SCILAB *à l'École nationale des ponts et chaussées*

http://cermics.enpc.fr/scilab

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Dynamic programing and Markov chains

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1 Stoping time problem

Let $(X_n)_{n\in\mathbb{N}}$ be a finite Markov chain with transition matrix $M^{(n)}$. We want to compute :

$$u_n(x) = \sup_{\tau\ \mathcal{F}_n t.a., n\leq\tau\leq N} \mathbb{E}\left[\sum_{k=n}^{\tau-1} c^{(k)}(X^k) + \psi^{\tau}(X^{\tau})|X^n = x\right].$$

The control is a stopping time $n \leq \tau \leq N$. As shown in the course the value function can be recursively computed by :

$$\begin{cases} u_n(x) & = \max(M^{(n)}u_{n+1} + c^{(n)}, \psi^n), \quad n = 0, \ldots, N-1 \\ u_N & = \psi^N. \end{cases} \tag{1}$$

And the optimal stopping time :

$$\tau_n = \inf\{N \geq k \geq n, u_k(X^k) = \psi^k(X^k)\}$$

is such that :

$$u_n(X^n) = \mathbb{E}\left[\sum_{k=n}^{\tau_n-1} c^{(k)}(X^k) + \psi^{\tau_n}(X^{\tau_n})|X^n\right].$$

Question 1 *Write a first routine which returns a stochastic matrix M of size* `NxN`.

Question 2 *Using the choosen matrix M generate ans plot some samples of the homogeneous Markov chain with states in $[1, N]$ described by M (using* `grand` *in Scilab).*

Question 3 *Choosing an instantaneous cost c and a final cost K recusively compute the value function $v^n(x)$ and draw the result as a surface (i.e $v(x,t)$). Compute also $u^n(x)$ which for a given state at time n returns 1 if the optimal strategy is to go on and 2 is the optimal strategy is to stop. Note that the* `max` *function can be used to get the maximum value of two quantities but also to get the indice of the value which realize the max.*

```
// A stoping time problem.
c=... // instantaneous cost
K=... // final cost

r=0.05;
C=ones(N,T); // a matrix to store the value function
U=2*ones(N,T); // a matrix to store the control (1 or 2)
```

```
C(:,T) = ... ;

for i=T-1:-1:1
  ....
  C(:,i)= Ci;
  U(:,i)= ki;
end

plot3d(1:N,1:T,C)
```

Question 4 *Using samples of the markov chain for a given starting state evaluate by Monte Carlo the cost function* $v_0(1)$ *and compare the results with previous question.*

```
m=10000; // number of samplings
Cm=0;    // we want to evaluate C(1,1);
for k=1:m  // loop on sampling
  // the k-th trajectory.
  X=.... // sample a trajectory
  // the control along the trajectory
  for i=1:T, u(i)=... end
  stop=... // the stopping time
  cost=0;
  ...
  cost = .. // evaluate the cost for the m-th trajectory
  Cm=Cm+cost/m; // compute the mean
end
```