SCILAB *à l'École nationale des ponts et chaussées*

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Dynamic programing and Markov chains

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1 Stoping time problem in infinite horizon

Let $(X_n)_{n\in\mathbb{N}}$ be a finite homogeneous Markov chain with transition matrix M. We want to compute :

$$v(x) = \sup_{\tau \text{s.t}} \mathbb{E}\left[\frac{1}{(1+r)^{\tau+1}} f(X_\tau)|X_0 = x\right].$$

the value function can be recursively computed by here can be computed as a fixed point

$$v(x) = \frac{1}{1+r}\max(Mv(x), f(x)) \tag{1}$$

In order to numerically solve such a problem we will use a value iteration method and a policy iteration method.

we will use on of the two following routines to generate a stochastic matrix.

```
function M= Mtrans(n)
  M=rand(n,n);
  s=sum(M,'c');
  M= M./(s*ones(1,n));
endfunction

function M= Mprom(n,d)
  A= 2*diag(ones(1,n)) -diag(ones(1,n-1),1) -diag(ones(1,n-1),-1);
  A(1,2) = 2*A(1,2) ;
  A($,$-1)=2*  A($,$-1);
  M = eye(A) - A*d
endfunction
```

Question 1 *Using one of the previous function, choose a state space dimension, generate a transition matrix and write a function for* `f`*. build a column vector xs giving the states of the Markov chain.*

Question 2 *The value iteration méthod to solve the fixed point problem work as follows. Choose an initial vector* `v0=v` *which gives $v_0(x)$ for each of the* `n` *possible states. Then iterate $v_{n+1} = \frac{1}{1+r}\max(Mv_n, f)$. Stop the alogorithm when the infinite norm of $v_{n+1} - v_n$ is small enough. Then, plot the evolution of norm$(v_{n+1} - v_n)$ during the evolution of the algorithm. Run the same problem with different values of r.*

Question 3 *The policy iteration algorithm evolves as follow. Choose a first value for v* `v0=v`*. Then given* v_n *compute a new policy,* $u_n x = 1$ *if* $Mv_n(x) \geq f(x)$ *and* $u_n x = 2$ *elsewhere. Given a new policy* u_n*, the value function* $v_{n+1}(x)$ *is computed as follows, whe must have* $v_{n+1}(x) = Mv(x)/(1+r)$ *for all the states for which the* u_n *policy is equal to* 1 *and* $v_{n+1}(x) = f(x)$ *for all the states for which the* u_n *policy is equal to* 2*. Thus we just have to write and solve a linear system to compute* v_{n+1}*. As for the value iteration, Stop the alogorithm when the infinite norm of* $v_{n+1} - v_n$ *is small enough. Then, plot the evolution of norm*$(v_{n+1} - v_n)$ *during the evolution of the algorithm and compare the two algorithms. Run the same problem with different values of* r*.*