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The results presented in Figure 5 of the published paper need to be corrected. The focus of that paper was the downstream propagation of the saturation front and its interaction with the interface separating the two rocks with distinct capillary pressure curves. These phenomena are correctly handled by the proposed numerical scheme. However, for the chosen physical parameters and the selected observation times in Figure 5, there should also be some upstream (relative to convection) propagation of the saturation front due to degenerate diffusion. The reason for having missed this effect comes from the discretization of the saturation equation, which is a (nonlinear) convection-diffusion equation with degenerate diffusion and rough initial data (piecewise constant). Using harmonic penalties in the discontinuous Galerkin (dG) method will not propagate the left saturation front upstream. Instead, the usual dG method with arithmetic averages and penalties will propagate this front accurately (at least for observation times that are not too small). However, the weighted dG method performs better than the arithmetic dG method, at least on moderately refined meshes, to capture the downstream propagation of the right saturation front and its interaction with the rock

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interface. On highly refined meshes, the performances of the two methods coincide. A more detailed analysis and comparison of the two methods is beyond the present scope. Herein, we simply present the corrected results in Figure 1. In the saturation equation, the arithmetic dG method has been used for $x \leq 0.5$ and the weighted dG method elsewhere. We emphasize that the modification of the weighted dG method is required only because of diffusion degeneracy and rough initial condition. In particular, for the pressure equation, the weighted dG method is still used everywhere in the computational domain. Finally, we observe that the other results reported in the published paper need not be modified. For test case 1, the arithmetic dG method was actually used. For test case 3, the chosen physical parameters and observation times are such that convective effects are always dominant, so that the upstream propagation of the saturation front is negligible.

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Figure 1: Corrected results for Figure 5 of published paper. Left column: saturation profiles at various times and for different mesh sizes and time steps (dotted: $h^{-1} = 80, \tau = 1 \times 10^{-3}$; dashed: $h^{-1} = 120, \tau = 5 \times 10^{-4}$; solid: $h^{-1} = 160, \tau = 2.5 \times 10^{-4}$). Right column: total pressure (solid), capillary pressure (dashed), and wetting-phase pressure (dotted) at different times; $h^{-1} = 160, \tau = 2.5 \times 10^{-4}$. Times from top to bottom are $t = 0.008, t = 0.015, t = 0.045$, and $t = 0.25$. 