Risk of Collusion:
Will Groups of 3 Ruin the FIFA World Cup?

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Risk of Collusion: Will Groups of 3 Ruin the FIFA World Cup?

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Abstract. In 2026, the FIFA World Cup will for the first time gather 48 men's national teams. It will consist of a group stage made of 16 groups of three, with the best two teams in each group advancing to the knockout stage. Using groups of three raises several fairness issues, including the risk of match fixing and schedule imbalance. In this article we examine the risk of collusion. The two teams who play the last game in the group know exactly what results will let them advance to the knockout stage. Risk of match fixing occurs when a result qualifies both of them at the expense of the third team of the group, and can seriously tarnish the tournament. We quantify how often this is expected to happen and explain how to build the match schedule so as to minimize the risk of collusion. We also quantify how the risk of collusion depends on competitive balance. Moreover, we show that forbidding draws during the group stage (a rule considered by FIFA) does not eliminate the risk of match fixing, and that, surprisingly, the 3-2-1-0 point system does not do a better job at decreasing the risk of collusion than the 3-0 point system. Finally we describe alternate formats for a 48-team World Cup that would eliminate or strongly decrease the risk of collusion.

Keywords: Sports, tournament design, FIFA World Cup, groups of three, match fixing, collusion, point system

1. Introduction

The soccer World Cup is the most popular sporting event in the world together with the Olympic Games (Wikipedia, 2018). It is organized every four years by

On January 10, 2017, the FIFA council unanimously decided that starting with the 2026 edition, 48 teams will qualify to the World Cup finals. Interestingly, the press release by FIFA (2017) does not motivate the decision. However, according to The

New York Times, June 11, 2018

The New York Times

TheUpshot

Why Groups of 3 Will Ruin the World Cup (So Enjoy This One)

Unless FIFA changes course, the risk of collusion will be much higher in future Cups.

A new format with 48 teams, starting in 2026 but possibly as soon as the next World Cup, raises a serious fairness issue in the chase for the World Cup trophy. Alexander Zemlianichenko/Associated Press

By Julien Guyon

June 11, 2018

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If you're a World Cup aficionado, you may want to take the time to savor this one, because it may be the last version with a format — groups of four teams — that has a fair and consistently exciting

Motivation

- Soccer World Cup: the **most popular sporting event in the world** together with the Olympic Games.
- Organized every 4 years by FIFA.
- Evolution of the format:

1930	1934–1978	1982–1994	1998–2022	2026–
13 teams	16 teams	24 teams	32 teams	48 teams
4 groups of 3 or 4	4 groups of 4	6 groups of 4	8 groups of 4	16 **groups of 3**

- Each group will play a single round-robin tournament, and **in each group the winner and runner-up will advance to the KO stage**.

Using groups of 3 raises several fairness issues

Match 1	Match 2	Match 3
A–B	A–C	B–C

- Problem 1: Schedule imbalance
- **A more serious issue is the subject of this talk: the risk of collusion (match fixing).**
- After Match 2, Teams B and C will know what results of Match 3 will let them advance to the KO stage. **Risk of collusion occurs when a result lets both of them advance, at the expense of Team A.**
- It can badly harm the tournament and more globally the game of soccer, **whether the match is actually fixed or not**, since outcome uncertainty is at the very root of sport's popularity.

Risk of collusion: The "Disgrace of Gijón"

- **"Disgrace of Gijón"** (1982): most famous example of match fixing in the history of soccer. West Germany beat Austria 1-0, both teams qualified at the expense of Algeria who had played the day before.
- To prevent this to happen again, FIFA decided that all teams in a given group would play their last group match at the same time. But with groups of 3?

Risk of collusion already exists in groups of 4

Even in groups of 4, playing the last two group games at the same time does not fully prevent the risk of collusion:

- Denmark vs Sweden (2-2) at UEFA Euro 2004
- Peru vs Colombia (1-1): last minutes of World Cup qualifier (1-1) in October 2017
- Denmark vs France (0-0) at the 2018 World Cup
- Japan vs Poland (0-1) at the 2018 World Cup: last 15 minutes

Risk of collusion will be worse in groups of 3. Our goal: quantify it.

The Sun, June 26, 2018

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WORLD CUP 2018
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'SO OBVIOUSLY FIXED' World Cup 2018: France and Denmark spark 'fix' claims on social media after first 0-0 draw of the tournament

Both teams are booed throughout by their own fans as dreary stalemate guarantees they qualify for the last 16

Ian Tuckey
26 Jun 2018, 16:59 | Updated: 27 Jun 2018, 0:58

Comment now

FRANCE and Denmark sparked "fix" claims on social media after both teams were booed by their own fans throughout the first 0-0 draw of the World Cup.

A dreary stalemate guaranteed the European rivals would reach the last 16, despite Peru beating Australia 2-0 in today's other Group C match.

Objectives

- **Quantify the risk of collusion** in groups of 3, when 2 teams advance to the next phase.
- Quantify **impact of match schedule** on the risk of collusion.
- Quantify **impact of competitive balance** on the risk of collusion.
- Quantify **impact of point system**: 3-1-0 vs alternate point systems that forbid draws (3-0 and 3-2-1-0).
- **Suggest alternate formats** for a 48-team World Cup that would decrease or even eliminate the risk of collusion.

Occurrences of possibility of match fixing

Assumptions:

- 3-1-0 point system.
- Classical tie-breaking rules: overall goal difference, overall goals scored, etc.

Definitions:

- **RMF** = Risk of Match Fixing
- We say that the RMF is **aggravated** when Team B or C can **win the group even after losing its last game** $\longrightarrow$ **RMF***

Situations of possible collusion after Match 2

Proposition

Risk of match fixing occurs exactly in the following cases:

1. *Team A has one draw and one loss.*
2. *Team A has two draws.*
3. *Team A has one win and one loss and $GD_A \leq 0$.*

Aggravated risk of match fixing occurs if and only if Team A has one win and one loss and

- $GD_A < 0$, *or*
- $GD_A = 0$ *and* $i < j$.

- $\mathrm{GD}_A =$ goal difference of Team A after Match 2.
- When A has one win and one loss and $\mathrm{GD}_A = 0$:
  $i =$ number of goals conceded by A in the match it won,
  $j =$ number of goals scored by A in the match it lost.
- For example, when A wins 1-0 against B and loses 3-2 against C, then $i = 0$ and $j = 2$.

The corresponding probabilities

Win prob.	A	B	C
A		p_{AB}	p_{AC}
B	p_{BA}		p_{BC}
C	p_{CA}	p_{CB}	

Table: Win probabilities: p_{XY} is the probability that Team X wins against Team Y; $d_{XY} := 1 - p_{XY} - p_{YX}$; $p_{<0}$ = probability that $\mathrm{GD}_A < 0$ *given that A has one win and one loss in the group stage*

Situation of Team A after Match 2	Probability	RMF	RMF*
Two wins	$p_{AB}p_{AC}$		
One win and one draw	$p_{AB}d_{AC} + d_{AB}p_{AC}$		
One win and one loss, $\mathrm{GD}_A > 0$	$p_{>0}(p_{AB}p_{CA} + p_{BA}p_{AC})$		
One win and one loss, $\mathrm{GD}_A = 0$, $i \geq j$	$p_{0,i\geq j}(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔	
One win and one loss, $\mathrm{GD}_A = 0$, $i < j$	$p_{0,i<j}(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔	✔
One win and one loss, $\mathrm{GD}_A < 0$	$p_{<0}(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔	✔
Two draws	$d_{AB}d_{AC}$	✔	
One draw and one loss	$p_{BA}d_{AC} + d_{AB}p_{CA}$	✔	
Two losses	$p_{BA}p_{CA}$		

Probability of match fixing opportunity

Proposition

The probability that there is a risk of match fixing in a given group of 3 is

$$p_{\mathrm{RMF}} := p_{BA} d_{AC} + d_{AB} p_{CA} + d_{AB} d_{AC} + p_{\leq 0} \left(p_{AB} p_{CA} + p_{BA} p_{AC} \right).$$

The probability of an aggravated risk of match fixing in a given group of 3 is

$$p^*_{\mathrm{RMF}} := \left(p_{<0} + p_{0,i<j} \right) \left(p_{AB} p_{CA} + p_{BA} p_{AC} \right).$$

- Assume perfect competitive balance:
  all win probabilities $= p \leq 1/2$
  all draw probabilities $= 1 - 2p$

$$p_{\mathrm{RMF}} = 2p(1-2p) + (1-2p)^2 + 2p_{\leq 0} p^2 = 1 - 2p + 2p_{\leq 0} p^2$$

- When $p = 3/8$ and $p_{\leq 0} = 0.6$, $p_{\mathrm{RMF}} \approx 42\%$.
- $\Longrightarrow$ **In the situation of perfect competitive balance, the risk of collusion is very high**.

Probability of match fixing opportunity

Corollary

The probability that there is a risk of match fixing is maximum, equal to 1, in the case where $d_{AB} = d_{AC} = 1$.

- This corollary explains why **FIFA has considered banning draws** during the group stage. All group stage matches would have a winner and a loser, possibly decided by a penalty shootout.
- **Forbidding draws does not eliminate the risk of collusion**. The situations where A has one win and one loss and $\mathrm{GD}_A \leq 0$ would still be prone to match fixing.

Probability of match fixing opportunity

Corollary

The probability that there is a risk of match fixing is minimum, equal to 0, if and only if one of those 3 conditions holds:

(i) $p_{AB} = 1$ *and* ($p_{CA} = 0$ *or* $p_{\leq 0} = 0$)*: A surely wins against B, and it cannot lose against C, or if it loses against C its global goal difference* GD_A *can only be positive.*

(ii) *Same with B* $\longleftrightarrow$ *C.*

(iii) $p_{BA} = p_{CA} = 1$*: A surely loses against B and C.*

- $\Longrightarrow$ To minimize probability of RMF, **Team A should be the a priori strongest team in the group or the a priori weakest team in the group, if very weak**. Team A should not be the middle team.
- Conditions (i), (ii), or (iii) are never satisfied in practice: **Risk of match fixing cannot be avoided**.

Probability of match fixing opportunity in at least one of the 16 groups

Proposition

Assume that the same values of p_{AB}, p_{BA}, p_{AC}, p_{CA}, $p_{<0}$, and $p_{\leq 0}$ apply to all 16 groups of the World Cup, and that the results in the 16 groups are all independent. Let N_{RMF} (resp. N^*_{RMF}) be the number of groups in which RMF (resp. aggravated RMF) occurs. Then for all $k \in \{0, 1, \ldots, 16\}$,

$$\begin{aligned}
\mathbb{P}(N_{\mathrm{RMF}} = k) &= \frac{16!}{k!(16-k)!} p^k_{\mathrm{RMF}} (1 - p_{\mathrm{RMF}})^{16-k} \\
\mathbb{P}(N^*_{\mathrm{RMF}} = k) &= \frac{16!}{k!(16-k)!} (p^*_{\mathrm{RMF}})^k (1 - p^*_{\mathrm{RMF}})^{16-k}.
\end{aligned}$$

In particular, the **probability that there is RMF for at least one group is**

$$p_{\mathrm{RMF}}(16) = 1 - (1 - p_{\mathrm{RMF}})^{16}, \quad p^*_{\mathrm{RMF}}(16) = 1 - (1 - p^*_{\mathrm{RMF}})^{16}.$$

There are on average $\mathbb{E}[N_{\mathrm{RMF}}] = 16\, p_{\mathrm{RMF}}$ (resp. $\mathbb{E}[N^*_{\mathrm{RMF}}] = 16\, p^*_{\mathrm{RMF}}$) groups in which RMF (resp. aggravated RMF) occurs.

Impact of match schedule

Win prob.	S	M	W
S (Strong)		$p_{SM} = 50\%$	$p_{SW} = 80\%$
M (Middle)	$p_{MS} = 20\%$		$p_{MW} = 50\%$
W (Weak)	$p_{WS} = 5\%$	$p_{WM} = 20\%$	

Table: Win probabilities for a reasonably imbalanced group.

A	S	M	W
$p_{\leq 0}$	30%	60%	90%
$p_{<0} + p_{0,i<j}$	18%	48%	84%
p_{RMF}	14.6%	47.4%	52.7%
p^*_{RMF}	3.3%	13.9%	15.5%
$p_{\text{RMF}}(16)$	91.9%	99.997%	99.999%
$p^*_{\text{RMF}}(16)$	41.8%	90.9%	93.3%
$\mathbb{E}[N_{\text{RMF}}]$	2.3	7.6	8.4
$\mathbb{E}[N^*_{\text{RMF}}]$	0.5	2.2	2.5

$\Longrightarrow$ **Team A should be the a priori strongest team in the group.**

Impact of competitive balance

	Perfect balance			Imbalance			Strong imbalance		
Win prob.	S	M	W	S	M	W	S	M	W
S (Strong)		37.5%	37.5%		50%	80%		70%	90%
M (Middle)	37.5%		37.5%	20%		50%	10%		70%
W (Weak)	37.5%	37.5%		5%	20%		2%	10%	

Table: Win probabilities: p_{XY} is the probability that Team X wins against Team Y.

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0} + p_{0,i<j}$	48%	18%	48%	84%	18%	48%	84%
p_{RMF}	41.9%	14.6%	47.4%	52.7%	5.9%	50.0%	34.6%
p^*_{RMF}	13.5%	3.3%	13.9%	15.5%	1.9%	24.0%	8.7%
$p_{\mathrm{RMF}}(16)$	100.0%	91.9%	100.0%	100.0%	62.3%	100.0%	99.9%
$p^*_{\mathrm{RMF}}(16)$	90.2%	41.8%	90.9%	93.3%	26.1%	98.8%	76.8%
$\mathbb{E}[N_{\mathrm{RMF}}]$	6.7	2.3	7.6	8.4	0.9	8.0	5.3
$\mathbb{E}[N^*_{\mathrm{RMF}}]$	2.2	0.5	2.2	2.5	0.3	3.8	1.4

Impact of competitive balance

	Perfect balance			Imbalance			Strong imbalance		
Win prob.	S	M	W	S	M	W	S	M	W
S (Strong)		37.5%	37.5%		50%	80%		70%	90%
M (Middle)	37.5%		37.5%	20%		50%	10%		70%
W (Weak)	37.5%	37.5%		5%	20%		2%	10%	

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	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0} + p_{0,i<j}$	48%	18%	48%	84%	18%	48%	84%
p_{RMF}	41.9%	14.6%	47.4%	52.7%	5.9%	50.0%	34.6%
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Impact of competitive balance

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M (Middle)	37.5%		37.5%	20%		50%	10%		70%
W (Weak)	37.5%	37.5%		5%	20%		2%	10%	

Table: Win probabilities: p_{XY} is the probability that Team X wins against Team Y.

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0} + p_{0,i<j}$	48%	18%	48%	84%	18%	48%	84%
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$\mathbb{E}[N^*_{\mathrm{RMF}}]$	2.2	0.5	2.2	2.5	0.3	3.8	1.4

Impact of competitive balance

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M (Middle)	37.5%		37.5%	20%		50%	10%		70%
W (Weak)	37.5%	37.5%		5%	20%		2%	10%	

Table: Win probabilities: p_{XY} is the probability that Team X wins against Team Y.

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0} + p_{0,i<j}$	48%	18%	48%	84%	18%	48%	84%
p_{RMF}	41.9%	14.6%	47.4%	52.7%	5.9%	50.0%	34.6%
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$\mathbb{E}[N_{\mathrm{RMF}}]$	6.7	2.3	7.6	8.4	0.9	8.0	5.3
$\mathbb{E}[N^*_{\mathrm{RMF}}]$	2.2	0.5	2.2	2.5	0.3	3.8	1.4

Impact of forbidding draws: the 3-0 point system

- Risk of match fixing **on penalties**: when Team B or C can win the group and eliminate Team A even after drawing its last game and losing on penalties $\longrightarrow$ **RMF$_{\text{pen}}$**
- B and C may agree on a draw, say 0-0, and **the team leading in the rankings can at no expense decide to eliminate Team A by losing the penalty shootout**—a situation FIFA surely wants to avoid by all means.

Proposition

(i) The situations of RMF in a given group of three in the 3-0 point system are exactly the following ones:

1. *Team A has one win and one loss and $GD_A \leq 0$.*
2. *Team A has two draws with one win and one loss on penalties.*
3. *Team A has one draw won on penalties and one loss.*

The corresponding probability is

$$p^{30}_{\mathrm{RMF}} := \frac{1}{2}\left(p_{BA}d_{AC} + d_{AB}p_{CA}\right) + \frac{1}{2}d_{AB}d_{AC} + p_{\leq 0}\left(p_{AB}p_{CA} + p_{BA}p_{AC}\right) \leq p_{\mathrm{RMF}}.$$

Impact of forbidding draws: the 3-0 point system

Proposition

(ii) The situations of aggravated RMF are exactly the following ones:

1. *Team A has one win $(m+i)$ - i and one loss j - $(n+j)$, and $GD_A := m-n<0$ or $\{GD_A=0$ and $i<j\}$.*
2. *Team A has one draw i - i won on penalties and one loss j - $(n+j)$, and we are in Case 1:*
   - *$n \geq 3$, or*
   - *$n=2$ and $i=0$, or*
   - *$n=2$, $i \neq 0$, and $n+j \geq i+2$.*

The corresponding probability is

$$p^{*30}_{\mathrm{RMF}} := (p_{<0}+p_{0,i<j})\,(p_{AB}p_{CA}+p_{BA}p_{AC}) + \tfrac{1}{2}p_{\mathrm{case1}}(p_{BA}d_{AC}+d_{AB}p_{CA})$$
$$= p^{*}_{\mathrm{RMF}} + \tfrac{1}{2}p_{\mathrm{case1}}(p_{BA}d_{AC}+d_{AB}p_{CA})$$

where p_{case1} denotes the probability of Case 1 given that Team A has one draw won on penalties and one loss. In particular, $p^{*30}_{\mathrm{RMF}} \geq p^{*}_{\mathrm{RMF}}$: **forbidding draws and adopting the 3-0 point system always increases the probability of an aggravated risk of match fixing**.

Impact of forbidding draws: the 3-0 point system

Proposition

(iii) The situations of aggravated RMF on penalties are exactly the following ones:

1. *Team A has one draw (won on penalties) and one loss.*
2. *Team A has two draws i - i (won on penalties) and j - j (lost on penalties), and $i < j$.*
3. *Team A has one win $(m+i)$ - i and one loss j - $(n+j)$, $GD_A := m - n < 0$, and $2m \leq n$.*

The corresponding probability is

$$p^{*30}_{\mathrm{RMF,pen}} := \frac{1}{2}(p_{BA}d_{AC} + d_{AB}p_{CA}) + \frac{1}{2}p_{\mathrm{2d},i<j}d_{AB}d_{AC} + p_{2m\leq n}p_{<0}(p_{AB}p_{CA} + p_{BA}p_{AC})$$

where $p_{\mathrm{2d},i<j}$ denotes the probability that $i < j$ given that Team A has two draws i - i (won on penalties) and j - j (loss on penalties), and $p_{2m\leq n}$ denotes the probability that $2m \leq n$ given that Team A has one win $(m+i)$ - i and one loss j - $(n+j)$ and that $GD_A := m - n < 0$.

Impact of forbidding draws: the 3-0 point system

Situation of Team A after Match 2	pts A	Probability	RMF	RMF*	$\text{RMF}^*_{\text{pen}}$
Two wins	6	$p_{AB}p_{AC}$			
One win and one draw, win on pen.	6	$\frac{1}{2}\left(p_{AB}d_{AC}+d_{AB}p_{AC}\right)$			
One win and one draw, loss on pen.	3	$\frac{1}{2}\left(p_{AB}d_{AC}+d_{AB}p_{AC}\right)$			
One win and one loss, GD$_A > 0$	3	$p_{>0}(p_{AB}p_{CA}+p_{BA}p_{AC})$			
One win and one loss, GD$_A = 0,\ i \geq j$	3	$p_{0,i\geq j}(p_{AB}p_{CA}+p_{BA}p_{AC})$	✔		
One win and one loss, GD$_A = 0,\ i < j$	3	$p_{0,i<j}(p_{AB}p_{CA}+p_{BA}p_{AC})$	✔	✔	
One win and one loss, GD$_A < 0$	3	$p_{<0}(p_{AB}p_{CA}+p_{BA}p_{AC})$	✔	✔	if $2m \leq n$
Two draws, two wins on pen.	6	$\frac{1}{4}d_{AB}d_{AC}$			
Two draws, one win and one loss on pen.	3	$\frac{1}{2}d_{AB}d_{AC}$	✔		if $i < j$
Two draws, two losses on pen.	0	$\frac{1}{4}d_{AB}d_{AC}$			
One draw and one loss, win on pen., case 1	3	$\frac{1}{2}p_{\text{case1}}(p_{BA}d_{AC}+d_{AB}p_{CA})$	✔	✔	✔
One draw and one loss, win on pen., case 2	3	$\frac{1}{2}p_{\text{case2}}(p_{BA}d_{AC}+d_{AB}p_{CA})$	✔		✔
One draw and one loss, loss on pen.	0	$\frac{1}{2}(p_{BA}d_{AC}+d_{AB}p_{CA})$			
Two losses	0	$p_{BA}p_{CA}$			

Impact of forbidding draws: the 3-0 point system

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
$p_{<0} + p_{0,i<j}$	48%	18%	48%	84%	18%	48%	84%
p_{case1}	60%	40%	60%	90%	40%	60%	90%
$p_{2\text{d},i<j}$	35%	35%	35%	35%	35%	35%	35%
$p_{2m\leq n}$	70%	70%	70%	70%	70%	70%	70%
p_{RMF}^{30}	29.4%	10.1%	32.4%	34.7%	4.5%	40.0%	22.0%
p_{RMF}^{*30}	19.1%	4.2%	20.2%	29.7%	2.1%	28.8%	19.4%
$p_{\text{RMF},\text{pen}}^{*30}$	18.3%	4.3%	20.2%	26.9%	1.6%	22.7%	17.9%
$p_{\text{RMF}}^{30}(16)$	99.6%	81.6%	99.8%	99.9%	52.3%	100.0%	98.1%
$p_{\text{RMF}}^{*30}(16)$	96.7%	49.9%	97.3%	99.6%	28.9%	99.6%	96.8%
$p_{\text{RMF},\text{pen}}^{*30}(16)$	96.1%	50.8%	97.3%	99.3%	22.8%	98.4%	95.7%
$\mathbb{E}[N_{\text{RMF}}^{30}]$	4.7	1.6	5.2	5.5	0.7	6.4	3.5
$\mathbb{E}[N_{\text{RMF}}^{*30}]$	3.1	0.7	3.2	4.8	0.3	4.6	3.1
$\mathbb{E}[N_{\text{RMF},\text{pen}}^{*30}]$	2.9	0.7	3.2	4.3	0.3	3.6	2.9

Impact of forbidding draws: the 3-0 point system

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
$p_{<0} + p_{0,i<j}$	48%	18%	48%	84%	18%	48%	84%
p_{case1}	60%	40%	60%	90%	40%	60%	90%
$p_{2\text{d},i<j}$	35%	35%	35%	35%	35%	35%	35%
$p_{2m\leq n}$	70%	70%	70%	70%	70%	70%	70%
p^{30}_{RMF}	29.4%	10.1%	32.4%	34.7%	4.5%	40.0%	22.0%
p^{*30}_{RMF}	19.1%	4.2%	20.2%	29.7%	2.1%	28.8%	19.4%
$p^{*30}_{\text{RMF,pen}}$	18.3%	4.3%	20.2%	26.9%	1.6%	22.7%	17.9%
$p^{30}_{\text{RMF}}(16)$	99.6%	81.6%	99.8%	99.9%	52.3%	100.0%	98.1%
$p^{*30}_{\text{RMF}}(16)$	96.7%	49.9%	97.3%	99.6%	28.9%	99.6%	96.8%
$p^{*30}_{\text{RMF,pen}}(16)$	96.1%	50.8%	97.3%	99.3%	22.8%	98.4%	95.7%
$\mathbb{E}[N^{30}_{\text{RMF}}]$	4.7	1.6	5.2	5.5	0.7	6.4	3.5
$\mathbb{E}[N^{*30}_{\text{RMF}}]$	3.1	0.7	3.2	4.8	0.3	4.6	3.1
$\mathbb{E}[N^{*30}_{\text{RMF,pen}}]$	2.9	0.7	3.2	4.3	0.3	3.6	2.9

Impact of forbidding draws: the 3-0 point system

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
$p_{<0} + p_{0,i<j}$	48%	18%	48%	84%	18%	48%	84%
p_{case1}	60%	40%	60%	90%	40%	60%	90%
$p_{2\text{d},i<j}$	35%	35%	35%	35%	35%	35%	35%
$p_{2m\leq n}$	70%	70%	70%	70%	70%	70%	70%
p^{30}_{RMF}	29.4%	10.1%	32.4%	34.7%	4.5%	40.0%	22.0%
p^{*30}_{RMF}	19.1%	4.2%	20.2%	29.7%	2.1%	28.8%	19.4%
$p^{*30}_{\text{RMF,pen}}$	18.3%	4.3%	20.2%	26.9%	1.6%	22.7%	17.9%
$p^{30}_{\text{RMF}}(16)$	99.6%	81.6%	99.8%	99.9%	52.3%	100.0%	98.1%
$p^{*30}_{\text{RMF}}(16)$	96.7%	49.9%	97.3%	99.6%	28.9%	99.6%	96.8%
$p^{*30}_{\text{RMF,pen}}(16)$	96.1%	50.8%	97.3%	99.3%	22.8%	98.4%	95.7%
$\mathbb{E}[N^{30}_{\text{RMF}}]$	4.7	1.6	5.2	5.5	0.7	6.4	3.5
$\mathbb{E}[N^{*30}_{\text{RMF}}]$	3.1	0.7	3.2	4.8	0.3	4.6	3.1
$\mathbb{E}[N^{*30}_{\text{RMF,pen}}]$	2.9	0.7	3.2	4.3	0.3	3.6	2.9

Impact of forbidding draws: the 3-0 point system

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
$p_{<0} + p_{0,i<j}$	48%	18%	48%	84%	18%	48%	84%
p_{case1}	60%	40%	60%	90%	40%	60%	90%
$p_{\mathrm{2d},i<j}$	35%	35%	35%	35%	35%	35%	35%
$p_{2m\leq n}$	70%	70%	70%	70%	70%	70%	70%
p^{30}_{RMF}	29.4%	10.1%	32.4%	34.7%	4.5%	40.0%	22.0%
p^{*30}_{RMF}	19.1%	4.2%	20.2%	29.7%	2.1%	28.8%	19.4%
$p^{*30}_{\mathrm{RMF,pen}}$	18.3%	4.3%	20.2%	26.9%	1.6%	22.7%	17.9%
$p^{30}_{\mathrm{RMF}}(16)$	99.6%	81.6%	99.8%	99.9%	52.3%	100.0%	98.1%
$p^{*30}_{\mathrm{RMF}}(16)$	96.7%	49.9%	97.3%	99.6%	28.9%	99.6%	96.8%
$p^{*30}_{\mathrm{RMF,pen}}(16)$	96.1%	50.8%	97.3%	99.3%	22.8%	98.4%	95.7%
$\mathbb{E}[N^{30}_{\mathrm{RMF}}]$	4.7	1.6	5.2	5.5	0.7	6.4	3.5
$\mathbb{E}[N^{*30}_{\mathrm{RMF}}]$	3.1	0.7	3.2	4.8	0.3	4.6	3.1
$\mathbb{E}[N^{*30}_{\mathrm{RMF,pen}}]$	2.9	0.7	3.2	4.3	0.3	3.6	2.9

Ignacio Palacios-Huerta, New Scientist, June 30, 2018

Penalties for fair play

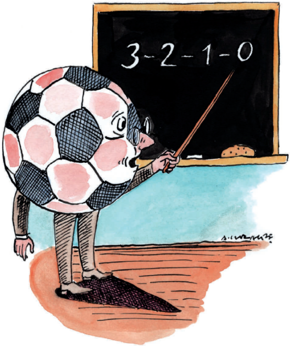
A new format for future football World Cups raises the risk of teams colluding. There is a simple fix, says **Ignacio Palacios-Huerta**

AS THE round-robin group stage of the World Cup draws to a close, football fans might want to pause for thought. This might be the last contest to feature a four teams per group format. What will replace it could make the contest less fair.

The 2026 World Cup will open with 16 groups of three teams, and this may also apply in 2022. Julien Guyon, a mathematician, has pointed out that this raises the incentive for teams to collude.

Collusion is already possible in a four-team group. Two teams playing each other in their last group game may know exactly what result will let both advance to the knockout stage at the expense of another team, based on earlier results. FIFA, the sport's governing body, stages final group games simultaneously to mitigate this.

Basic calculations show that moving to three teams per group will in principle worsen the risk



of collusion. Three teams means fewer games, but a proportionally greater number of problematic outcomes are possible. Matches will also have to be sequential.

What to do? FIFA could play the entire tournament on a knockout basis. It would solve the problem, but also mean half the countries playing just one game. If FIFA insists on three-team groups, though, another suggestion is to ban draws by introducing group stage penalty shoot-outs. This would eliminate many scenarios that create incentives to collude.

Unfortunately, this is not fair. Winning a shoot-out after a drawn game does not seem equivalent to winning a game. But there is a simple and sensible variant.

Currently, a win is worth 3 points to the winner and 0 to the loser, while in drawn games both teams get a point. This makes drawn games less valuable in

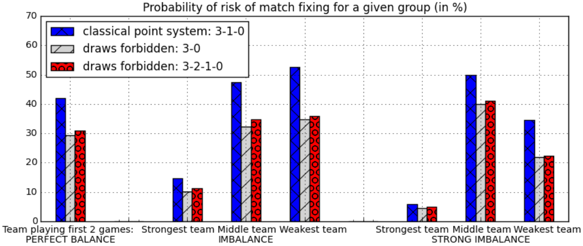
Impact of the point system: the 3-2-1-0 point system

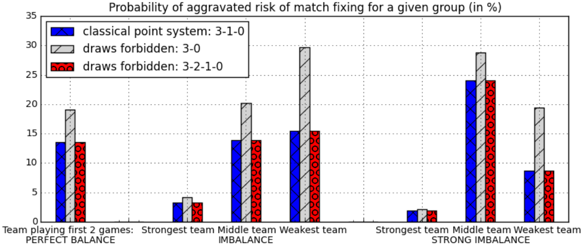
Situation of Team A after Match 2	Probability	RMF	RMF*	$\text{RMF}^*_{\text{pen}}$
Two wins	$p_{AB}p_{AC}$			
One win & one draw	$p_{AB}d_{AC} + d_{AB}p_{AC}$			
One win & one loss, $\text{GD}_A > 0$	$p_{>0}(p_{AB}p_{CA} + p_{BA}p_{AC})$			
One win & one loss, $\text{GD}_A = 0$, $i \geq j$	$p_{0,i\geq j}(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔		
One win & one loss, $\text{GD}_A = 0$, $i < j$	$p_{0,i<j}(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔	✔	
One win & one loss, $\text{GD}_A < 0$	$p_{<0}(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔	✔	
Two draws, two wins on penalties	$\frac{1}{4}d_{AB}d_{AC}$			
Two draws, one win & one loss on pen	$\frac{1}{2}d_{AB}d_{AC}$	✔		if $i < j$
Two draws, two losses on penalties	$\frac{1}{4}d_{AB}d_{AC}$	✔		
One draw & one loss, win on pen	$\frac{1}{2}(p_{BA}d_{AC} + d_{AB}p_{CA})$	✔		✔
One draw & one loss, loss on pen	$\frac{1}{2}(p_{BA}d_{AC} + d_{AB}p_{CA})$			
Two losses	$p_{BA}p_{CA}$			

Impact of the point system: the 3-2-1-0 point system

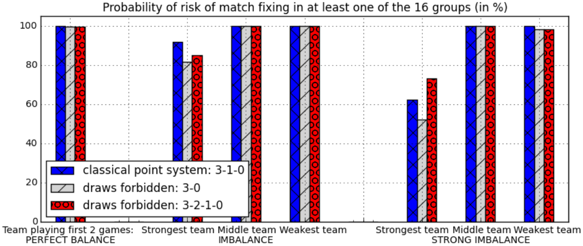
	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
$p_{<0} + p_{0,i<j}$	48%	18%	48%	84%	18%	48%	84%
$p_{2\mathrm{d},i<j}$	35%	35%	35%	35%	35%	35%	35%
p_{RMF}^{3210}	30.9%	11.2%	34.7%	35.8%	4.9%	41.0%	22.4%
p_{RMF}^{*3210}	13.5%	3.3%	13.9%	15.5%	1.9%	24.0%	8.7%
$p_{\mathrm{RMF,pen}}^{*3210}$	10.5%	3.0%	12.1%	16.5%	0.9%	8.7%	12.1%
$p_{\mathrm{RMF}}^{3210}(16)$	99.7%	85.0%	99.9%	99.9%	73.2%	99.9%	98.3%
$p_{\mathrm{RMF}}^{*3210}(16)$	90.2%	41.8%	90.9%	93.3%	26.1%	98.8%	76.8%
$p_{\mathrm{RMF,pen}}^{*3210}(16)$	83.0%	39.0%	87.2%	94.5%	13.2%	76.7%	87.3%
$\mathbb{E}[N_{\mathrm{RMF}}^{3210}]$	5.0	1.8	5.6	5.7	0.8	6.6	3.6
$\mathbb{E}[N_{\mathrm{RMF}}^{*3210}]$	2.2	0.5	2.2	2.5	0.3	3.8	1.4
$\mathbb{E}[N_{\mathrm{RMF,pen}}^{*3210}]$	1.7	0.5	1.9	2.6	0.1	1.4	1.9

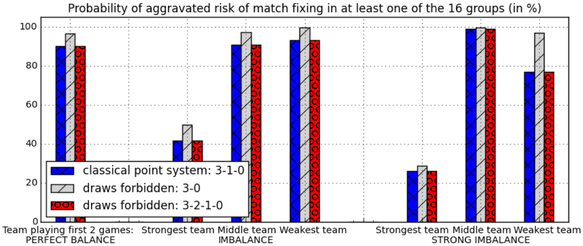
Summary


Probability of risk of match fixing for a given group (in %)
classical point system: 3-1-0
draws forbidden: 3-0
draws forbidden: 3-2-1-0
Team playing first 2 games: PERFECT BALANCE
Strongest team Middle team Weakest team IMBALANCE
Strongest team Middle team Weakest team STRONG IMBALANCE

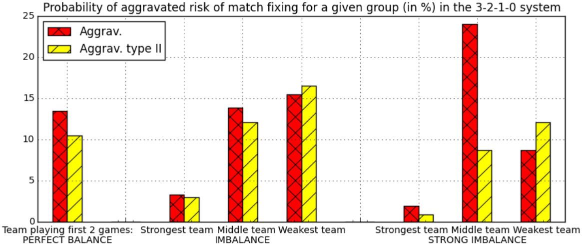

Probability of aggravated risk of match fixing for a given group (in %)
classical point system: 3-1-0
draws forbidden: 3-0
draws forbidden: 3-2-1-0
Team playing first 2 games: PERFECT BALANCE
Strongest team Middle team Weakest team IMBALANCE
Strongest team Middle team Weakest team STRONG IMBALANCE

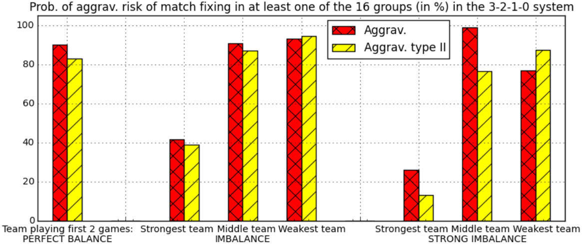
Summary


Probability of risk of match fixing in at least one of the 16 groups (in %)
100
80
60
40
20
0
classical point system: 3-1-0
draws forbidden: 3-0
draws forbidden: 3-2-1-0
Team playing first 2 games:
PERFECT BALANCE
Strongest team Middle team Weakest team
IMBALANCE
Strongest team Middle team Weakest team
STRONG IMBALANCE


Probability of aggravated risk of match fixing in at least one of the 16 groups (in %)
100
80
60
40
20
0
classical point system: 3-1-0
draws forbidden: 3-0
draws forbidden: 3-2-1-0
Team playing first 2 games:
PERFECT BALANCE
Strongest team Middle team Weakest team
IMBALANCE
Strongest team Middle team Weakest team
STRONG IMBALANCE

Summary


Probability of aggravated risk of match fixing for a given group (in %) in the 3-2-1-0 system
Aggrav.
Aggrav. type II
25
20
15
10
5
0
Team playing first 2 games:
PERFECT BALANCE
Strongest team Middle team Weakest team
IMBALANCE
Strongest team Middle team Weakest team
STRONG IMBALANCE


Prob. of aggrav. risk of match fixing in at least one of the 16 groups (in %) in the 3-2-1-0 system
Aggrav.
Aggrav. type II
100
80
60
40
20
0
Team playing first 2 games:
PERFECT BALANCE
Strongest team Middle team Weakest team
IMBALANCE
Strongest team Middle team Weakest team
STRONG IMBALANCE

Summary

- **The most important factor impacting risk of collusion is the schedule**: the RMF is minimized when it is the *a priori* strongest team that plays the first 2 games.
- **Forbidding draws and adopting the 3-0 point system decreases the risk of collusion, but increases the probability of aggravated RMF.**
- Surprisingly, **compared to the 3-0 point system, the RMF is slightly larger in the 3-2-1-0 point system**; but, compared to the classical 3-1-0 point system, it is smaller.
- The probability of aggravated RMF in the 3-2-1-0 point system is exactly the same as in the 3-1-0 point system. However, **banning draws introduces very problematic situations where a team may decide to eliminate another team by deliberately losing the penalty shootout**.

New York Times, June 26, 2018

The New York Times

TheUpshot

FIFA, We Fixed Your World Cup Collusion Problem for You

Readers tackle a math puzzle on how to devise a fair tournament with 48 teams.

Can FIFA find its way to a fair tournament format before the next World Cup? Spain goalkeeper David De Gea looked lost after giving up a goal June 15 against Portugal. Thanassis Stavrakis/Associated Press

By Toni Monkovic

June 26, 2018

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The World Cup hardly seems broken, and yet FIFA seems desperate to fix it.

FIFA, world soccer's governing body, has decided that the World

Alternate 48-team formats

- Assuming a 48-team World Cup, what alternate formats would significantly decrease, or even eradicate, the risk of collusion?
- Constraint: total number of matches ≤ 100. Current: 64. 2026: 80.
- Precludes the classical round-robin format with 8 groups of 6—120 games just in group stage!

1. **12 groups of 4, 32 teams advance**: $12 + 12 + 8$
2. **12 groups of 4, 16 teams advance**: $12 + 4$
3. **16 groups of 3, only group winners advance**
4. **16 groups of 3, all teams advance**, playoff 2nd vs 3rd
5. **16 groups of 3, seed KO bracket based on performance across groups** (Guyon, 2018)
6. **8 groups of 6 but each team plays only 3 teams in their group**
7. **16 groups of 3 followed by 8 groups of 4**, results are carried over

Alternate formats

Format	FIFA	1	2	3	4	5	6	7
Nb groups	16	12	12	16	16	16	8	16 then 8
Nb teams per group	3	4	4	3	3	3	6	3 then 4
Nb of match days in group stage	3	3	3	3	3	3	3	3 then 2
Full round robin in each group	✔	✔	✔	✔	✔	✔		✔
Total nb group matches	48	72	72	48	48	48	72	$48 + 32 = 80$
Nb teams in knockout round	32	32	16	16	48	32	16	16
Total nb matches	80	104	88	64	96	80	88	96
Predetermined bracket routes	✔	✔	✔	✔	✔		✔	✔
Minimal nb matches per team	2	3	3	2	3	2	3	2
Maximal nb matches per team	7	8	7	6	8	7	7	8

Conclusion

- We have quantified the risk of collusion in a group of 3 teams playing a single round-robin tournament, where 2 teams advance to the next phase.
- We have shown that **the best way to minimize the risk of collusion is that the team that plays the first two group matches is the a priori strongest team in the group**, especially if group is strongly imbalanced.
- We have quantified how competitive imbalance within a group impacts the risk of collusion.
- We have also quantified by how much the risk of collusion would decrease if FIFA does not use the traditional 3-1-0 point system but adopts alternate point systems that forbid draws: 3-0 and 3-2-1-0.
- Even though it looks appealing on paper, 3-2-1-0 does not do a better job at decreasing the risk of collusion than 3-0.
- When banning draws, FIFA would introduce new problematic situations of possible collusion where a team may decide to eliminate another team by deliberately losing the penalty shootout (these would be more likely with the 3-0 point system).

Conclusion

- 16 groups of 3 $\Longrightarrow$ the risk of collusion in at least one group very high, even in the most favorable case where all groups are strongly imbalanced and in every group Team A is the *a priori* strongest team in the group.
- **The introduction of groups of 3 is a terrible step back in the history of the World Cup**. Not only it makes the "disgrace of Gijón" possible again, but it makes the risk of its repetition very high.
- Therefore, we have also described practical alternate formats for a 48-team World Cup that would eliminate or strongly decrease the risk of collusion, with groups of 3, 4, or 6 teams.
- It is FIFA's responsibility to build a fair World Cup. **It is not too late for FIFA to review the format of the 2026 World Cup**.
- **We encourage the FIFA Council to realize the danger posed by groups of 3 and opt for one of the better formats we suggested.**

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New York Times, June 11, 2018

The New York Times

TheUpshot

Why Groups of 3 Will Ruin the World Cup (So Enjoy This One)

Unless FIFA changes course, the risk of collusion will be much higher in future Cups.

A new format with 48 teams, starting in 2026 but possibly as soon as the next World Cup, raises a serious fairness issue in the chase for the World Cup trophy. Alexander Zemlianichenko/Associated Press

By Julien Guyon

June 11, 2018

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If you're a World Cup aficionado, you may want to take the time to savor this one, because it may be the last version with a format — groups of four teams — that has a fair and consistently exciting