



# Performing 2D Hamiltonian-Exchange method for Biological Free Energy Calculations on Petascale Supercomputer

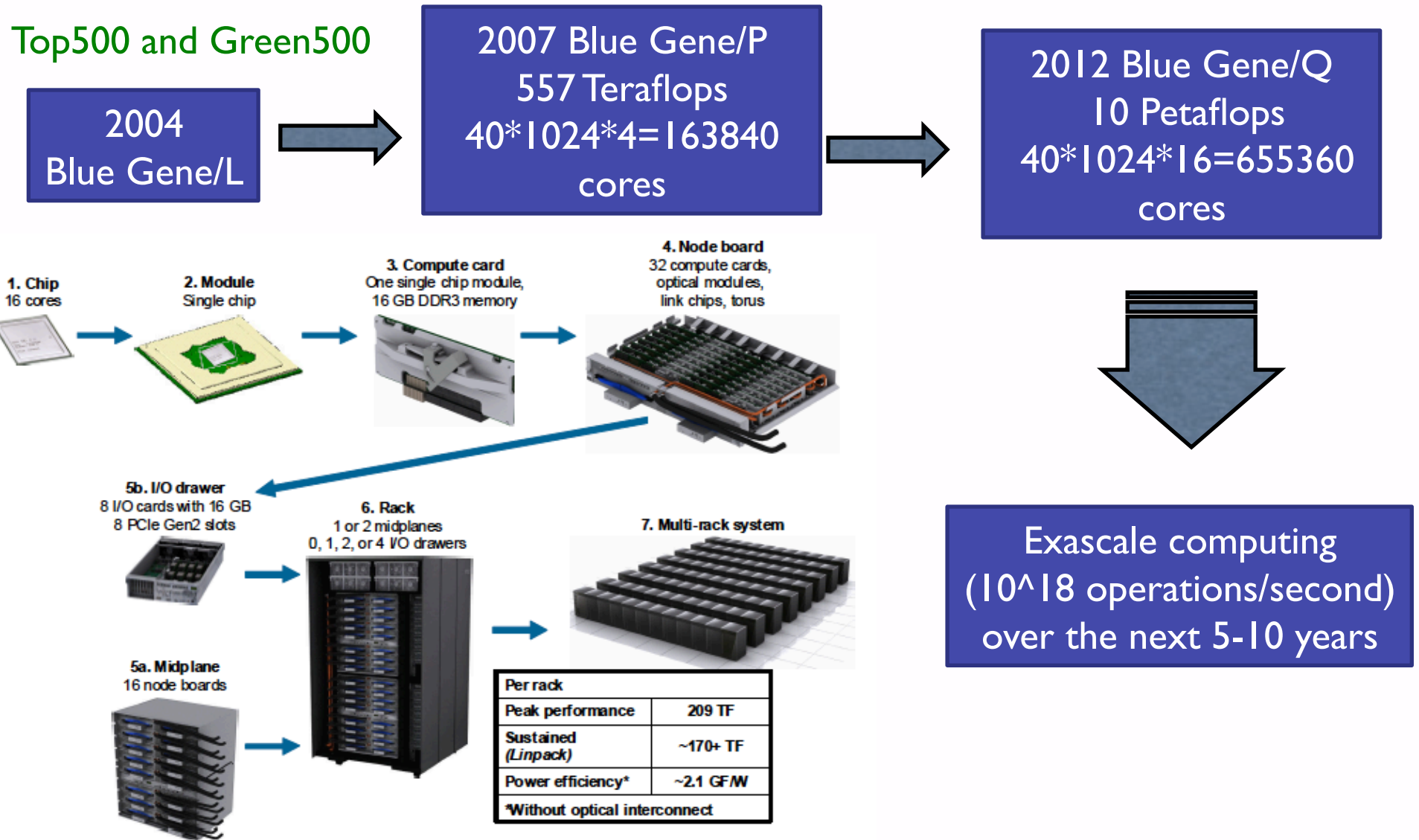
Yun Lyna Luo, Wei Jiang, Benoît Roux\*

Argonne Leadership Computing Facility, Argonne National Laboratory

Department of Biochemistry and Molecular Biology  
University of Chicago

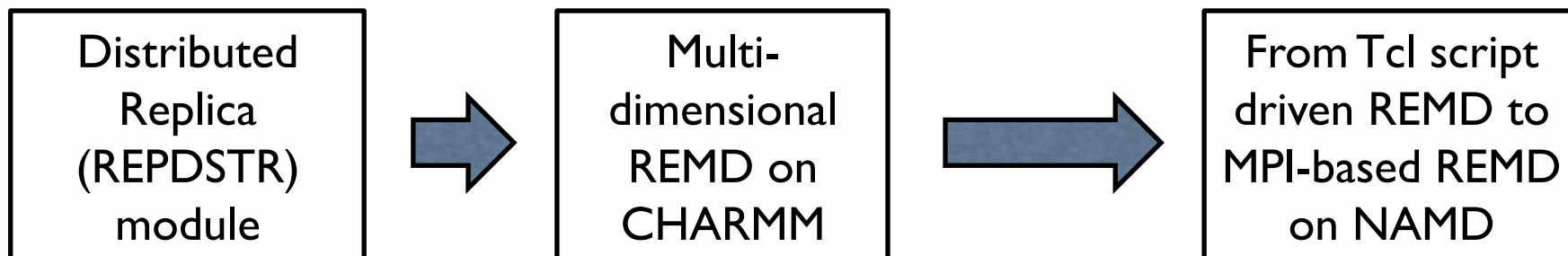
# Why replica-exchange:

- Suitable for massively distributed supercomputer.
- Free energy calculations for complex biomolecular system.

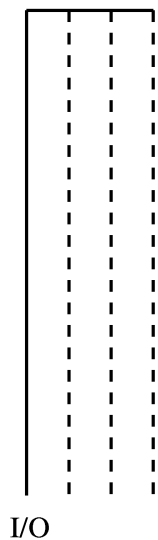


# Hamiltonian-Exchange: Umbrella sampling (US) and FEP

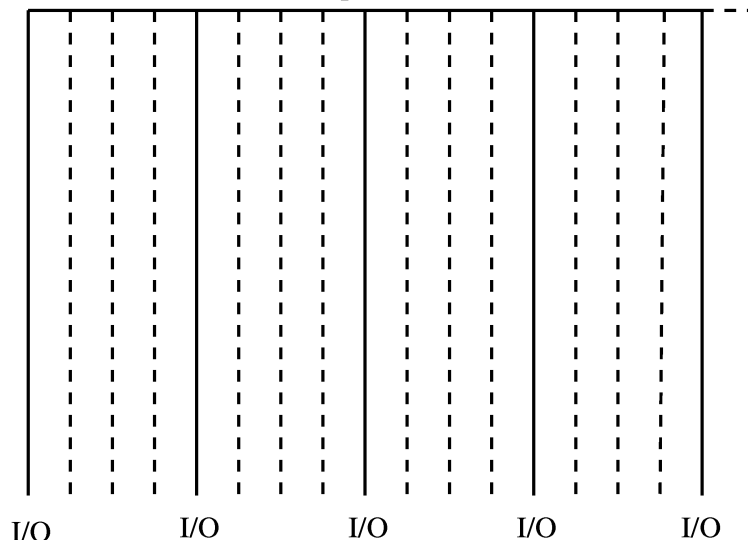
$$\exp \left[ \frac{1}{k_B T} (E(\lambda_m, \mathbf{r}_m) + E(\lambda_n, \mathbf{r}_n) - E(\lambda_m, \mathbf{r}_n) - E(\lambda_n, \mathbf{r}_m)) \right] \geq \text{random}(0,1)$$



Normal Parallel

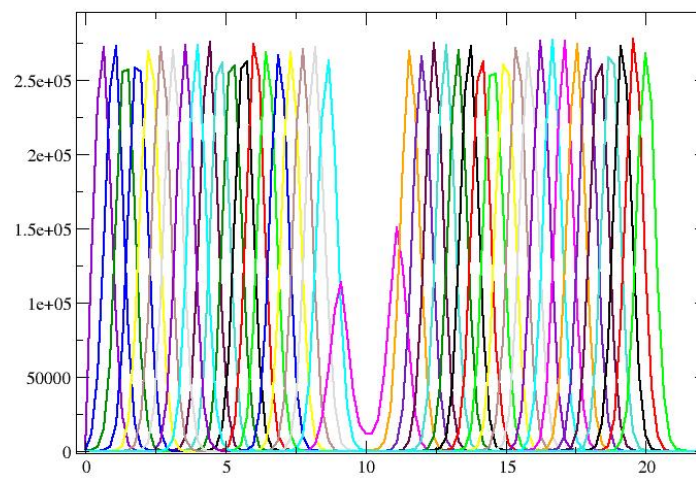
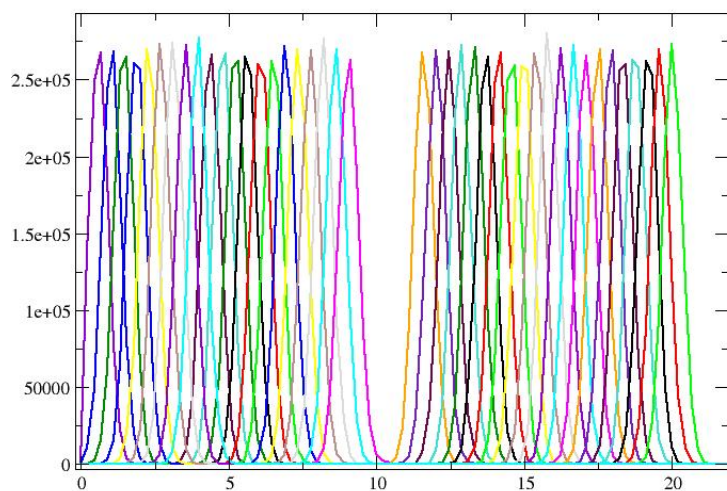
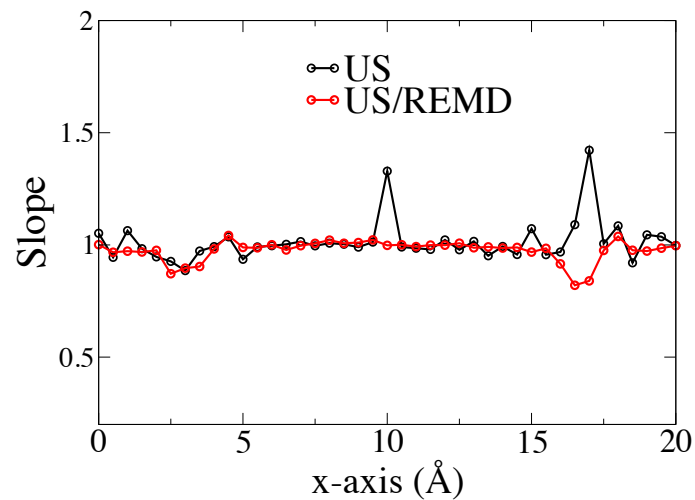
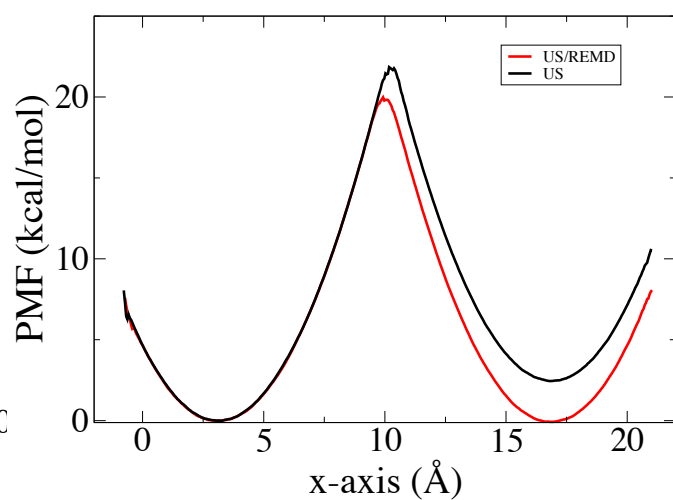
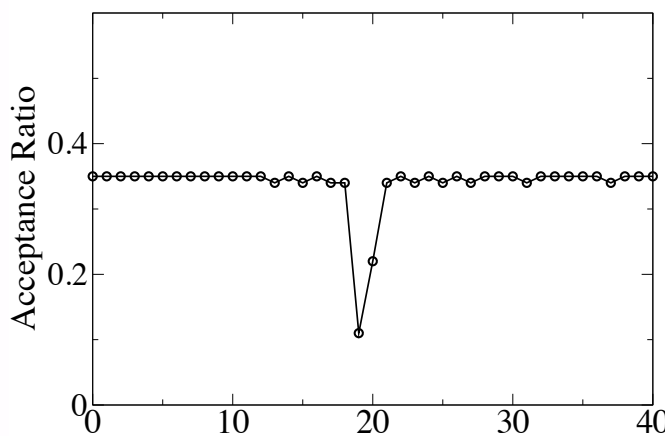
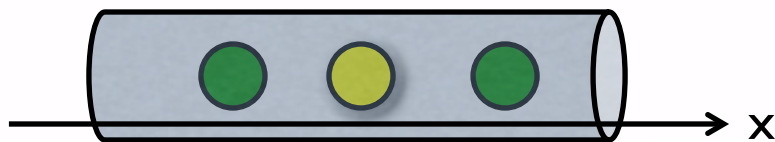


Distributed Replica Parallel



Postprocessing:  
OpenMP or POSIX Thread  
weighted histogram analysis  
on shared memory nodes

# PMF/REMD Toy model analysis: *necessity but not sufficiency?*



# Binding Free Energy of Calbindin D<sub>9k</sub>

$$k_{eq} = \frac{\int_{\text{site}} d1 \int dX e^{-\beta U}}{\int_{\text{bulk}} d1 \delta(r_1 - r_1^*) \int dX e^{-\beta U}}$$

$$= \frac{\int_{\text{site}} d1 \int dX e^{-\beta U}}{\int_{\text{site}} d1 \int dX e^{-\beta[U+u_a]}}$$

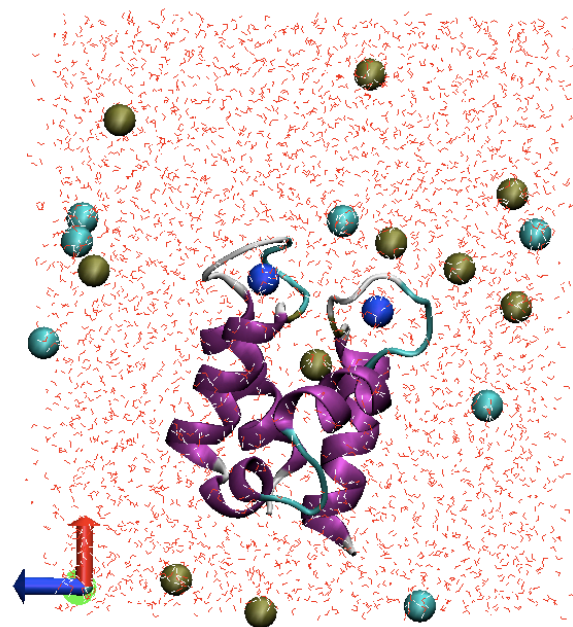
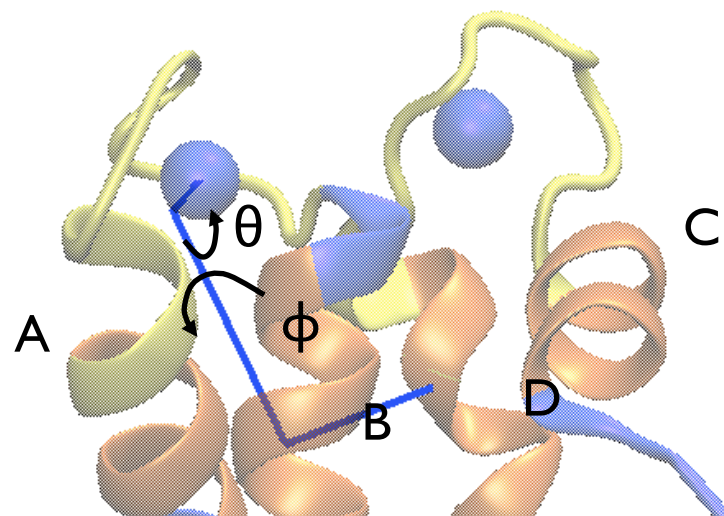
$$\times \frac{\int_{\text{site}} d1 \int dX e^{-\beta[U+u_a]}}{\int_{\text{bulk}} d1 \delta(r_1 - r_1^*) \int dX e^{-\beta U}}$$

$$= e^{-\beta(-\Delta G_a^{\text{site}})} * I^* S^*$$

$$I^* = \int_{\text{site}} dr e^{-\beta[w(r) - w(r^*)]}$$

$$S^* = (r^*)^2 \int_0^\pi \sin(\theta) d\theta \int_0^{2\pi} d\phi e^{-\beta u_a(\theta, \phi)}$$

$$\Delta G_{\text{bind}}^0 \equiv -k_B T \ln[k_{eq} C^0]$$

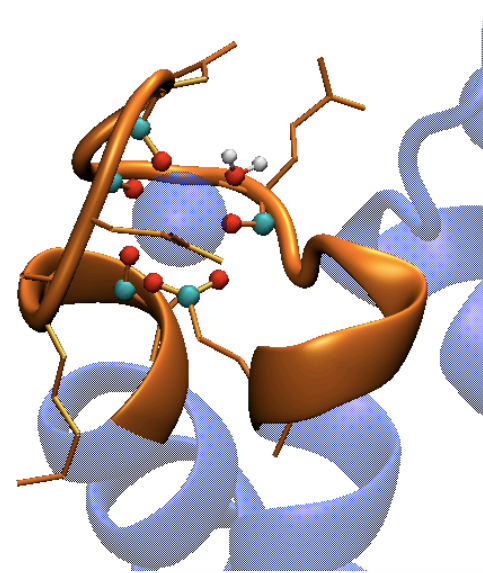
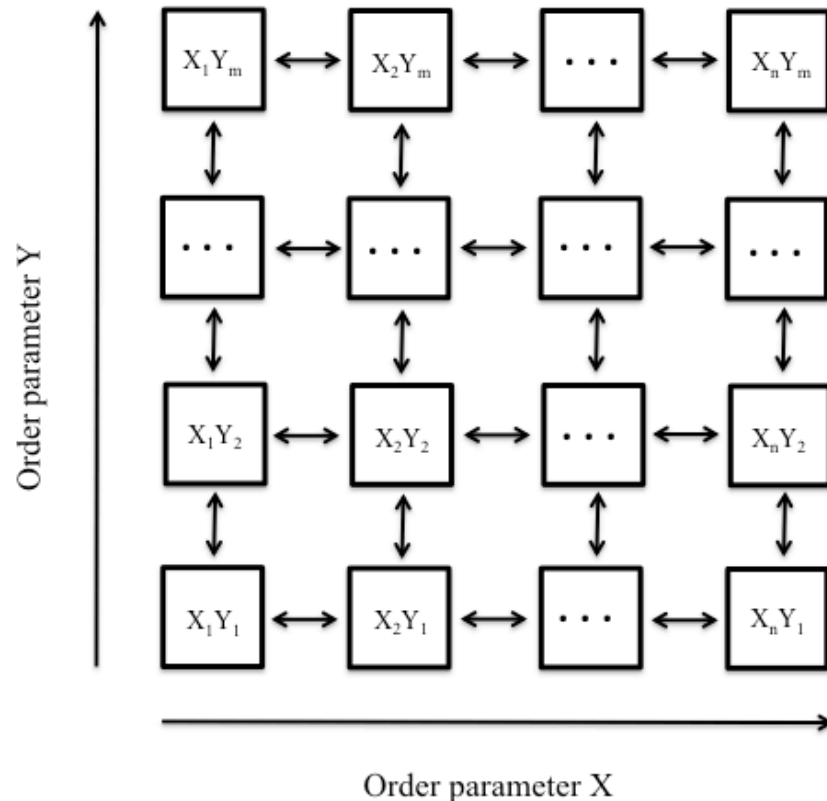


## 2D US/H-REMD

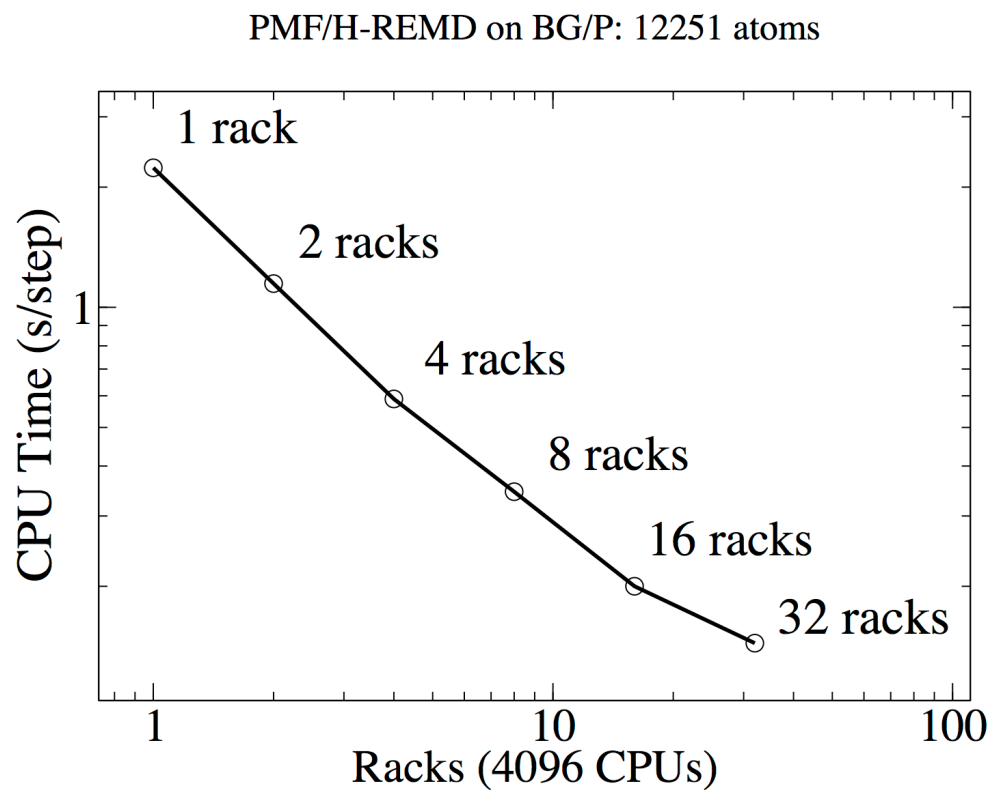
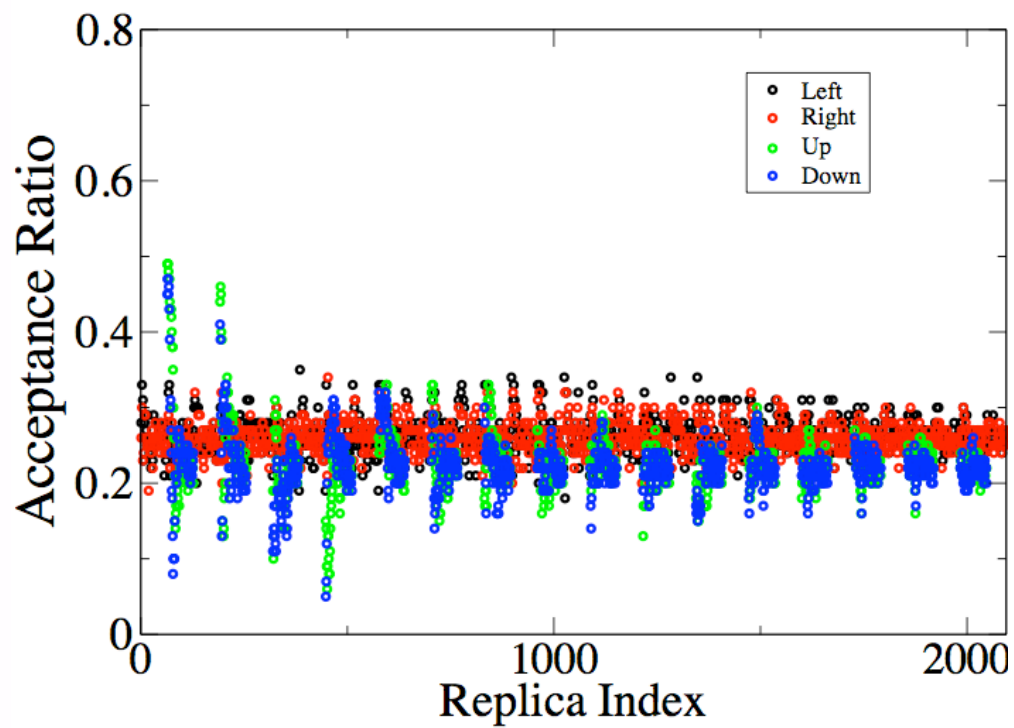
Order list of replicas is defined by  $k_1(x-p_1)^2+k_2(y-p_2)^2$   $\{p_1^i, k_1^i, p_2^i, k_2^i\}$

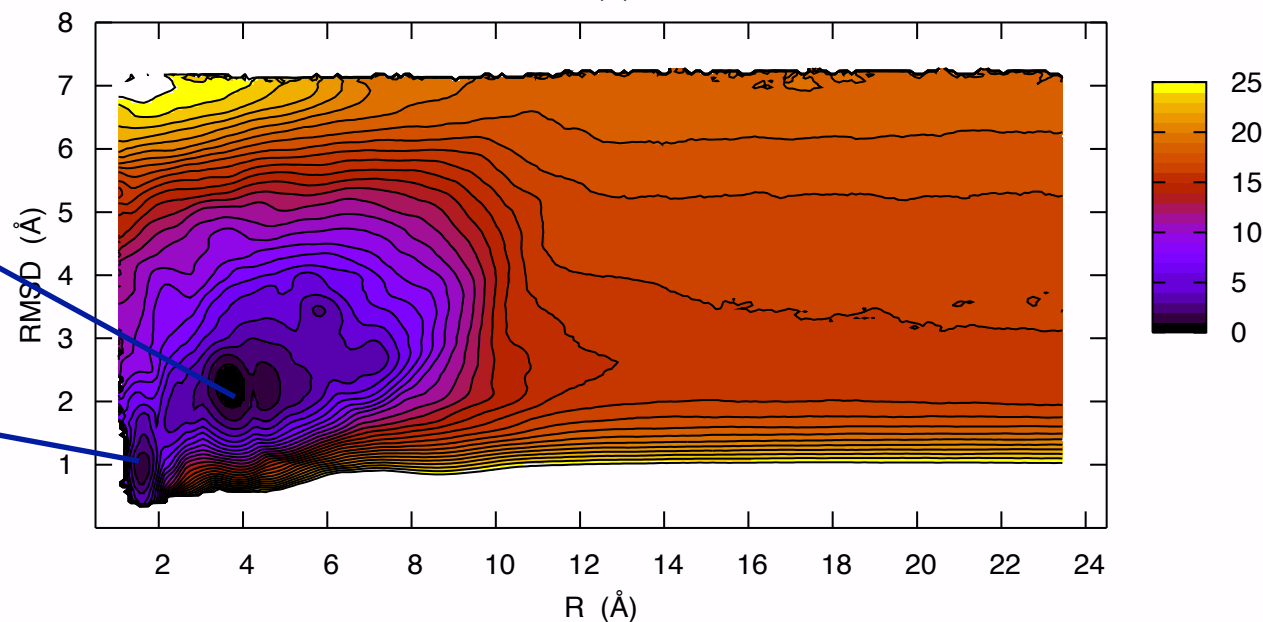
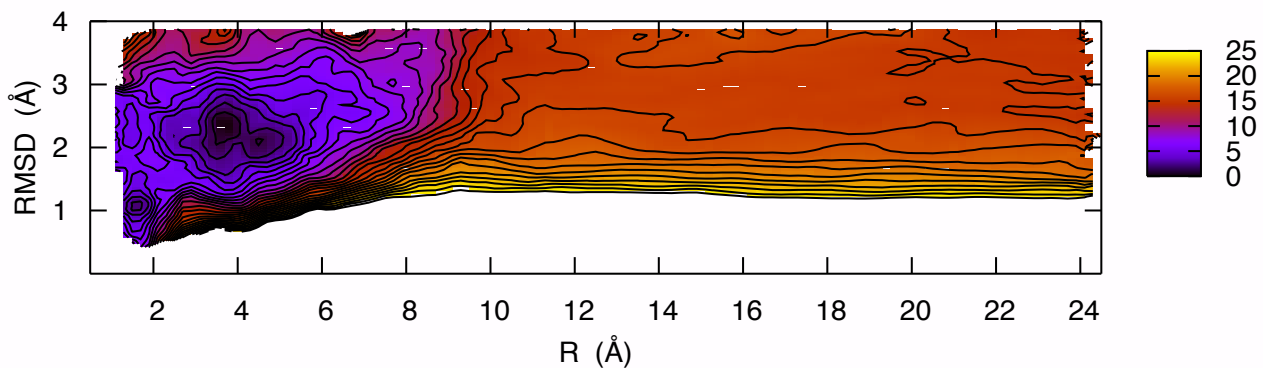
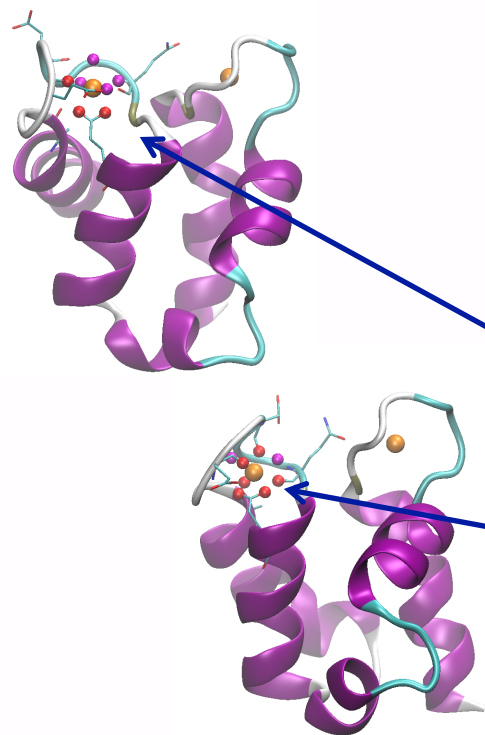
$$P(p_1^i \rightarrow p_1^j, p_2^l) = \min \left\{ 1, e^{-[U(p_1^i, p_2^l, \mathbf{r}_{i,l}) + U(p_1^j, p_2^l, \mathbf{r}_{j,l}) - U(p_1^j, p_2^l, \mathbf{r}_{j,l}) - U(p_1^i, p_2^l, \mathbf{r}_{i,l})] / k_B T} \right\}$$

$$P(p_2^m \rightarrow p_2^n, p_1^k) = \min \left\{ 1, e^{-[U(p_1^k, p_2^m, \mathbf{r}_{k,m}) + U(p_1^k, p_2^n, \mathbf{r}_{k,n}) - U(p_1^k, p_2^m, \mathbf{r}_{k,m}) - U(p_1^k, p_2^n, \mathbf{r}_{k,n})] / k_B T} \right\}$$



# Acceptance Ratio & Strong Scaling





$$I^* = \int_{\text{site}} dr e^{-\beta[w(r)-w(r^*)]} = 1.35 \times 10^{10} \text{ \AA}$$

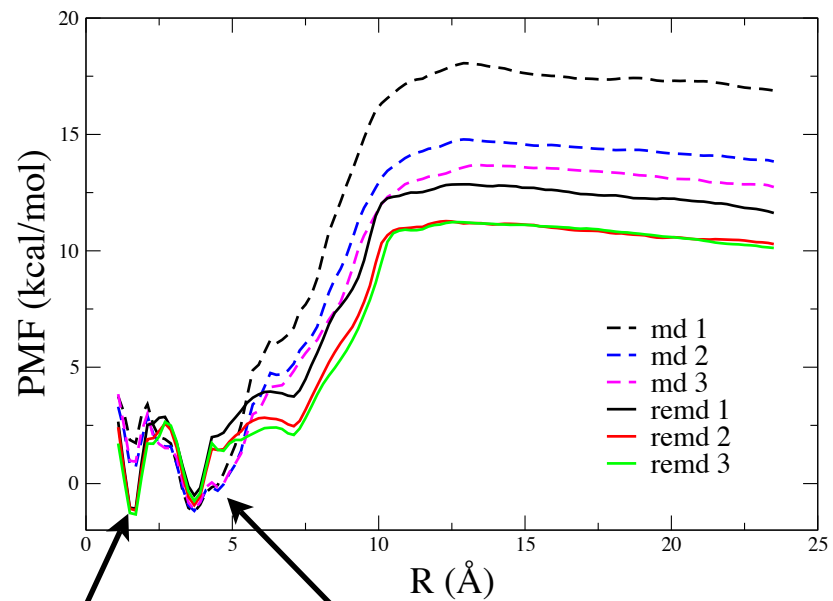
$$S^* = (r^*)^2 \int_0^\pi \sin(\theta) d\theta \int_0^{2\pi} d\phi e^{-\beta u_a(\theta, \phi)} = 15.35 \text{ \AA}^2$$

$$\begin{aligned} \Delta G_{\text{binding}}^0 &= -k_B T \ln(I^* S^* C^0) - \Delta G_a^{\text{site}} \\ &= -11.1 \text{ kcal/mol} \end{aligned}$$

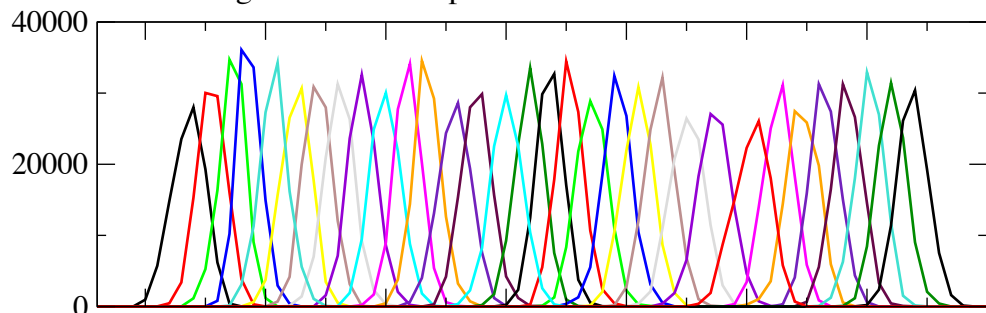
$$C^0 = 1/1661 \text{ \AA}^{-3} \quad \Delta G_a^{\text{site}} = 0.19 \text{ kcal/mol}$$

(Experimental value -9.4 kcal/mol)

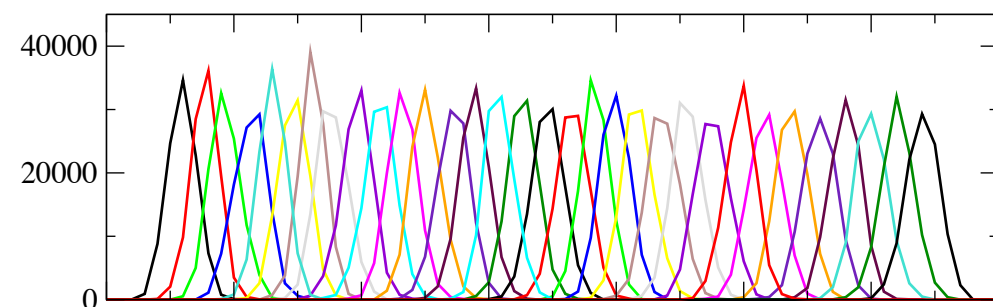




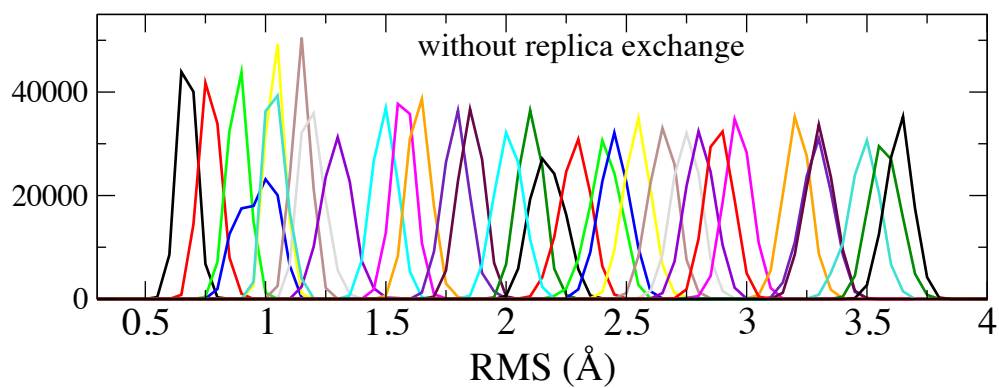
Histogram of RMS replicas at distance 1.5 Å with REMD



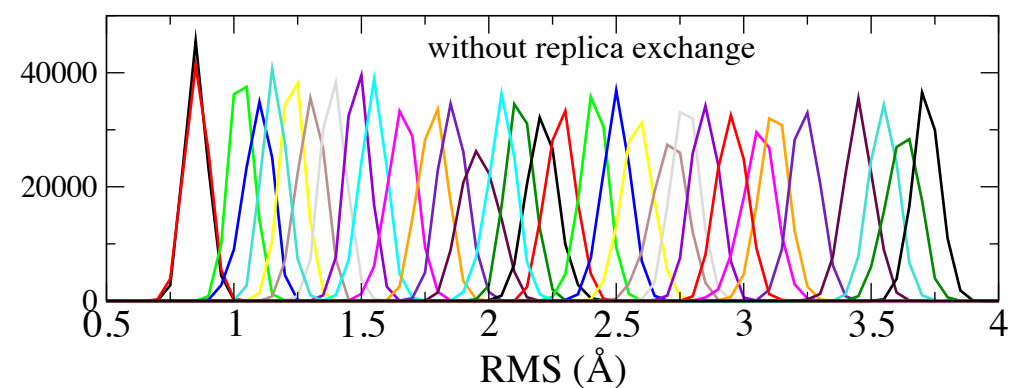
Histogram of RMS replicas at distance 4.25 Å with REMD



without replica exchange



without replica exchange



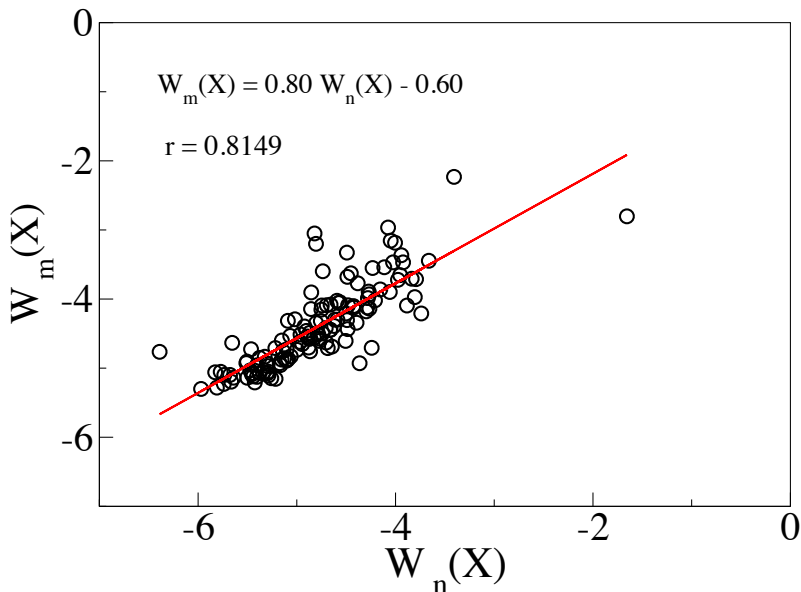
# Analysis of PMF convergence

Slope  $W_m = MW_n + B$

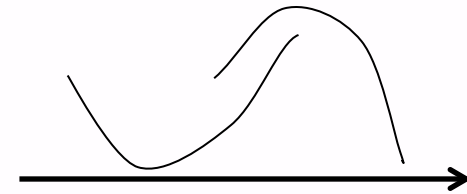
$$M_{nm} = \frac{\left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_n(\mathbf{x}) W_m(\mathbf{x}) \right) - \left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_n(\mathbf{x}) \right) \left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_m(\mathbf{x}) \right)}{\left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_n(\mathbf{x}) W_n(\mathbf{x}) \right) - \left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_n(\mathbf{x}) \right) \left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_n(\mathbf{x}) \right)}$$

$$p_{nm}(\mathbf{x}) = \frac{\rho_n(\mathbf{x}) \rho_m(\mathbf{x})}{\sum_{\mathbf{x}} \rho_n(\mathbf{x}) \rho_m(\mathbf{x})}$$

Variences:  $\sigma_{nm}^2 = \left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_n(\mathbf{x}) W_n(\mathbf{x}) \right) - \left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_n(\mathbf{x}) \right)^2 + \left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_m(\mathbf{x}) W_m(\mathbf{x}) \right) - \left( \sum_{\mathbf{x}} p_{nm}(\mathbf{x}) W_m(\mathbf{x}) \right)^2$



Variation big



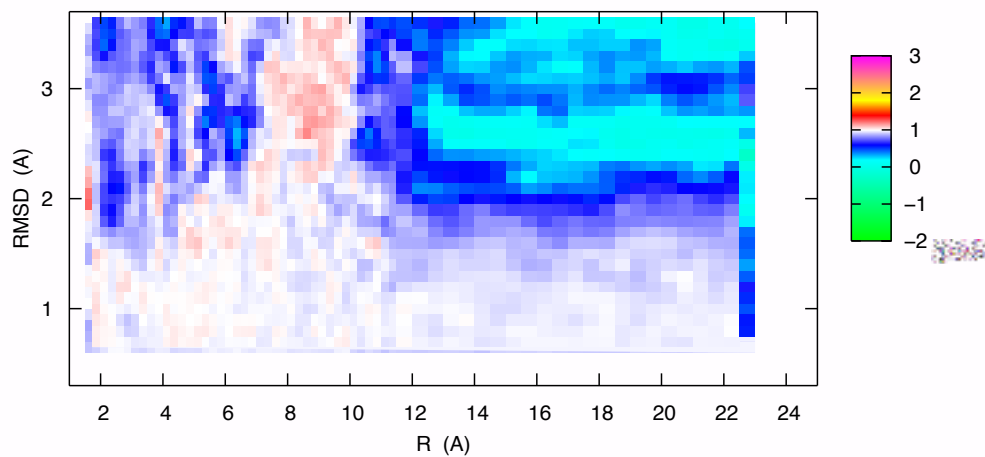
Variation small



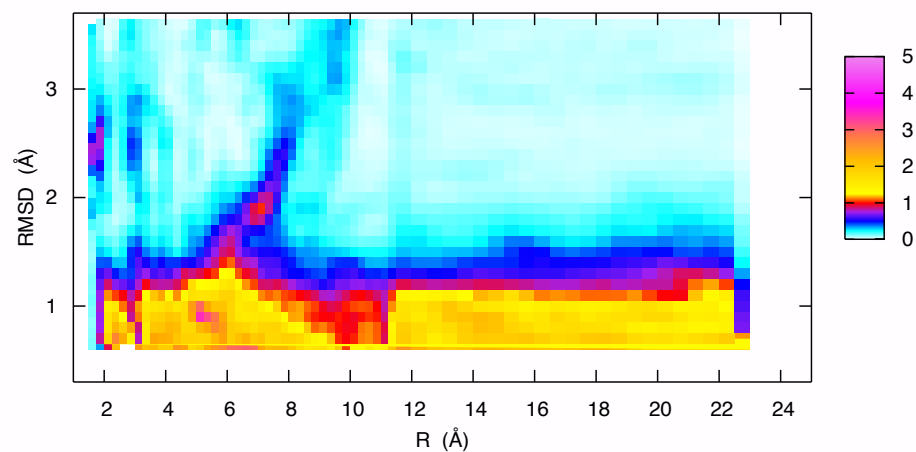
# Analysis of PMF convergence: US/REMD vs. US

US/REMD

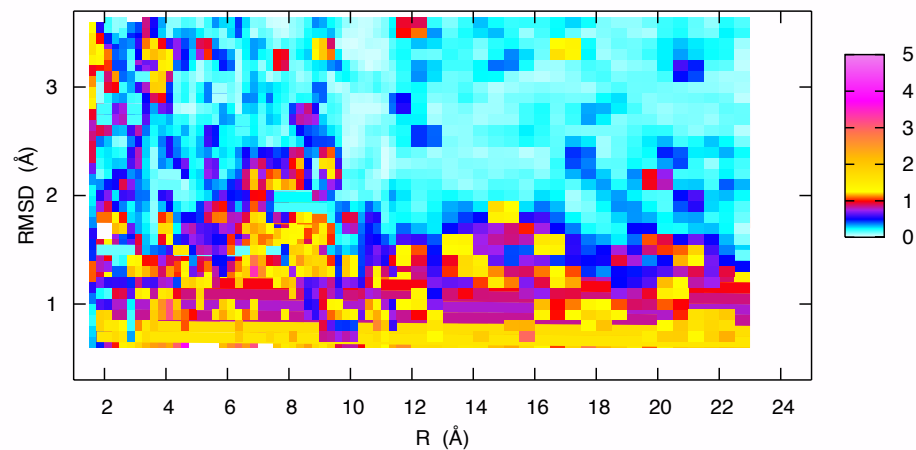
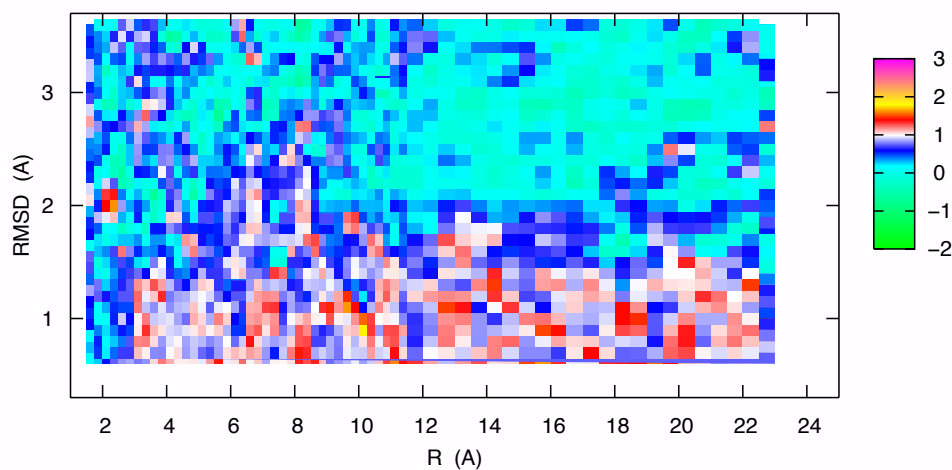
Slope

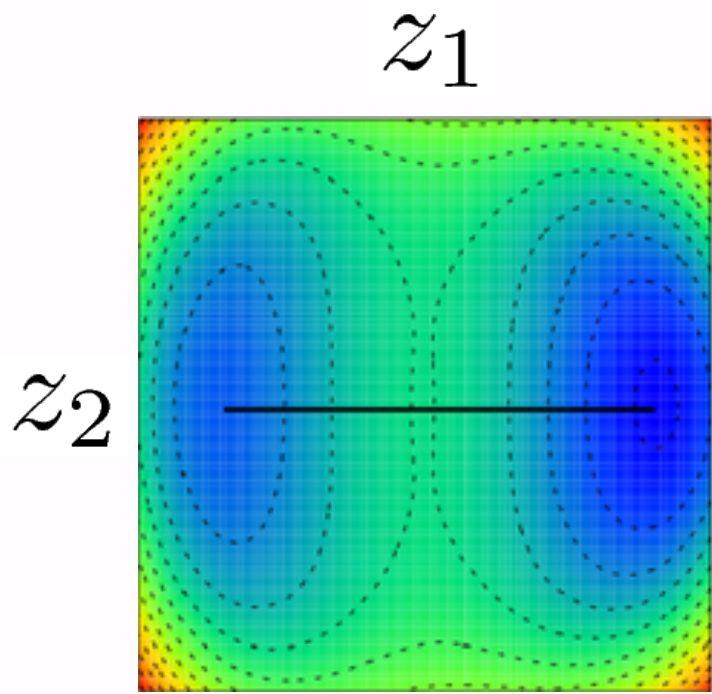


Variations

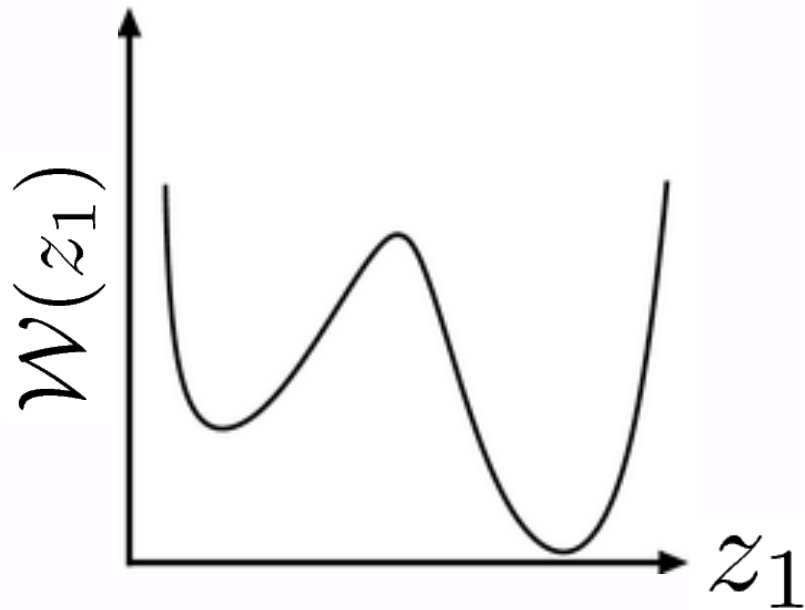


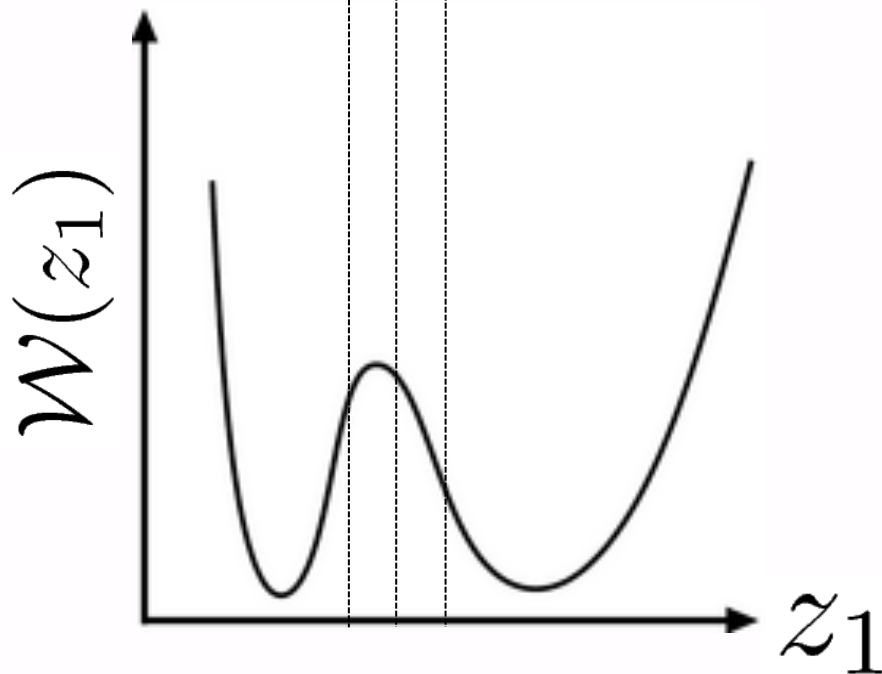
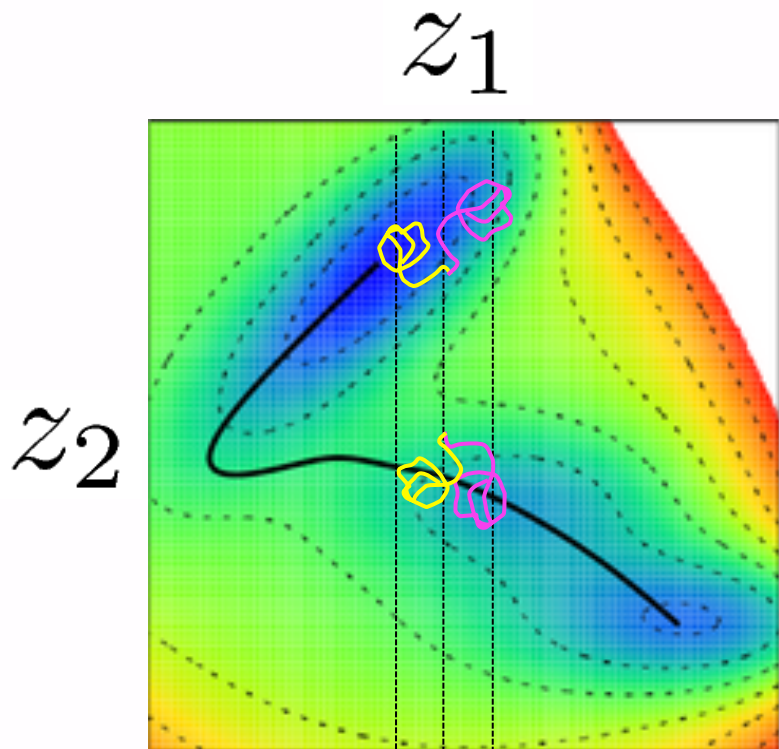
US





Trivial umbrella sampling in 1D





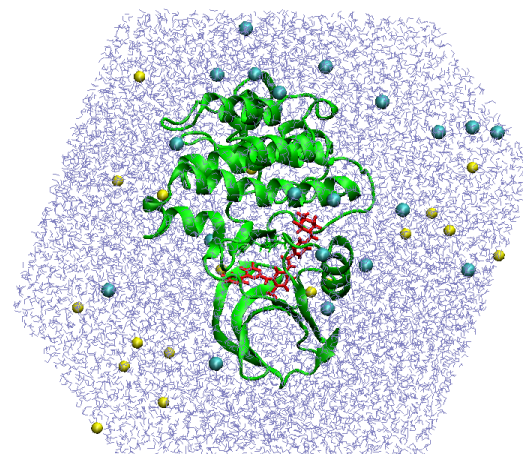
Sampling problems when all the collective variables relevant to the dynamical transition pathway are not included

Slowly varying orthogonal degrees of freedom

Degeneracy of  $Z_1$

# Binding Free Energy of Gleevec in Abl, c-Kit, Lck, and c-Src

$$U(\lambda_{\text{rep}}, \lambda_{\text{dis}}, \lambda_{\text{elec}}, \lambda_{\text{rstr}}) = U_0 + U_{\text{rep}}(\lambda_{\text{rep}}) + \lambda_{\text{dis}} U_{\text{dis}} + \lambda_{\text{elec}} U_{\text{elec}} + \lambda_{\text{rstr}} U_{\text{rstr}}$$



## FEP/ $\lambda$ -REMD scheme

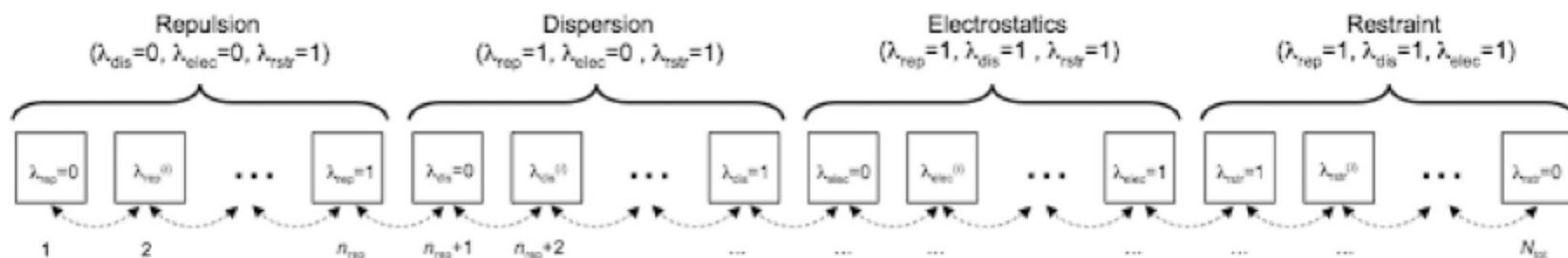
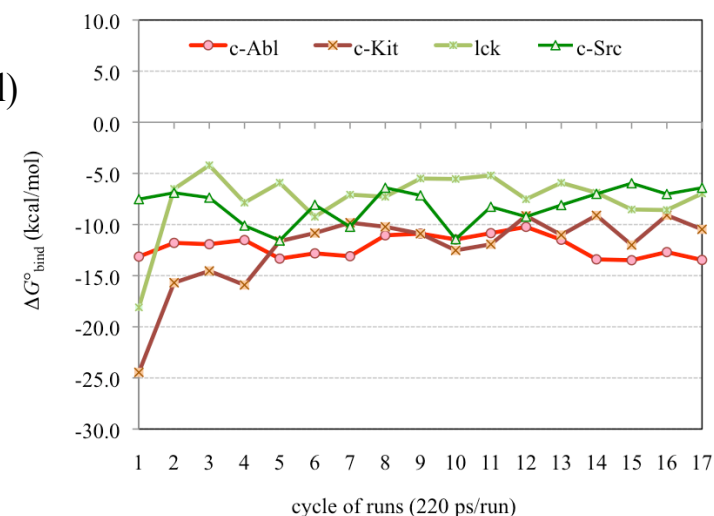
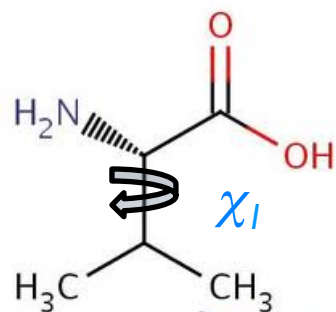
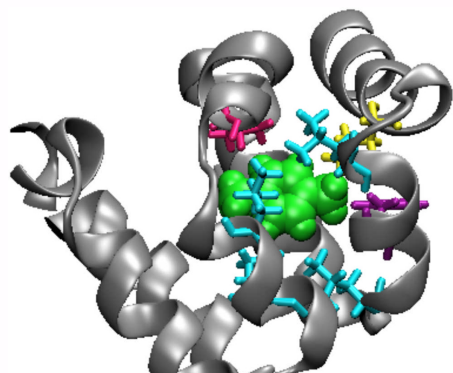


Table: Absolute Binding Free Energy of Gleevec With Src-family kinases (unit: kcal/mol)

	$\Delta G^{\circ}_{\text{bind}}$	$\Delta G_{\text{expt}}$	
		ref. 6	ref. 7
bulk $\rightarrow$ Abl	-11.8	-10.8	-8.2
bulk $\rightarrow$ c-Kit	-1.4	—	-7.4
bulk $\rightarrow$ Lck	-7.3	-8.7	-6.9
bulk $\rightarrow$ c-Src	-6.8	-6.2	-4.1



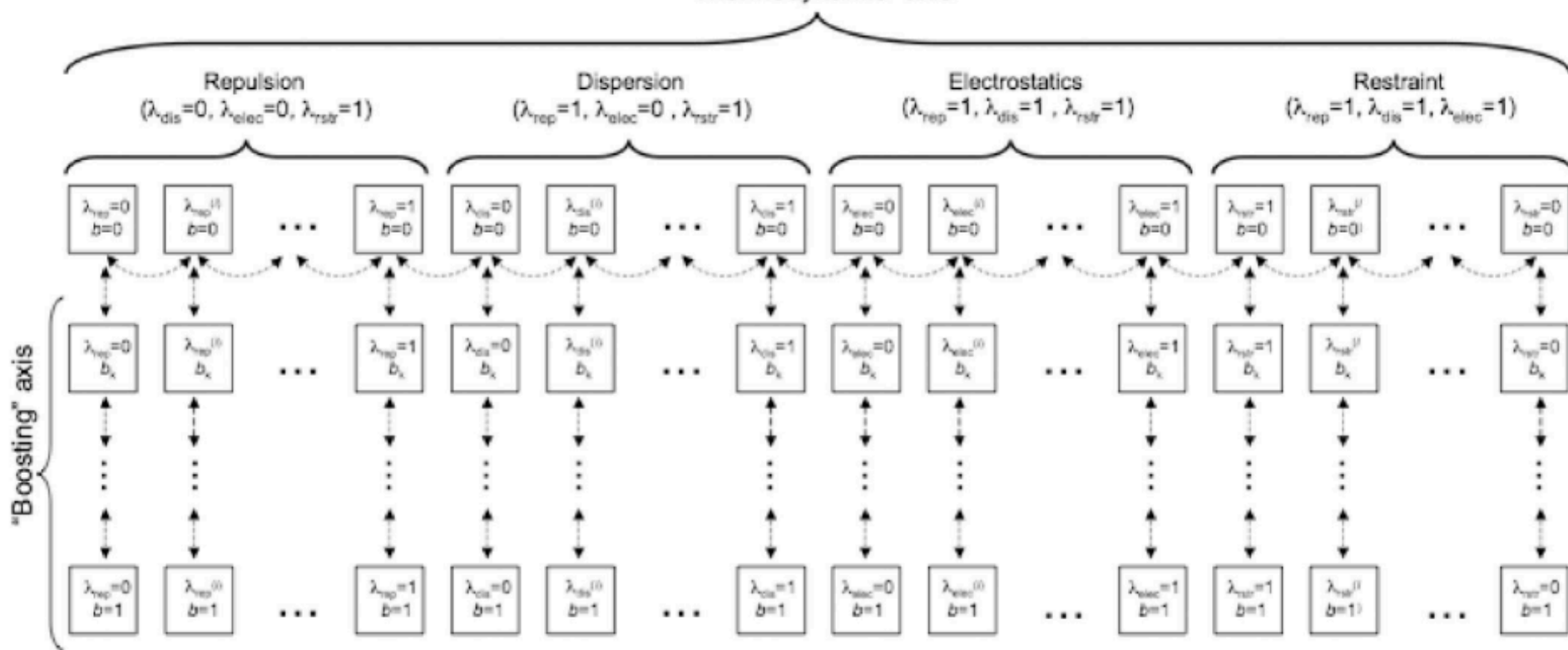
## 2-Dimensional FEP/H-REMD



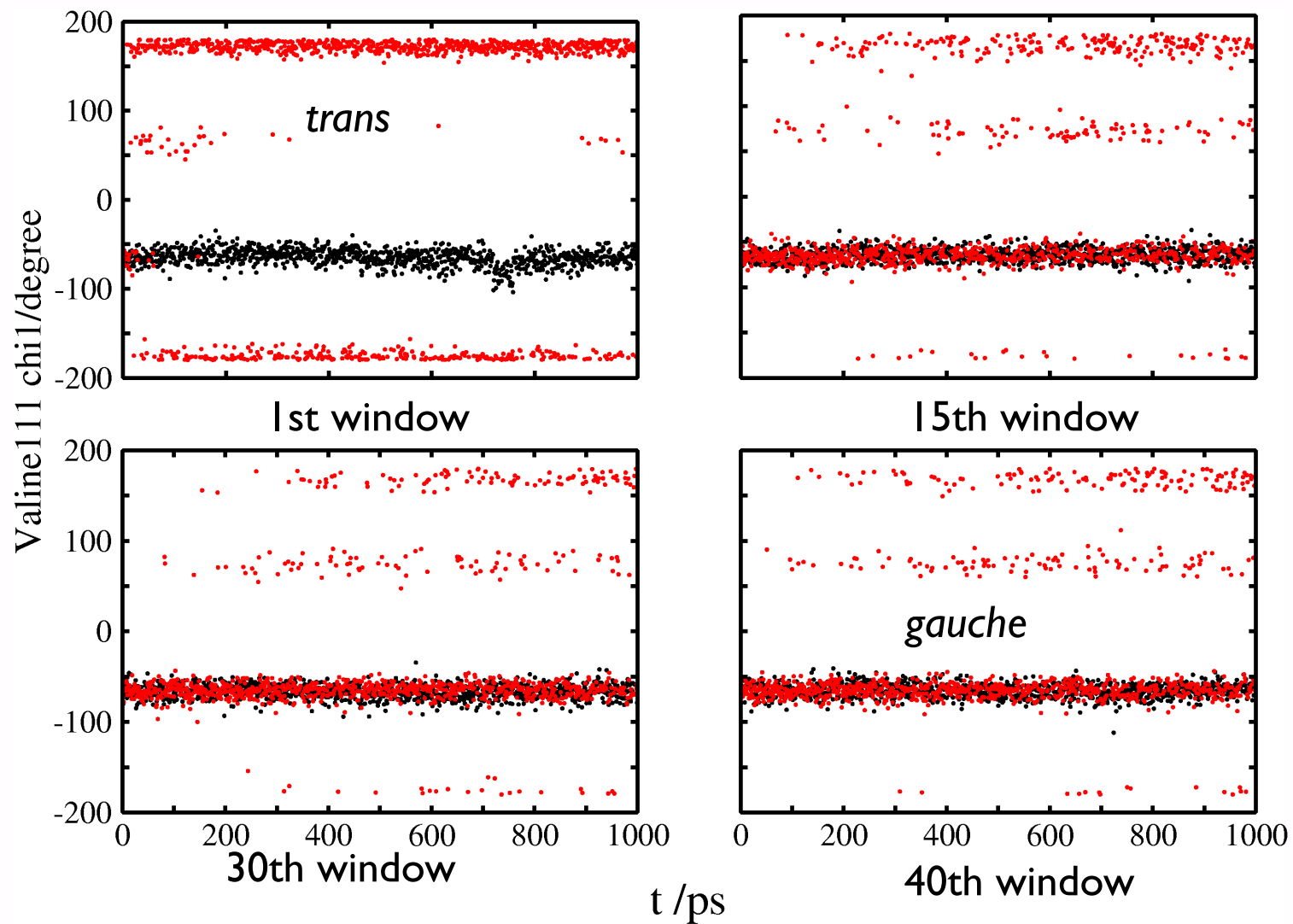
Binding of p-xylene to  
T4 lysozyme

$$V(\phi) = \sum_{n=1}^3 K_n (1 + \cos(n(\phi - \phi_{\min})))$$

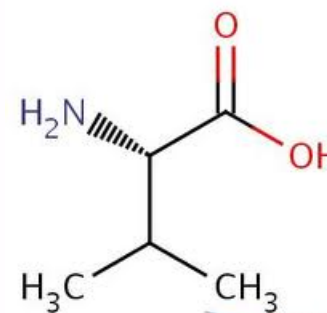
"Thermodynamic" axis



# Enhanced $\chi_1$ Sampling by FEP/H-REMD



- FEP/ $\lambda$ -REMD
- FEP/H-REMD



$$\Delta G_{\text{bind}} = -5.1 \text{ kcal/mol, (exp : } -4.7 \text{ kcal/mol)}$$



## **Conclusions and Perspectives**

MPI level REMD is implemented in CHARMM and NAMD (including threaded version), suitable for petascale supercomputing platform

Two case studies prove the enhance sampling and extremely scalability of two free energy calculation methods coupled with REMD.

2D US/H-REMD: Calcium ion binding Calbindin

2D FEP/H-REMD: *p*-xylene binding T4 Lysozyme

## **Undergoing Improvements:**

- Irregular shape multi-dimensional US/REMD by assigning an array of neighboring replicas
- String methods or coarse grain transition path search to define several initial path, and expanding replicas along the path way.
- Performing PCA to determine few order parameters out of high dimensional space



Benoit Roux

Wei Jiang

Yen-lin Lin

Lei Huang

Luca Maragliano

Roux Group members

Klaus Schulten

James Phillips



ANL Leadership Computing Facility

The Early Science Program (ESP)



Chris Chipot

Tony Lelièvre

Andrew Pohorille

