# Towards 2D overland flow simulations

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# Problem context

Preventing overland flow and erosion

*From upstream...*

(Photos : Yves Le Bissonnais, INRA)

*...to downstream.*

Downstream zones modifications (watersheds)

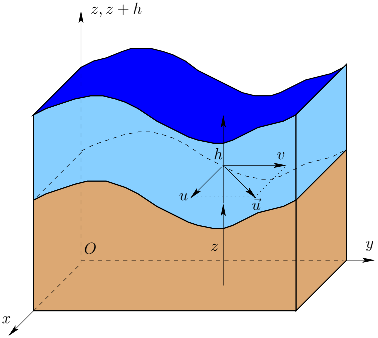
- Where is the water coming from ?
- Where is it flowing ?

Use of physical models is required to :

- simulate flow (volumes and location)
- suggest changes (grass strip).

$\rightsquigarrow$ to carry out improvements

# Shallow Water (Saint-Venant) system



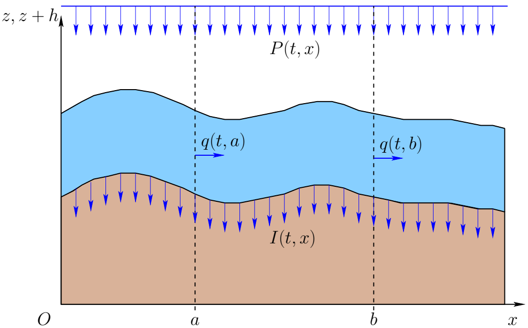

Data : topography $z$, rain $P$, infiltration $I$
Unknowns : velocities $u$, $v$, water height $h$

$$
\begin{cases}
\partial_t h + \partial_x (hu) + \partial_y (hv) = P - I \\
\partial_t (hu) + \partial_x \left(hu^2 + gh^2/2\right) + \partial_y (huv) = gh(-\partial_x z - S_{f\,x}) \\
\partial_t (hv) + \partial_x (huv) + \partial_y \left(hv^2 + gh^2/2\right) = gh(-\partial_y z - S_{f\,y})
\end{cases}
$$

# Strategy

- Properties of the 1D Shallow Water system
- Choice of the method depending on the properties
- Validation : analytical solutions and laboratory experiment
- Application : field data

# 1D Shallow Water system




Data : topography $z$, rain $P$, infiltration $I$

Unknowns : velocities $u$, water height $h$

A system of conservation laws

$$\begin{cases} \partial_t h + \partial_x(hu) = P - I \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) = gh\,(-\partial_x z - S_f) \end{cases} \tag{1}$$

# System properties (I) : **Hyperbolicity**

Setting $q = hu$

$$U = \begin{pmatrix} h \\ q \end{pmatrix}, \; F(U) = \begin{pmatrix} q \\ q^2/h + gh^2/2 \end{pmatrix}, \; B = \begin{pmatrix} P - I \\ gh(-\partial_x z - S_f) \end{pmatrix},$$

compact form

$$\partial_t U + \partial_x F(U) = \partial_t U + F'(U)\partial_x U = B,$$

Hyperbolicity if $h > 0$ :

$$\lambda_-(U) = u - \sqrt{gh}, \quad \lambda_+(U) = u + \sqrt{gh}$$

| **Saint-Venant** | **gaz dynamic** |
|---|---|
| Froude number $Fr = \frac{\lvert u \rvert}{c}$<br>$c = \sqrt{gh}$ free surface waves celerity | Mach number $\frac{\lvert u \rvert}{c}$<br>$c = \sqrt{p'(\rho)}$ sound speed [1] |
| subcritical $Fr < 1$ | subsonic |
| supercritical $Fr > 1$ | supersonic |

1. $p(\rho) = \rho RT$ perfect gaz

# System properties (II) : **Conservation laws**

Integral of equation (1) in x

$$\partial_t h + \partial_x q = P - I,$$

gives

$$\frac{d}{dt}\int_a^b h(t,x)\,dx = q(t,a) - q(t,b) + \int_a^b P(t,x) - I(t,x)dx,$$

Mass conservation of water.

Second equation : momentum equation

# System properties (III) : **Steady states**

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$$\left\{ \begin{array}{l} \partial_t h + \partial_x(hu) = P - I \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) = gh\,(-\partial_x z - S_f) \end{array} \right. \tag{2}$$

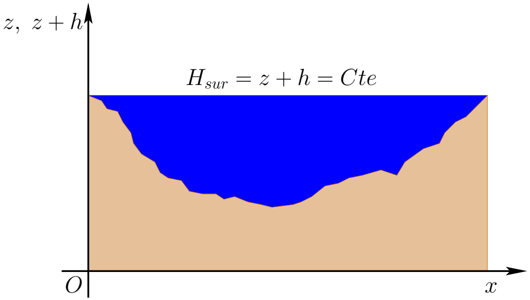
$$\partial_t h = \partial_t u = \partial_t q = 0$$

$$\left\{ \begin{array}{l} \partial_x hu = P - I \\ \partial_x(hu^2 + gh^2/2) = gh\,(-\partial_x z - S_f) \end{array} \right. .$$

# System properties (III) : **Steady states**

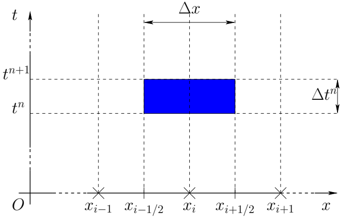
Lac at rest equilibrium

$$\begin{cases} u = 0 \\ g(h + z) = Cst \end{cases} .$$



# Numerical method (I)

Finite volume method




We integrate
$$\partial_t U + \partial_x F(U) = 0$$
on the volume
$$[t^n, t^{n+1}[\times]x_{i-1/2}, x_{i+1/2}[,$$
and we set
$$U_i^n = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} U(t^n, x)\, dx$$

we get
$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^n - F_{i-1/2}^n\right],$$

with the interface flux approximation

$$F_{i+1/2}^n = \mathcal{F}(U_i^n, U_{i+1}^n) \sim \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} F(U(t, x_{i+1/2}))\, dt.$$

# Numerical method (I)

- For each choice of $\mathcal{F}(U_G, U_D)$ we have a different finite volume scheme :

  HLL, kinetic, Rusanov, VFRoe-ncv, suliciu, ...

- second Order
  - in space : MUSCL, ENO, modified ENO
  - in time : Heun

# Numerical method (I)

- For each choice of $\mathcal{F}(U_G, U_D)$ we have a different finite volume scheme :

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- second Order
  - in space : MUSCL, ENO, modified ENO
  - in time : Heun
- Coupling with the source term (topography $\partial_x z$)
  Necessity : compatibility with steady states

# Steady states (II)

$$\begin{cases} \partial_t h + \partial_x(hu) = 0 \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) = -gh\partial_x z \end{cases} \tag{3}$$

$$\partial_t h = \partial_t u = \partial_t q = 0$$

$$\begin{cases} hu = Cst \\ u^2/2 + g(h+z) = Cst \end{cases} .$$

We consider

$$\begin{cases} u = Cst \\ g(h+z) = Cst \end{cases} .$$

# Hydrostatic reconstruction (II) [Audusse et al., 2004]

We define

$$z^* = max(z_G, z_D)$$

and

$$\begin{cases} U_G^* = (h_G^*, h_G^* u_G), \ U_D^* = (h_D^*, h_D^* u_D) \\ h_G^* = max(h_G + z_G - z^*, 0) \\ h_D^* = max(h_D + z_D - z^*, 0) \end{cases} .$$

Thus, we have

$$\begin{cases} \mathcal{F}_G(U_G, U_D, \Delta Z) = \mathcal{F}(U_G^*, U_D^*) + \begin{pmatrix} 0 \\ g({h_G}^2 - (h_G^*)^2)/2 \end{pmatrix} \\ \mathcal{F}_D(U_G, U_D, \Delta Z) = \mathcal{F}(U_G^*, U_D^*) + \begin{pmatrix} 0 \\ g({h_D}^2 - (h_D^*)^2)/2 \end{pmatrix} \end{cases} ,$$

where $\mathcal{F}(U_G, U_D)$ is the numerical flux.

# Friction treatment

Shallow Water system with friction $f$

$$\begin{cases} \partial_t h + \partial_x(hu) = 0, \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) + h\partial_x z = -hf, \end{cases} \qquad (4)$$

$f = f(h, u)$ friction force (on the bottom)

Several friction laws possible

- Manning : $\quad f = n^2 \dfrac{u|u|}{h^{4/3}}$
- Darcy-Weisbach : $\quad f = F \dfrac{u|u|}{8gh}$

# Friction treatment

- Apparent topography [Bouchut, 2004]

$$\text{We consider :} \quad z_{app} = z + b^n$$

with $\partial_x b^n = S_f^n$

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- Apparent topography [Bouchut, 2004]

$$\text{We consider :} \quad z_{app} = z + b^n$$

with $\partial_x b^n = S_f^n$

- Semi-implicit [Bristeau and Coussin, 2001]

$$q_i^{n+1} + F\frac{|q_i^n|q_i^{n+1}}{8h_i^n h_i^{n+1}}\Delta t = q_i^n + \frac{\Delta t}{\Delta x}\left(F_{i+1/2} - F_{i-1/2}\right)$$

with $q_i^{n+1*}$ for the right part, we have

$$q_i^{n+1} = \frac{q_i^{n+1*}}{1 + \Delta t\frac{F|u_i^n|}{8h_i^{n+1}}}$$

# Validation on analytical solutions – SWASHES

New test cases :

- Saint-Venant/shallow water :
  - data $z$
  - unknowns $h$ et $u$ (and so $q$)

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- Saint-Venant/shallow water :
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- test cases
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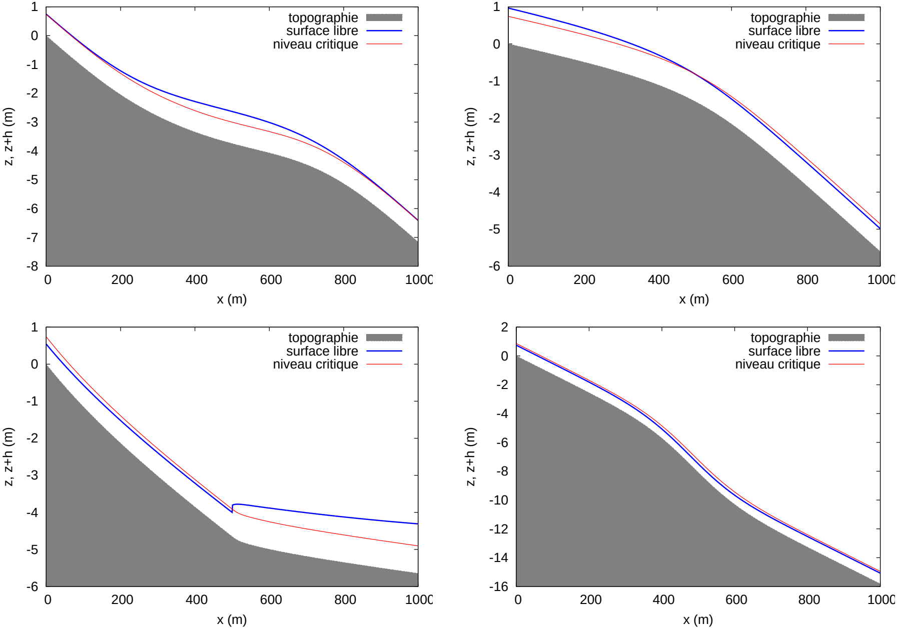
  - unknown $z$

# Validation on analytical solutions – SWASHES

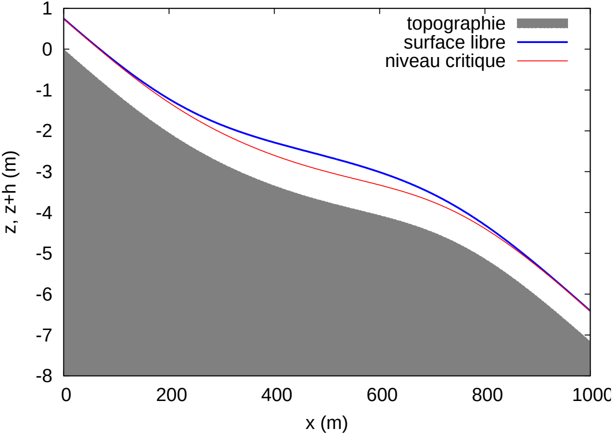
New test cases :

- Saint-Venant/shallow water :
  - data $z$
  - unknowns $h$ et $u$ (and so $q$)
- test cases
  - data $h$ and $q$ (and so $u$)
  - unknown $z$
- Several possibilities
  - several friction laws
  - diffusion source term [Delestre and Marche, 2010]
  - rain source term


topographie
surface libre
niveau critique
z, z+h (m)
x (m)
0
200
400
600
800
1000
1
0
-1
-2
-3
-4
-5
-6
-7
-8
2
-2
-4
-6
-8
-10
-12
-14
-16

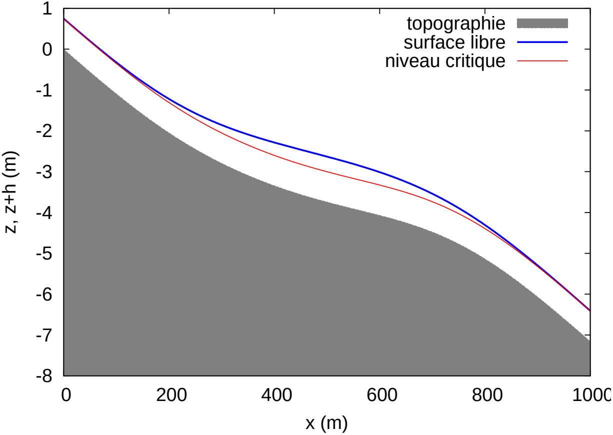
# Validation on analytical solutions – SWASHES

Apparent topography (subcritical-subcritical)



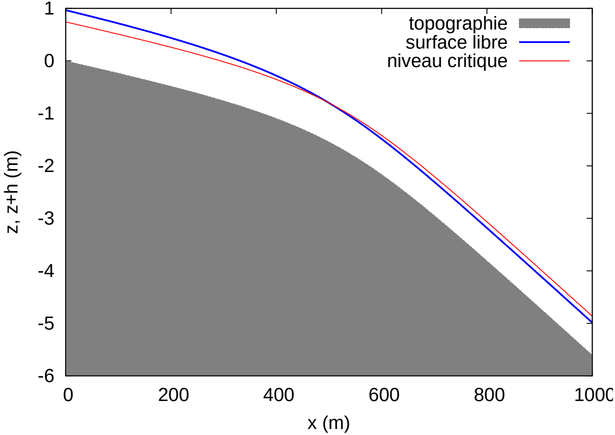
# Validation on analytical solutions – SWASHES

Semi-implicit (subcritical-subcritical)



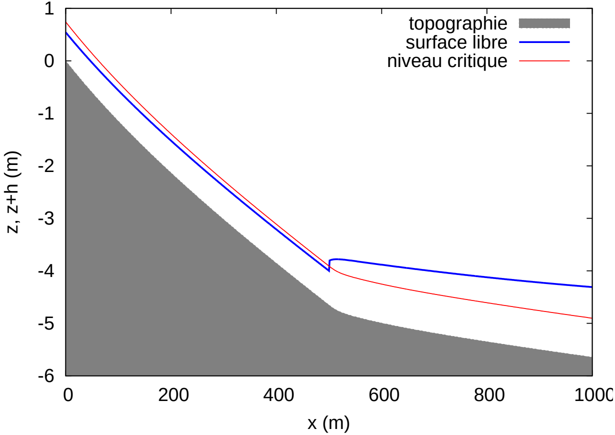
# Validation on analytical solutions – SWASHES

Semi-implicit (subcritical-supercritical)



# Validation on analytical solutions – SWASHES

Semi-implicit (supercritical-subcritical)



# Summary of the chosen numerical method

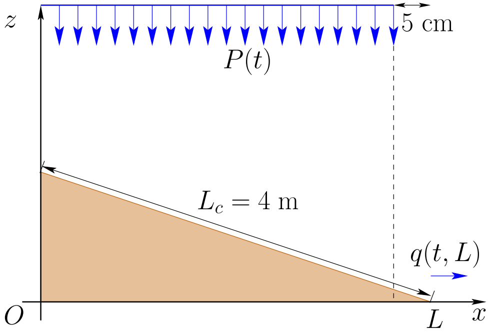
- Numerical flux : HLL
- Second order scheme : MUSCL
- Friction : semi-implicit treatment
- Shallow Water system with rain $P$

$$\left\{ \begin{array}{l} \partial_t h + \partial_x (hu) = P \\ \partial_t (hu) + \partial_x (hu^2 + gh^2/2) + h\partial_x z = -hf \end{array} \right. \tag{5}$$

time splitting/explicit treatment

# Validation on experiments – INRA rain simulator

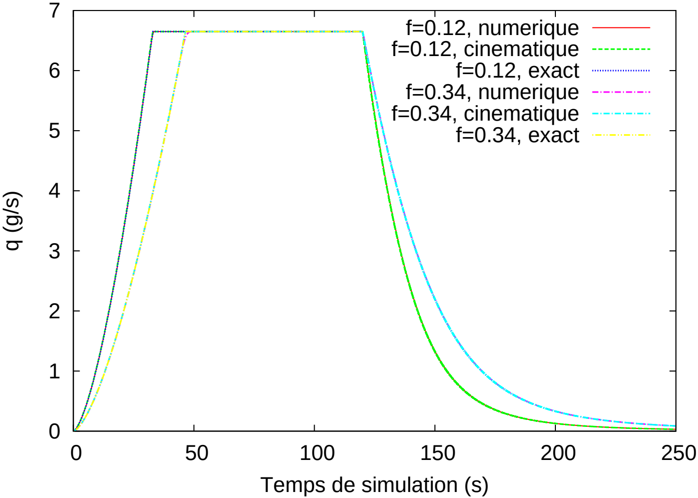
# Settings of the experiment



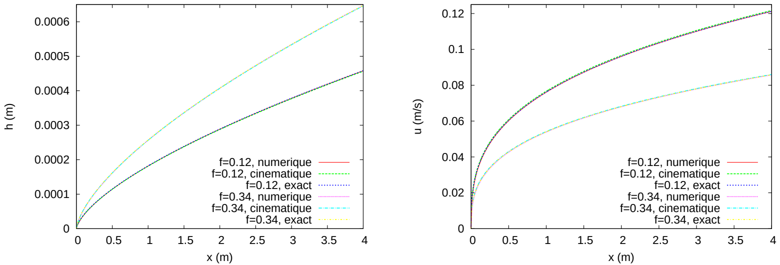

$$0 \leq t \leq 250\text{s}$$

$$R(x,t) = \begin{cases} 50 \text{ mm/h if } (x,t) \in [0,L] \times [5,125] \\ 0 \text{ else} \end{cases}$$

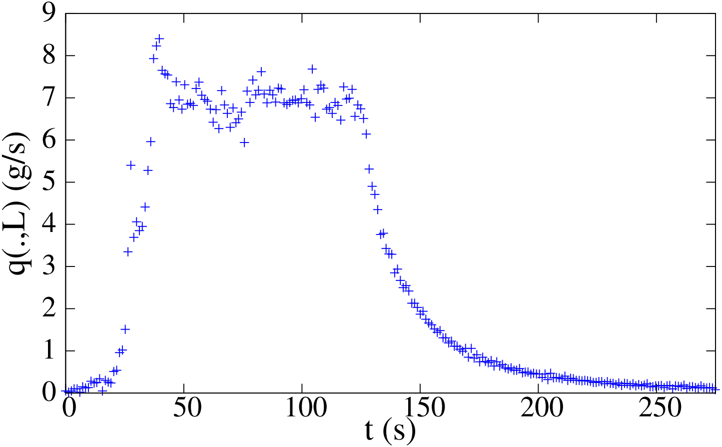
# Analytical solutions and simulations



# Water height and velocity at equilibrium

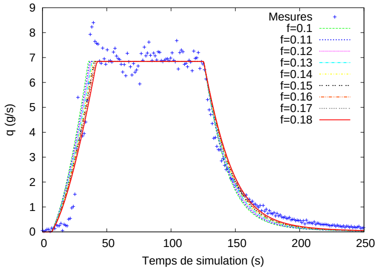


# What about reality ?

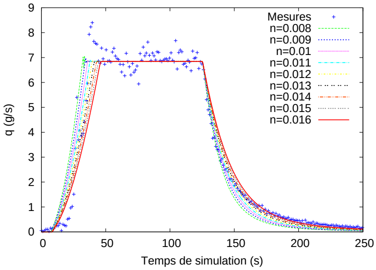


# "Calibration"

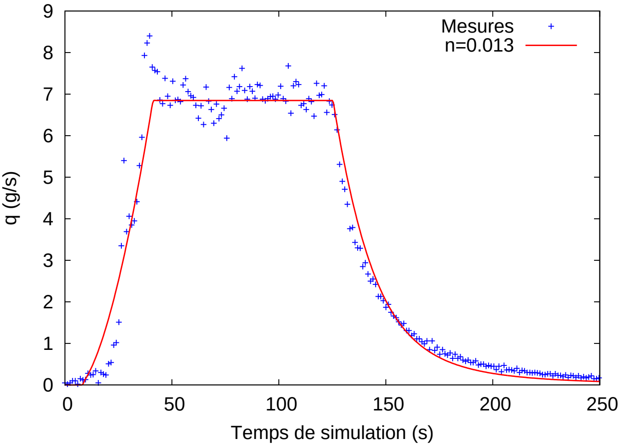
Darcy-Weisbach



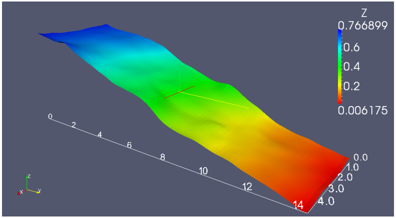

Manning

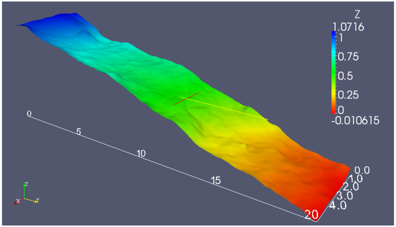


# A simulation result (Manning)

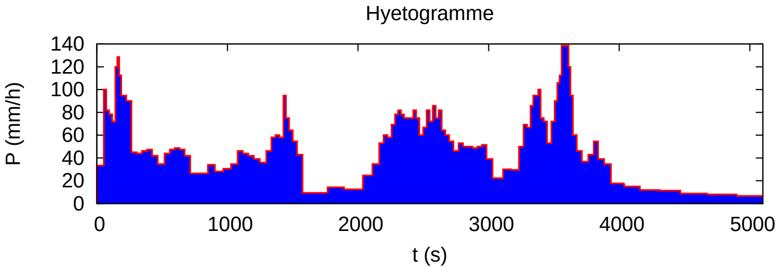


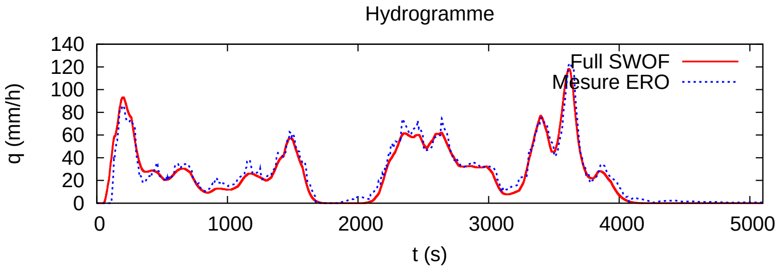
# Parcels in Niger ([Esteves et al., 2000], IRD)



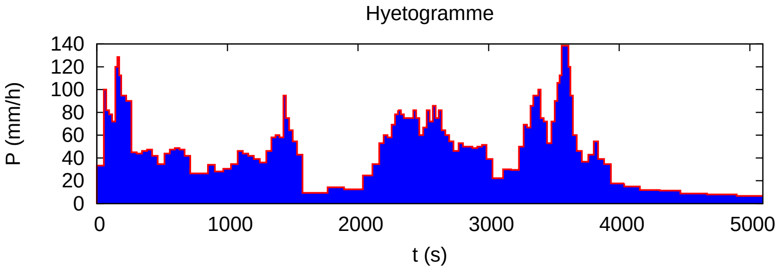



# Darcy-Weisbach

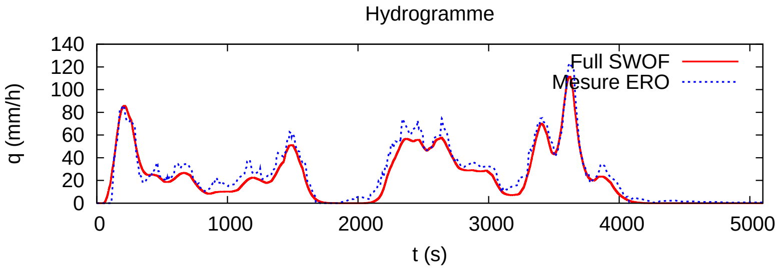


# Manning



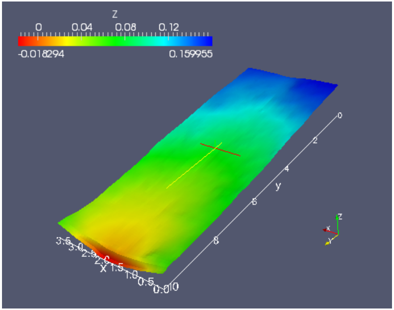
Hyetogramme
P (mm/h)
140
120
100
80
60
40
20
0
0
1000
2000
3000
4000
5000
t (s)

Hydrogramme
q (mm/h)
Full SWOF
Mesure ERO
140
120
100
80
60
40
20
0
0
1000
2000
3000
4000
5000
t (s)

# Thies parcel – Senegal ([Tatard et al., 2008], IRD)

Velocity measures by SVG [Planchon et al., 2005]
Number of cells : 40 $\times$ 100 (4 m $\times$ 10 m)



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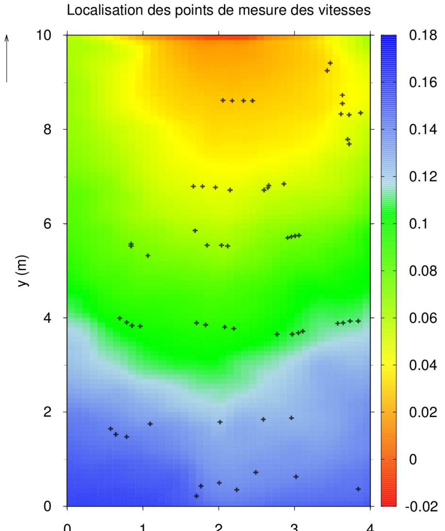

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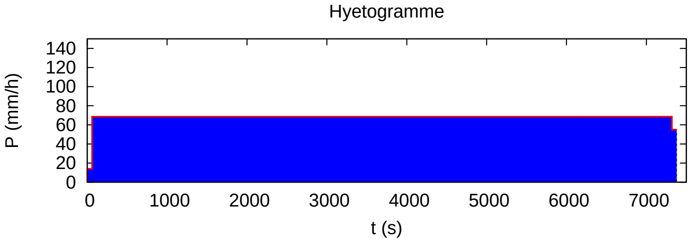
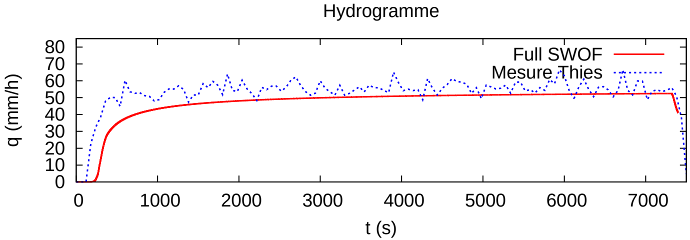
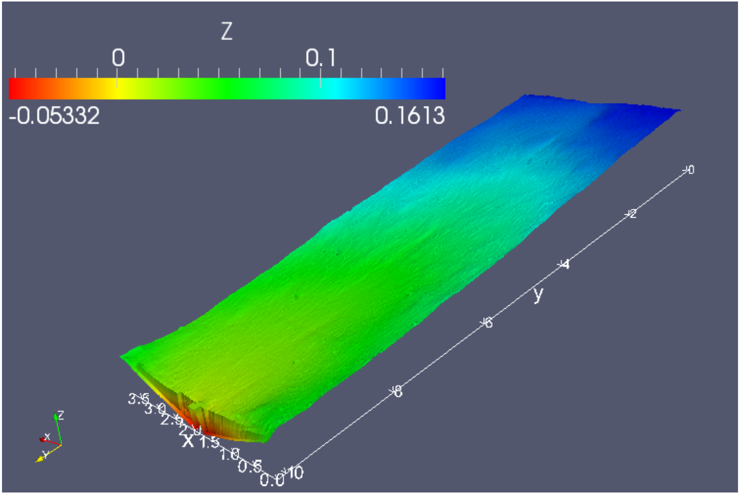


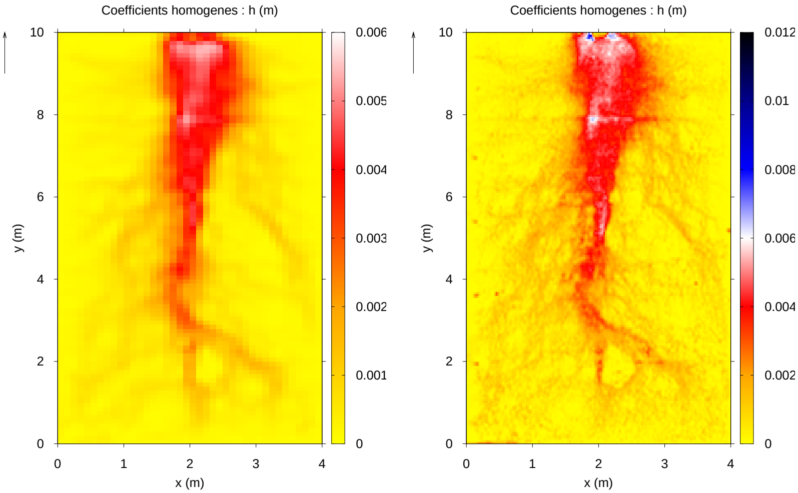
# Thies parcel – Senegal ([Tatard et al., 2008], IRD)

Number of cells : 160 × 200



# Thies parcel – Senegal ([Tatard et al., 2008], IRD)

Homogeneous coefficients



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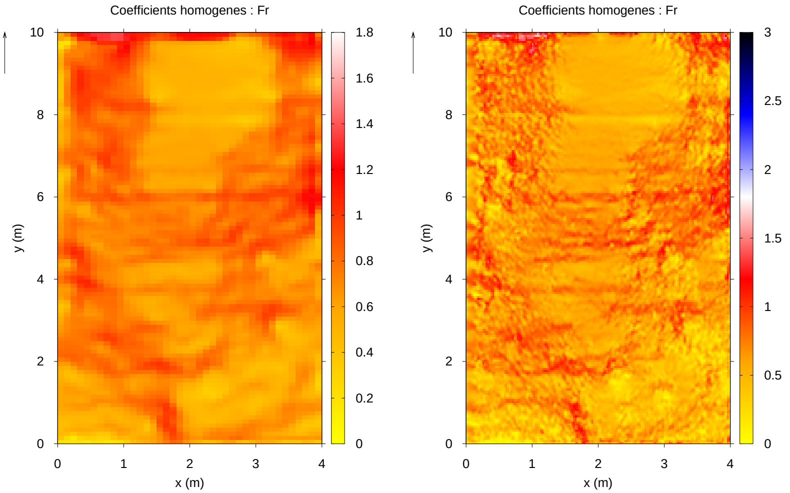
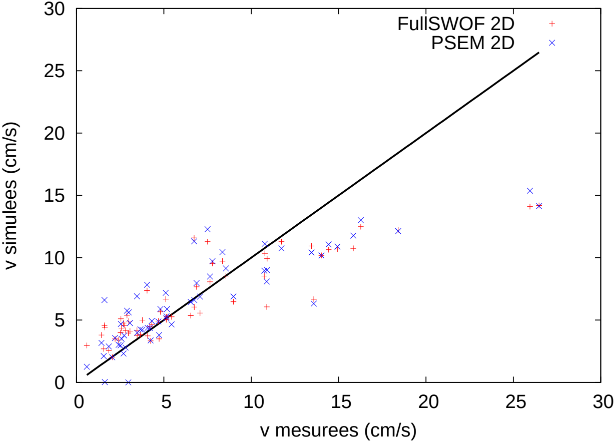


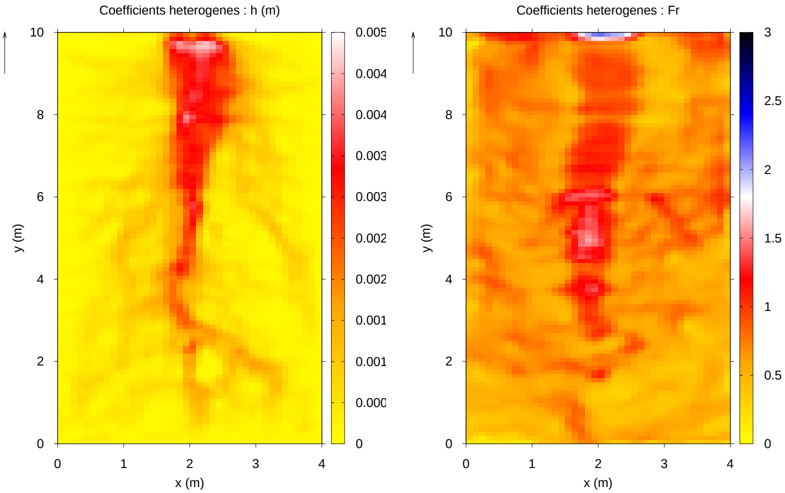
# Thies parcel – Senegal ([Tatard et al., 2008], IRD)

Velocities



# Thies parcel – Senegal ([Tatard et al., 2008], IRD)

Heterogeneous coefficients



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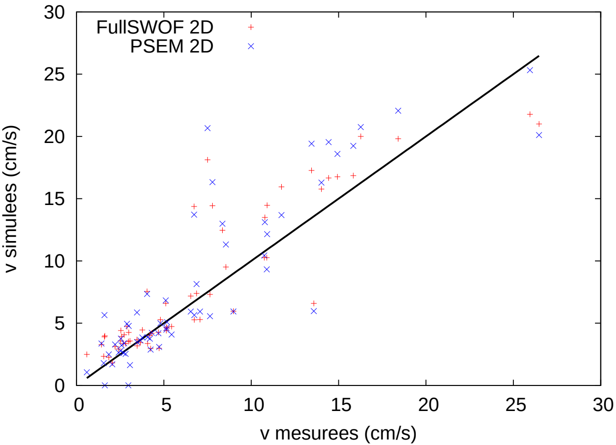
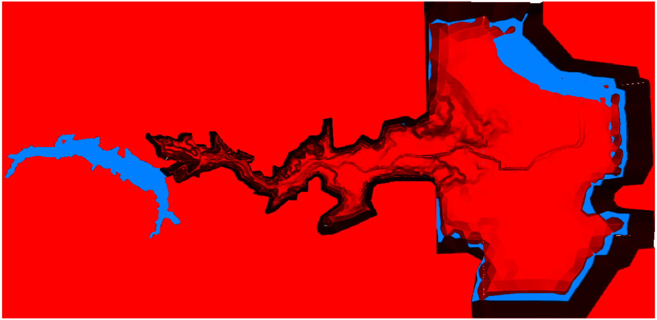


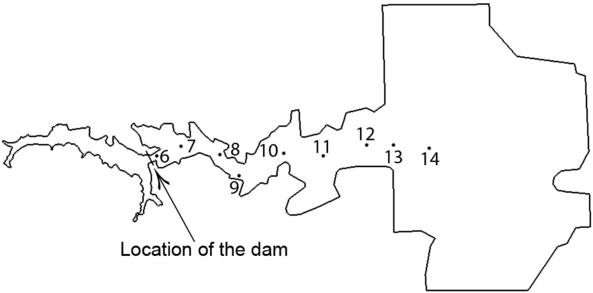
# FullSWOF

- object and inheritance
- variables encapsulation
- vector class (2d)
- objects "distributor"
- fixed CFL and fixed $\Delta t$
- Doxygen documentation
- Free open source software

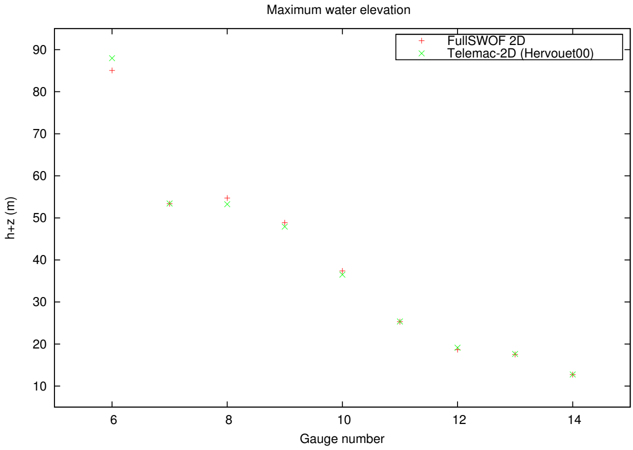
# Malpasset dam break simulation (Cordier et al., CEMRACS 2012)



# Malpasset dam break simulation



# Malpasset dam break simulation



Thank You !

## HLL flux

$$\mathcal{F}(U_G, U_D) = \begin{cases} F(U_G) \text{ if } 0 < c_1 \\ F(U_D) \text{ if } c_2 < 0 \\ \dfrac{c_2 F(U_G) - c_1 F(U_D)}{c_2 - c_1} + \dfrac{c_1 c_2 (U_D - U_G)}{c_2 - c_1} \text{ else} \end{cases},$$

with two parameters

$$c_1 < c_2.$$

For $c_1$ and $c_2$, we take

$$c_1 = \inf_{U = U_G, U_D} \left( \inf_{j \in \{1,2\}} \lambda_j(U) \right) \text{ and } c_2 = \sup_{U = U_G, U_D} \left( \sup_{i \in \{1,2\}} \lambda_i(U) \right).$$

with $\lambda_1(U) = u - \sqrt{gh}$ and $\lambda_2(U) = u + \sqrt{gh}$. retour

Audusse, E., Bouchut, F., Bristeau, M.-O., Klein, R., and Perthame, B. (2004).
A fast and stable well-balanced scheme with hydrostatic reconstruction for shallow water flows.
SIAM J. Sci. Comput., 25(6) :2050–2065.

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Esteves, M., Faucher, X., Galle, S., and Vauclin, M. (2000).
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Planchon, O., Silvera, N., Gimenez, R., Favis-Mortlock, D., Wainwright, J., Le Bissonnais, Y., and Govers, G. (2005).
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