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TD, 14 October 2025 Cours : Principes de Modelisation

Consider the SIR system

$$S' = -r\frac{SI}{N}, \quad I' = r\frac{SI}{N} - aI, \quad R' = aI. \tag{1}$$

with initial conditions $S(t=0) = S_0 > 0$, $I(t=0) = I_0 > 0$, $R(t=0) = 0$.
1. Explain what S, I, R, N and the constants a and r represent in this model.
2. Show that the total population is constant.
3. Can the system (1) be reduced to a system with less equations and, if so, which one ?
4. Determin the conditions on S_0 to have an epidemic outbreak.
5. Déetermine the maximum value of I during the epidemics.
6. determoine the number of individuals that are contaminated during the epidemics
7. How to change system (1) to take into account birth and death (supposing that new-borns don't have the disease but can catch it afterwards) ?
8.a) Consider a human sexually transmissible disease that can be transmitted only thrrough heterosexial relations, has no incubation period, has a recovery tyme of two weeks and does not grant immunity. Propose a system of ordinary differential equations to model the population dynamics of healthy and infected individuals. You can denote α_{HF} the translmission rate from women to men and α_{FH} that of men to women.
8b) How should you modify the model if you have an incubation period of a week for the transmission from men to women but no incubation period in the other sense ?
8c) Determine the basic reproduction number of the dynamics in question 8b) as a function of the model parameters.