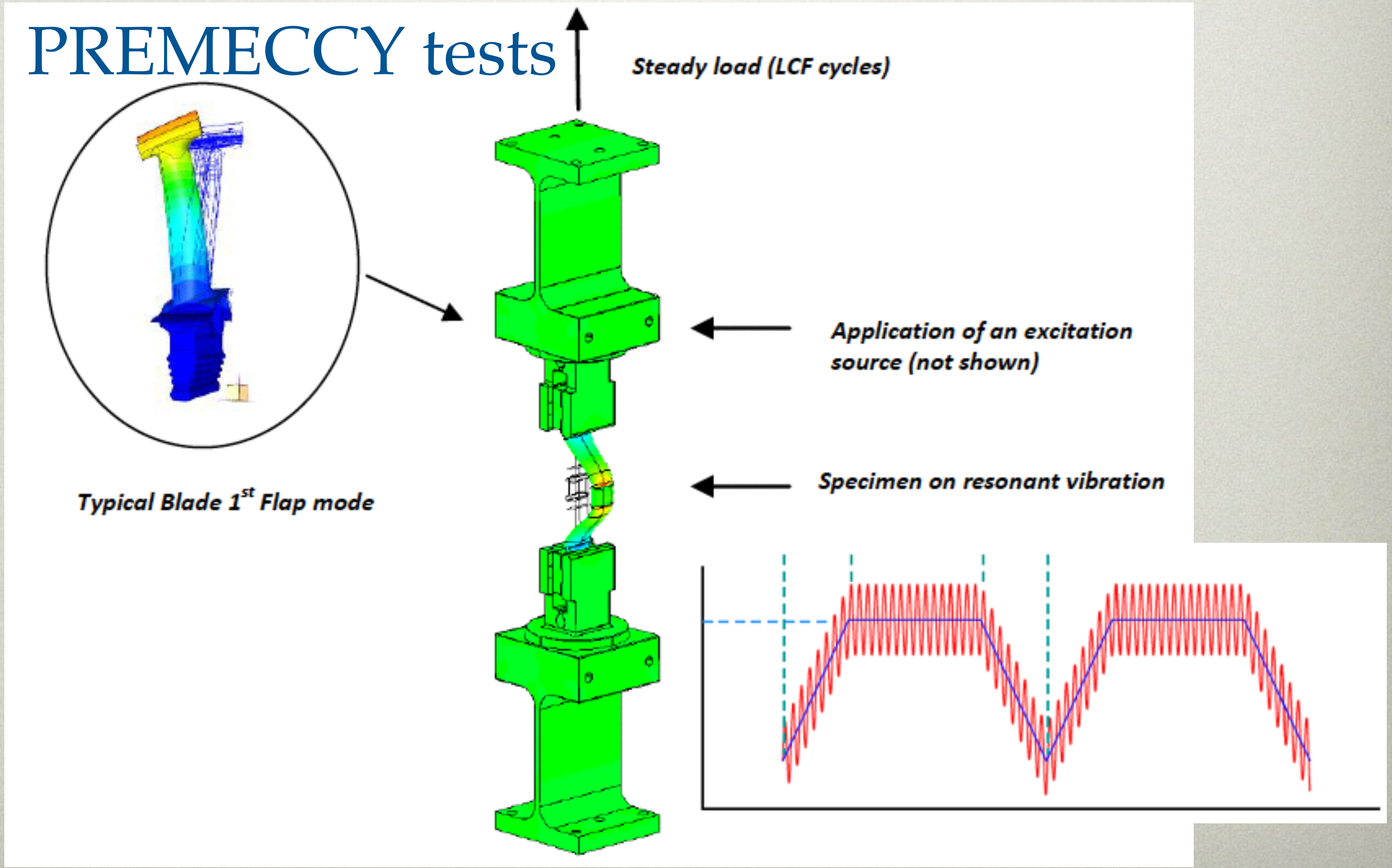
Other material laws

- Material fatigue simulation: CCF
  - viscoplasticity + damage / dynamic case



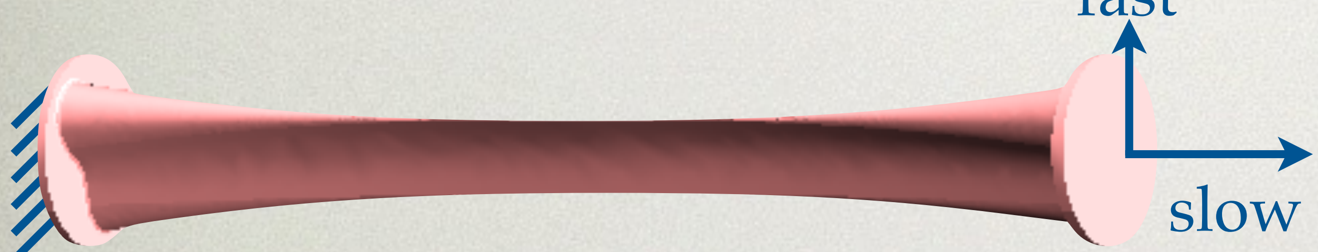
Back to context

- PREMECCY tests

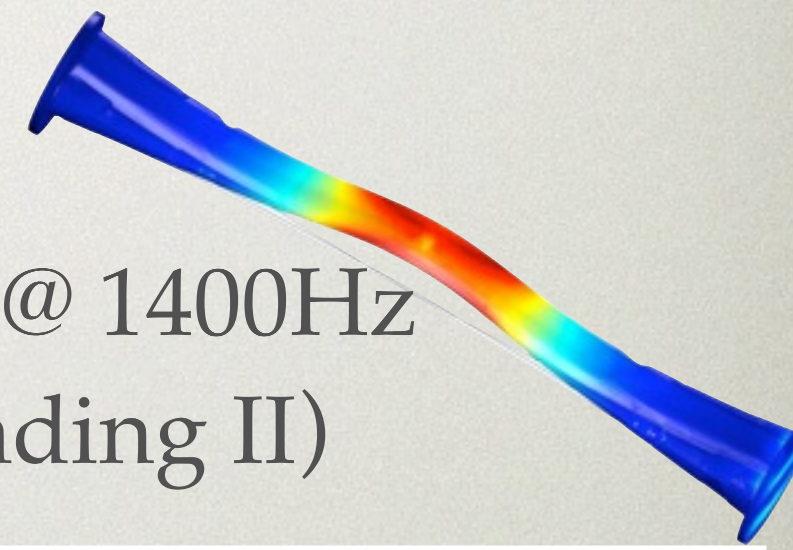


Back to context

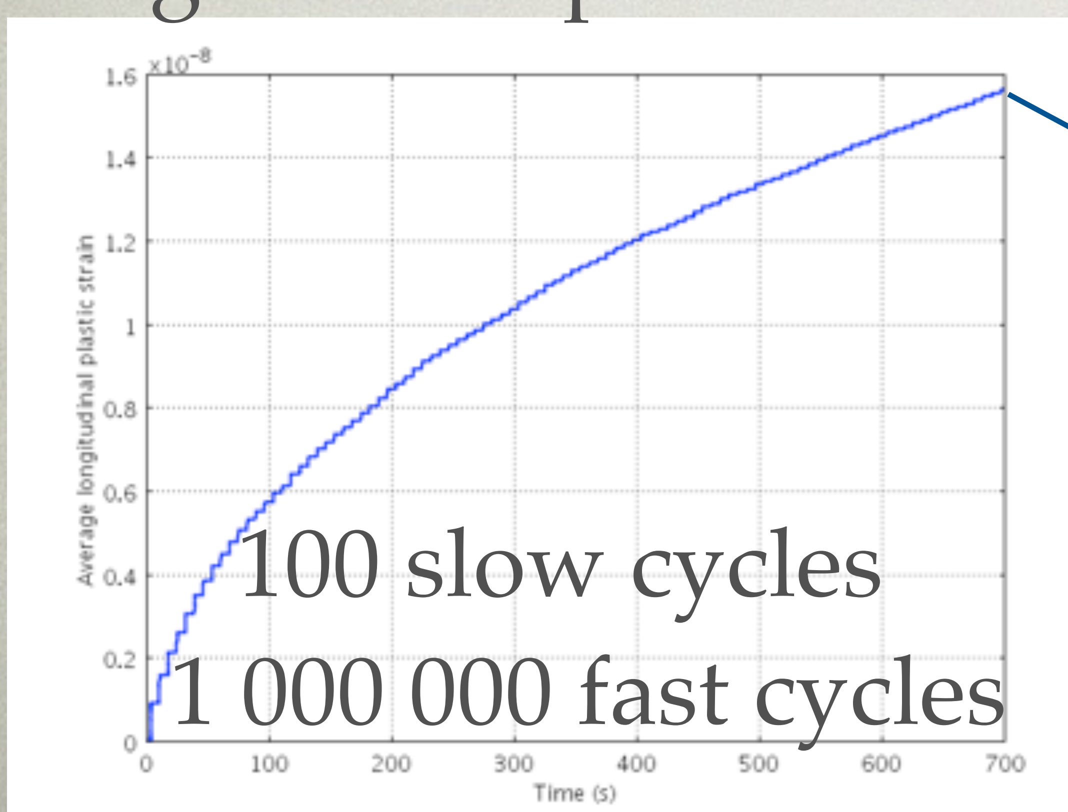
- Blade-shaped specimen: [Puel & Aubry IJMCE 2014]
  - CCF testing: 0.14Hz / 1400Hz

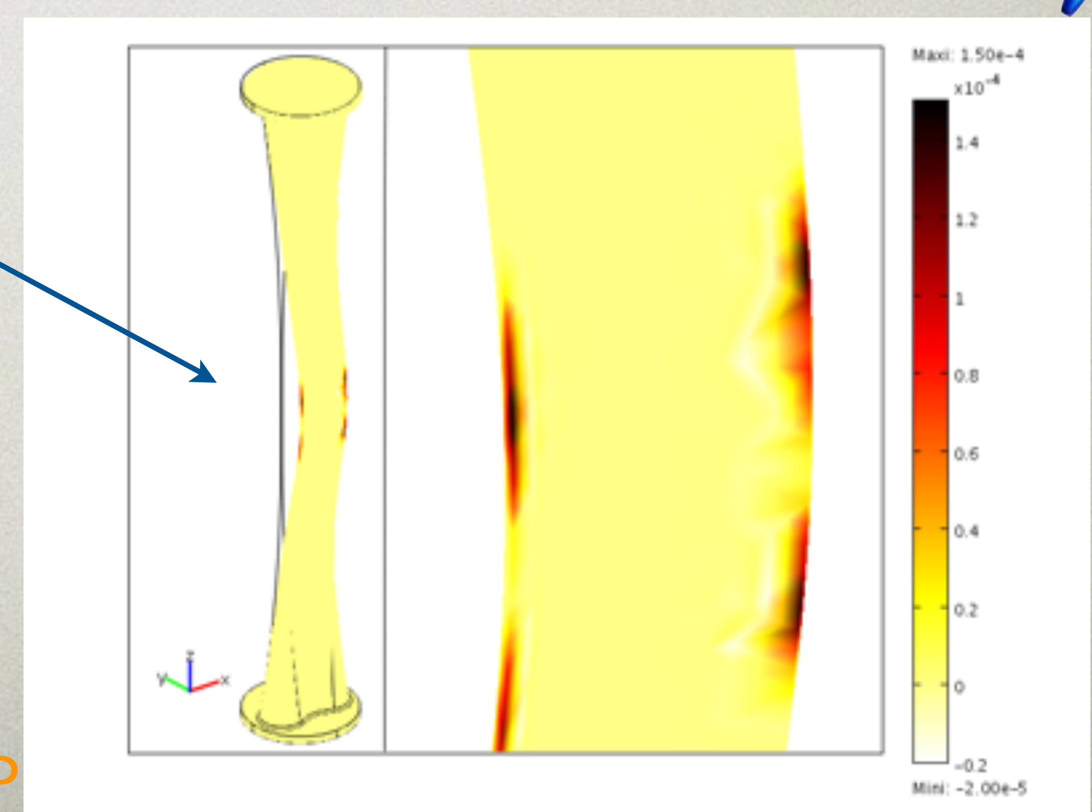


longitudinal plastic strain



mode @ 1400Hz
(bending II)





WORKSHO

Outline

- Periodic time homogenization
- Time-multi-scale parameter identification
  - generic framework
  - application to a simple case
- Prospects

Generic nonlinear model

- Time-dependent forward state equation:

$$\boxed{\mathcal{F}\left(\mathbf{u}(t), \mathbf{v}(t), \mathbf{a}(t), \mathbf{p}, t\right) = \mathbf{0}}$$

$$\mathbf{v}(t) = \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t}(t) \quad \mathbf{a}(t) = \frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}t^2}(t)$$

- = an ODE with initial conditions

$$\mathbf{u}(0) = \mathbf{U}_0 \qquad \mathbf{v}(0) = \mathbf{V}_0$$

- Model with scalar parameters $\mathbf{p}$:

➡ forward state $\mathbf{u}(t; \mathbf{p})$

  - $\mathbf{u}$ of size N = number of DOFs (FE discretization)

Experimental data

- Measurements:
  - associated with some given points only
  - assumption: $\mathbf{A}\mathbf{u}_{exp}(t)$

    with $\mathbf{A}$ linear operator
- Matching DOFs: $\mathbf{A}\mathbf{u}(t;\mathbf{p})$
- Misfit function:
  - discrepancy between model and experiments

Inverse problem

- Misfit function:

$$\mathcal{J}(\mathbf{p}) = \frac{1}{2}\int_0^T |\mathbf{A}(\mathbf{u}(t;\mathbf{p}) - \mathbf{u}_{exp}(t))|^2 \,\mathrm{d}t + \frac{\alpha}{2}|\mathbf{p}-\mathbf{p}_0|^2$$

➡ Tikhonov regularization

- Constrained minimization:
  - equivalent to the stationarity of the Lagrangian:

$$\mathcal{L}(\mathbf{u},\mathbf{p},\mathbf{z}) = \frac{1}{2}\int_0^T |\mathbf{A}(\mathbf{u}(t) - \mathbf{u}_{exp}(t))|^2 \,\mathrm{d}t + \frac{\alpha}{2}|\mathbf{p}-\mathbf{p}_0|^2$$

$$- \int_0^T \mathcal{F}\left(\mathbf{u}(t),\mathbf{v}(t),\mathbf{a}(t),\mathbf{p},t\right)^{\mathrm{T}} \mathbf{z}(t)\,\mathrm{d}t$$

$$-(\mathbf{u}(0)-\mathbf{U}_0)^{\mathrm{T}}\mathbf{z}(0) - (\mathbf{v}(0)-\mathbf{V}_0)^{\mathrm{T}}\frac{\mathrm{d}\mathbf{z}}{\mathrm{d}t}(0)$$

➡ independent variables

Adjoint state

- Stationarity of $\mathcal{L}$ with respect to $\mathbf{u}$:

$$\int_0^T \delta\mathbf{u}(t)^{\mathrm{T}}\Big(\mathbf{A}^{\mathrm{T}}\mathbf{A}(\mathbf{u}(t)-\mathbf{u}_{exp}(t))\Big)\,\mathrm{d}t$$

$$-\int_0^T \Big(\nabla_{\mathbf{u}}\mathcal{F}\delta\mathbf{u}(t)+\nabla_{\mathbf{v}}\mathcal{F}\delta\mathbf{v}(t)+\nabla_{\mathbf{a}}\mathcal{F}\delta\mathbf{a}(t)\Big)^{\mathrm{T}}\mathbf{z}(t)\,\mathrm{d}t$$

$$-\delta\mathbf{u}(0)^{\mathrm{T}}\mathbf{z}(0)-\delta\mathbf{v}(0)^{\mathrm{T}}\frac{\mathrm{d}\mathbf{z}}{\mathrm{d}t}(0)=0$$

  - $\mathbf{z}$ = adjoint state
  - $\nabla_{\mathbf{u}}\mathcal{F}$, $\nabla_{\mathbf{v}}\mathcal{F}$ and $\nabla_{\mathbf{a}}\mathcal{F}$ directional derivatives (↔differentiated forward equation)

Adjoint state

- Adjoint state problem:

$$\nabla_{\mathbf{u}}\mathcal{F}^{\mathrm{T}}\mathbf{z} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\nabla_{\mathbf{v}}\mathcal{F}^{\mathrm{T}}\mathbf{z}\right) + \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\nabla_{\mathbf{a}}\mathcal{F}^{\mathrm{T}}\mathbf{z}\right) = \mathbf{A}^{\mathrm{T}}\mathbf{A}(\mathbf{u} - \mathbf{u}_{exp})$$

  - time-backward ODE
    with two final conditions:

$$\nabla_{\mathbf{a}}\mathcal{F}^{\mathrm{T}}\mathbf{z}_{|t=T} = 0$$

$$\nabla_{\mathbf{v}}\mathcal{F}^{\mathrm{T}}\mathbf{z}_{|t=T} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\nabla_{\mathbf{a}}\mathcal{F}^{\mathrm{T}}\mathbf{z}\right)_{|t=T} = 0$$

  - solved with a change in variables $\tau = T - t$

Optimality conditions

- Misfit function's gradient:

$$\begin{aligned} \nabla_{\mathbf{p}}\mathcal{J}(\mathbf{p}) &= \nabla_{\mathbf{p}}\mathcal{L}(\mathbf{u}(t;p),\mathbf{p},\mathbf{z}(t)) \\ &= \alpha(\mathbf{p}-\mathbf{p}_0) - \int_0^T \nabla_{\mathbf{p}}\mathcal{F}^{\mathrm{T}}\mathbf{z}(t)\,\mathrm{d}t \end{aligned}$$

- Solving the problem:
  - gradient-based minimization algorithm

    $\Delta\mathbf{p}_k$

    $\mathbf{u}(t;\mathbf{p}_k)$ ➡ $\mathbf{z}(t)$ ➡ $\nabla_{\mathbf{p}}\mathcal{J}(\mathbf{p}_k)$

    1 ODE — 1 ODE ($\tau = T - t$) — P integrations
  - optimal computational cost and accuracy

Outline

- Periodic time homogenization
- Time-multi-scale parameter identification
  - generic framework
  - application to a simple case
- Prospects

Synthetic data

- Description:
  - cylindrical bar

  - two-frequency tensile load (0.05Hz / 500Hz) amplitudes ratio = 1/4
  - quasistatic calculation
- Material: steel wih Norton's law
  - parameters: (E, K, n)
- 'Experimental' data: $u_{exp}(t) = u_{synth}(L, t; E, K, n)$

MISFIT FUNCTION: 1ST CHOICE

- Homogenized misfit function:

$$\mathcal{J}^0(E,K,n) = \frac{1}{2}\int_0^T |< u_0 > (L,t;E,K,n) - < u_{exp} > (t)|^2 \,\mathrm{d}t$$

- Experimental data:
  - for the homogenized model: $< u_0 >$

  $< u_0 >$ only known on 'macro' time steps t_k

  - definition of a homogenized experimental quantity: for each t_k

$$< u_{exp} > (t_k) = \frac{F}{\xi}\int_{t_k}^{t_k+\frac{\xi}{F}} u_{exp}(t)\,\mathrm{d}t$$

MISFIT FUNCTION: 1ST CHOICE

- Associated adjoint state:

$$\frac{\mathrm{d}z^0}{\mathrm{d}t}(t) = (< u_0 > (L,t;E,K,n) - < u_{exp} > (t))\, L$$

$$z^0(T) = 0$$

  - solved on 'macro' time steps only

- Misfit function gradient:
  - integrals computed on 'macro' time steps

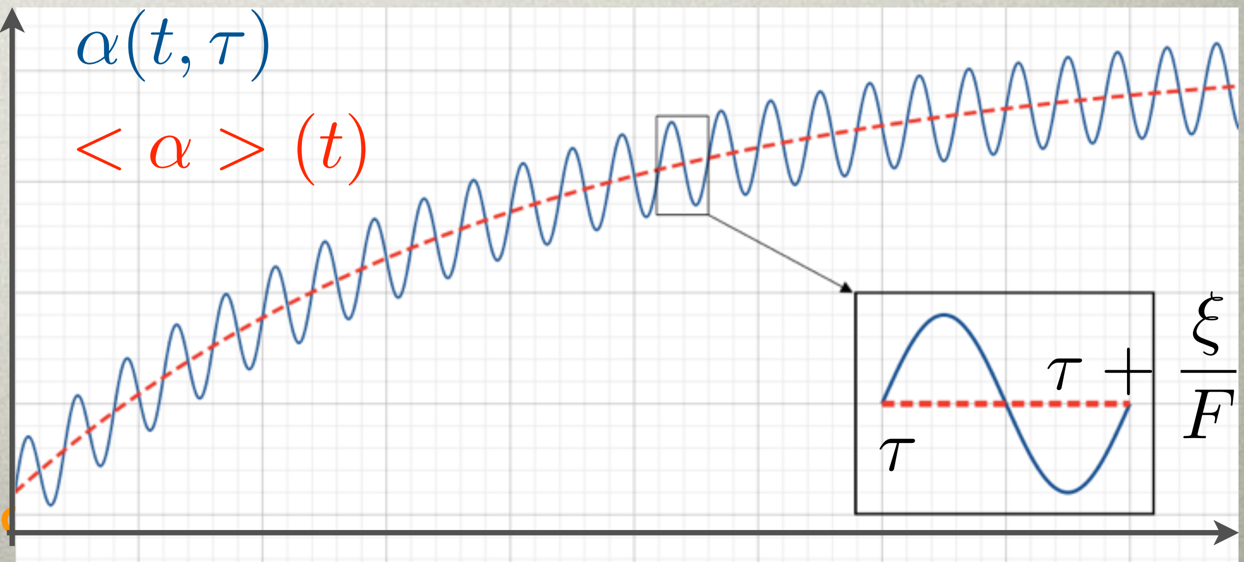
$$\nabla_{\mathbf{p}} \mathcal{J}(\mathbf{p}) \quad = \quad -\int_0^T \nabla_{\mathbf{p}} \mathcal{F}^{\mathrm{T}} \mathbf{z}(t)\, \mathrm{d}t$$

MISFIT FUNCTION: 2ND CHOICE

- ‘Instantaneous’ misfit function:
  - using $u_0^*(L,t,\tau;E)$ with $\tau = t/\xi$

$$\mathcal{J}^{0*}(E,K,n) = \frac{1}{2}\int_0^T |< u_0 > (L,t;E,K,n) + \frac{f_s^*(t/\xi)L}{E} - u_{exp}(t)|^2\,\mathrm{d}t$$

- Evaluation:
  - should be at the fast time scale
  - really necessary? (quasi-periodicity)



MISFIT FUNCTION: 2ND CHOICE

- Associated adjoint state:

$$\frac{\mathrm{d}z^{0*}}{\mathrm{d}t}(t) = \left(< u_0 > (L, t; E, K, n) + \frac{f_s^*(t/\xi)L}{E} - u_{exp}(t) \right) L$$

$$z^{0*}(T) = 0$$

  - should be solved at the fast time scale

- Time-homogenized version:

  - in order to use 'macro' time steps only

$$\dot{z}_0^{0*}(t) = (< u_0 > (L, t; E, K, n) - < u_{exp} > (t))\, L$$

$$z_0^{0*}(T) = 0$$

identical to the first choice!

Time-homogenized inverse problem

- Solving the id. problem: (for the two choices)
  - using the general strategy presented before
  - identical adjoint states eventually:
    adjoint state of homogenized inverse problem

    $$=$$

    homogenized adjoint state of original inv. pb.
  - similar to optimal control problems
    with space periodic homogenization
    [Kesavan & Saint Jean Paulin 1997], [Mahadevan & Muthukumar 2009]

Identification results

- **Parameter identification:** (for the two choices)
  - no regularization, ~20 iterations for each optim.

$\mathcal{J}^0$	exp.	init.	id.
E (GPa)	200	220	200
K (MPa)	100	110	97.6
n	10	11	10.2

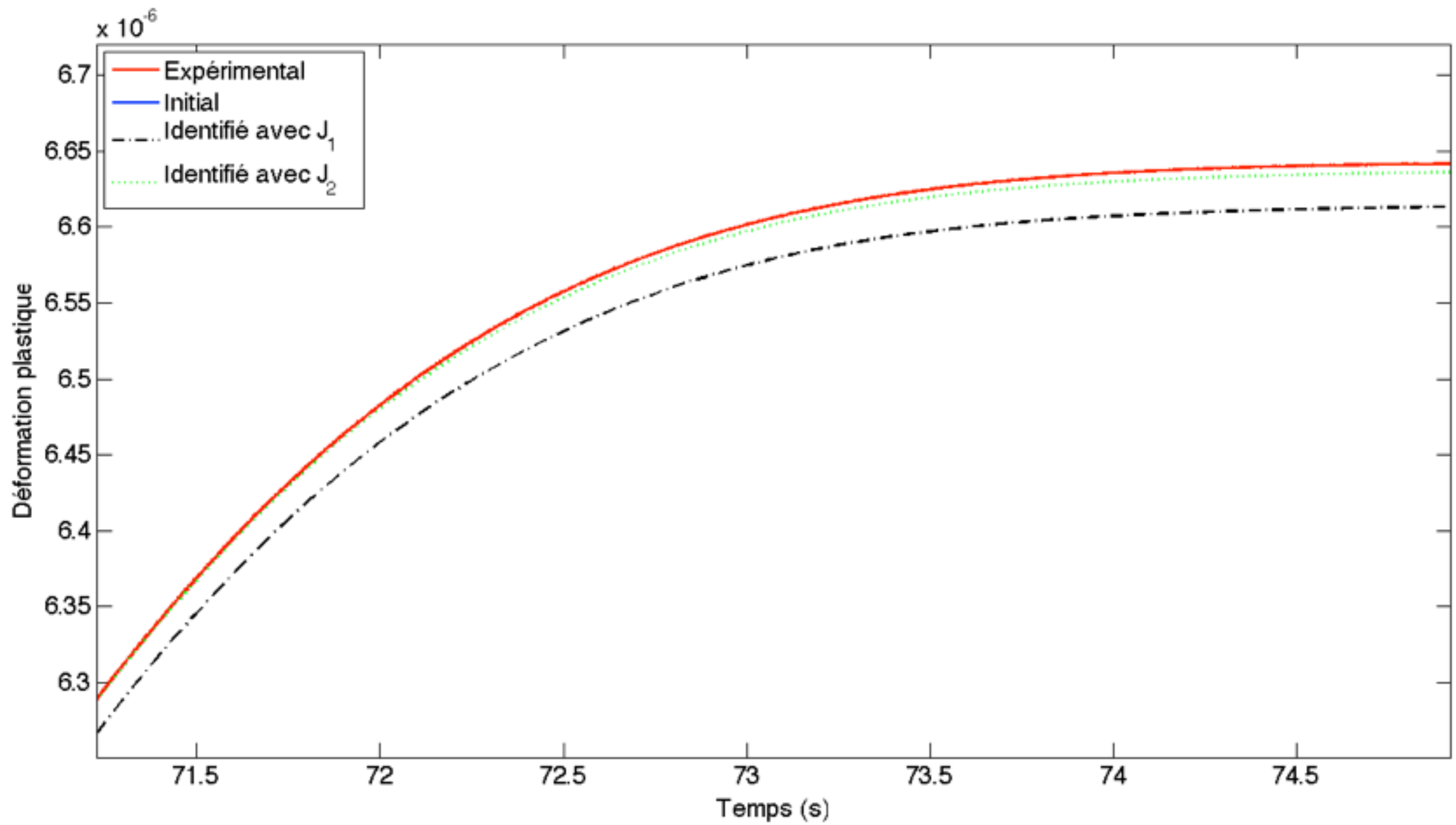
$\mathcal{J}^{0*}$	exp.	init.	id.
E (GPa)	200	220	200
K (MPa)	100	110	97.9
n	10	11	10.1

8 000 time steps for each ODE
- each ODE solution: 800 time steps for each integral
- each integral evaluation: 'general' time steps instead

- 8 000 time steps
- each ODE solution
- → each integral evaluation

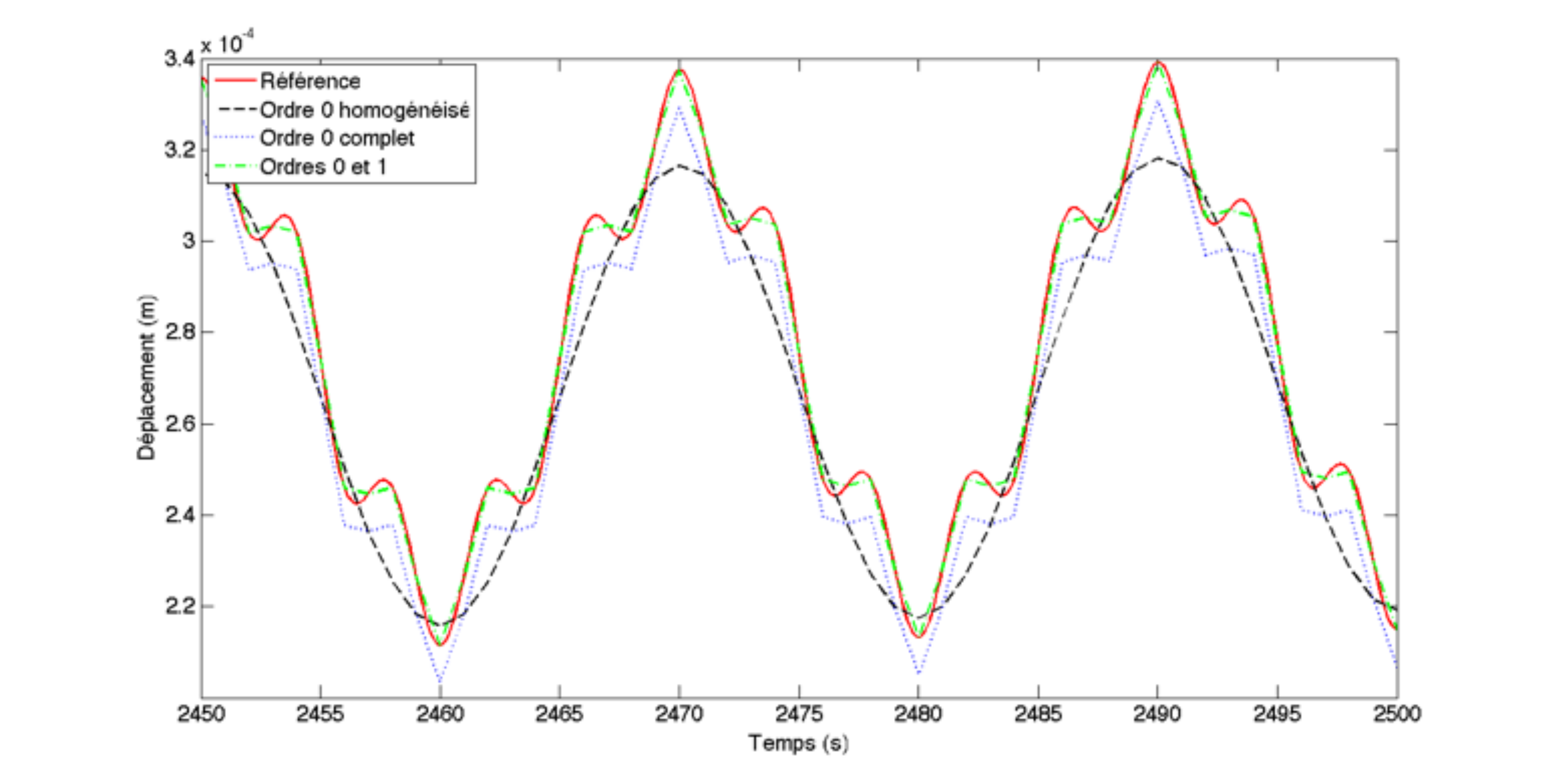
Identification results

- **Parameter identification:** (for the two choices)



Less-separated scales

- What if a bigger ξ?
  - ex. with $\xi = 1/5$: need of an additional order



Less-separated scales

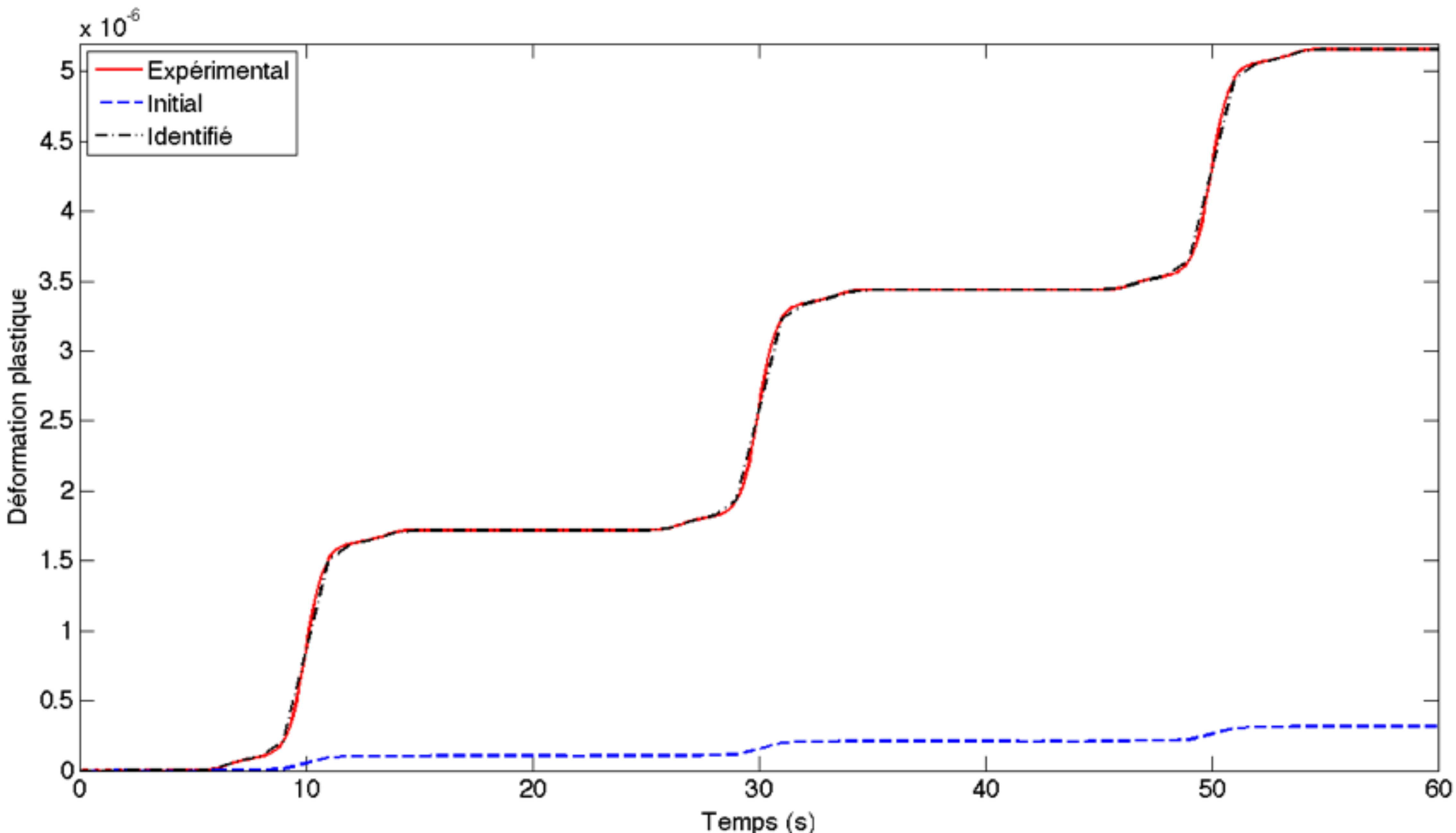
- Associated parameter identification
  - misfit function

$$\mathcal{J}_1(E,K,n) = \frac{1}{2}\int_0^T \Big| < u_0 > (L,t;E,K,n) + \frac{f_s^*(t/\xi)L}{E}$$
$$+\xi u_1(L,t,t/\xi;E,K,n) - u_{exp}(t)\Big|^2 \mathrm{d}t$$

  - additional adjoint state $z_1(t)$ to be used along with $z_0^{0\,*}(t)$ for the gradient estimates

Identification results

- Multi-order parameter identification:



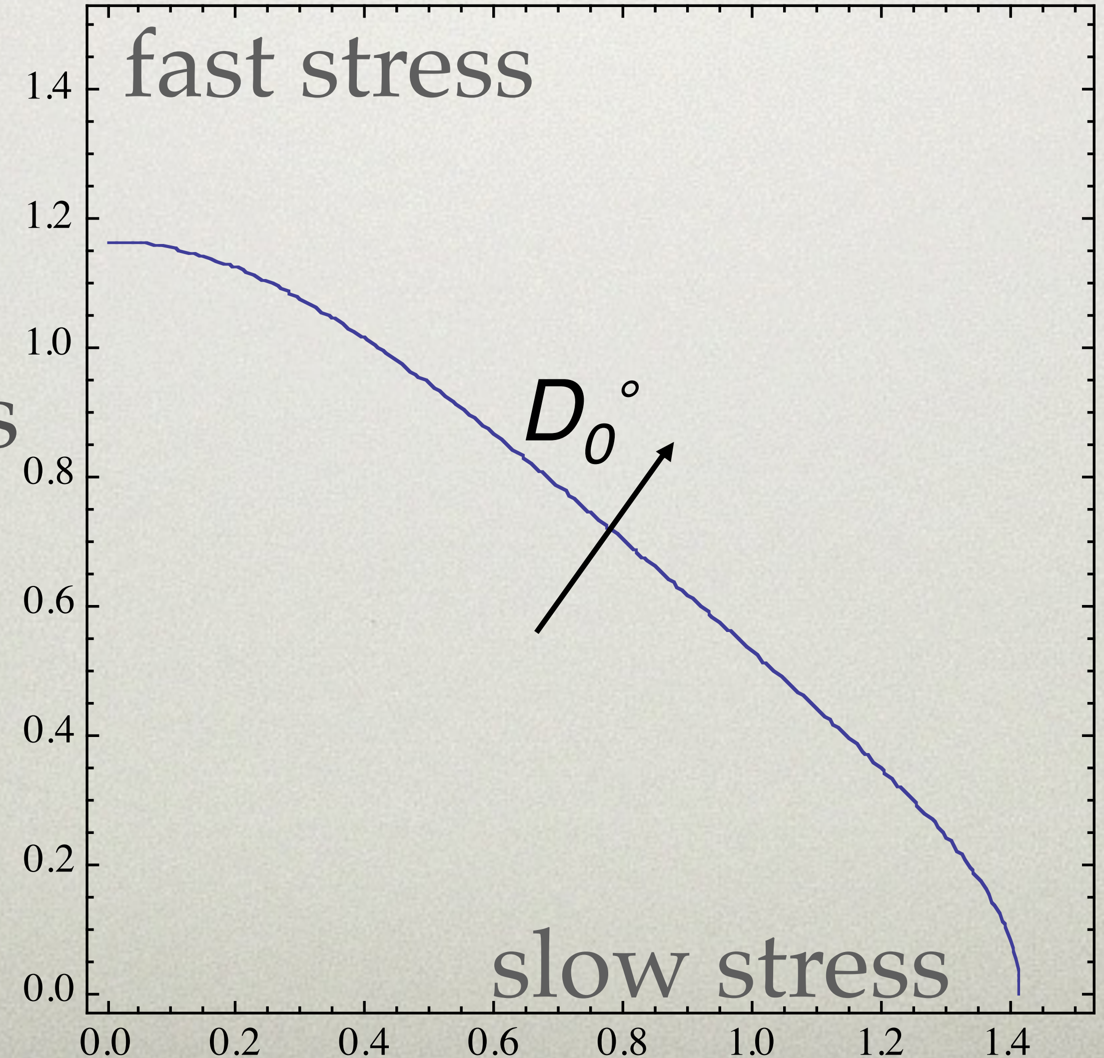
Summary

- Time homog. for material fatigue simulation
  - reduced cost for high numbers of cycles
  - various possible material laws
  - complex loading history taken into account
- Parameter identification of homog. models
  - based on a robust strategy
  - first encouraging methods

Prospects

- Fatigue life estimation:
  - predictions using the identified model

Goodman diagrams for CCF tests (N cycles)



contour plots of limit damage

Prospects

- Use of a 3rd time scale:
  - to further speed up the calculations
  - 'meso' time scale θ

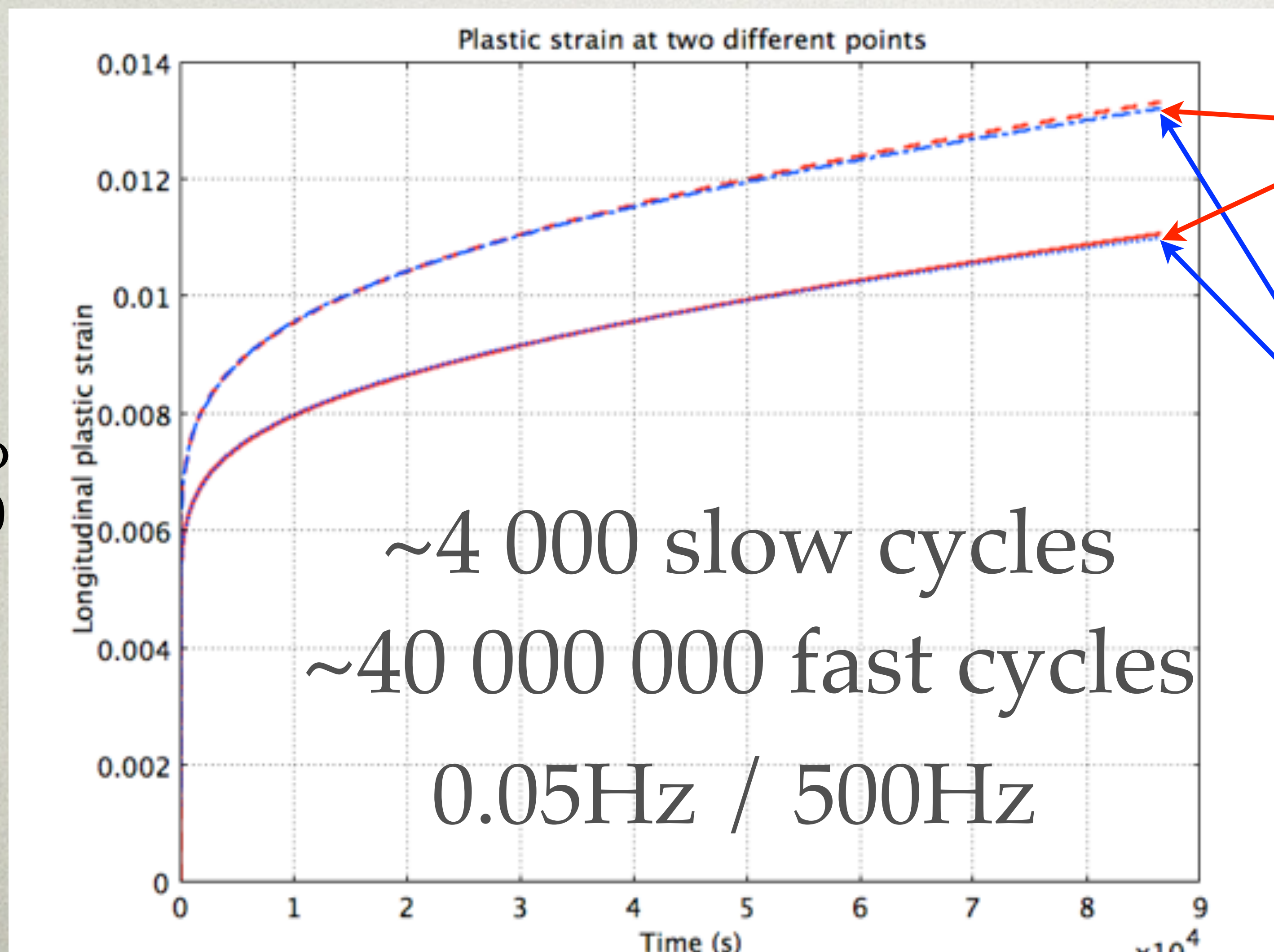
$$\xi = \frac{\theta}{\tau} \ll 1 \qquad \eta = \frac{t}{\theta} \ll 1$$

  - periodicity w.r.t. fast time τ and 'meso' time θ (CCF loading)
  - starting point: initial time-homogenized model

Prospects

- **Validation:** for 3 time scales
  - comparison for a 1-day sim.

ε_0^{p}



~4 000 slow cycles

~40 000 000 fast cycles

0.05Hz / 500Hz

3 time scales

~500 time steps

2 time scales

~200 000 time steps

reference

~10^9 time steps

Prospects

- Towards predictive maintenance
  - for structures in operation
  - no periodicity assumption
  - use alternate frameworks

    e. g. stochastic homogenization ➡ in time

    [Sab 1992]
  - associated identification process

References

- A. Devulder, D. Aubry and G. Puel : Two-time scale fatigue modelling: application to damage. *Computational Mechanics*, 45:6, pp. 637–646, 2010.
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SMIP workshop

Time-multi-scale parameter identification of models describing material fatigue

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