Why does water fall from an inverted glass ?

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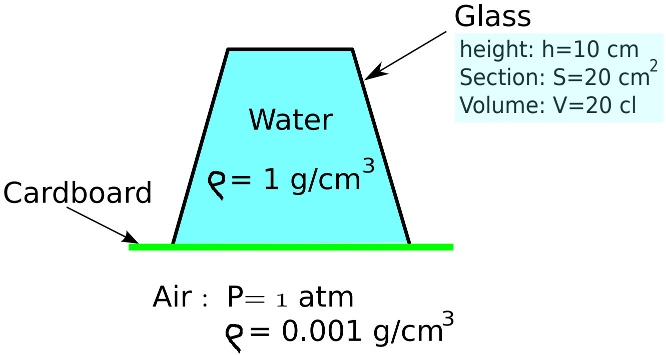
- Experiment:



Water is “heavier” than air ?

- Mass of water:

$$M = \rho_W V = 200g$$

  - with $V = 20cl$, $\rho_W = 1g/cm^3$

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- What mass can air at atmospheric pressure sustain on the surface S of the glass ?

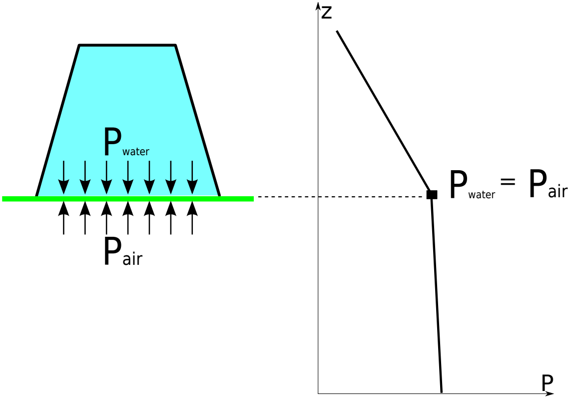
$$M_{max} = \frac{P_{air} S}{g} = 20\ kg$$

  - with earth gravity $g = 10\ m/s^2$, $P_{air} = 1atm$, $S = 20\ cm^2$

- Equivalent of a 10 m water column

But why does not the card fall ?

- Hydrostatic equilibrium → in water $P = P_{air} - \rho_W g z$.



- The force exerted by water on the card is (almost) equal to that exerted by air.

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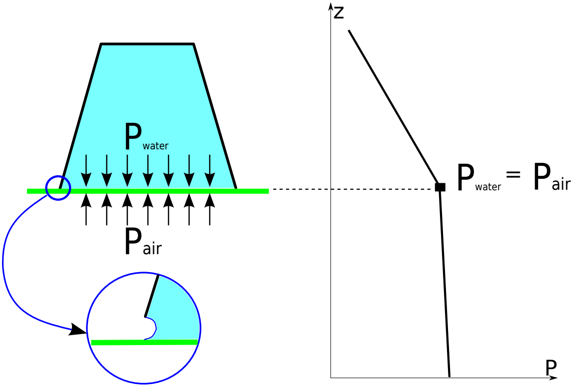

- The force exerted by water on the card is (almost) equal to that exerted by air.
- Surface tension effects stabilize the configuration and compensate for the mass of the card.

Hydrostatic equilibrium without a card

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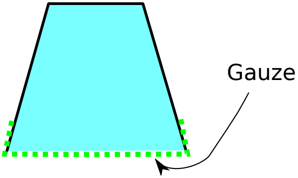

- What is the purpose of gauze ?
  - Not a mechanical barrier : no strength, porous

Hydrostatic equilibrium without a card

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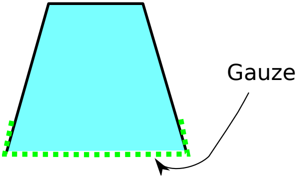
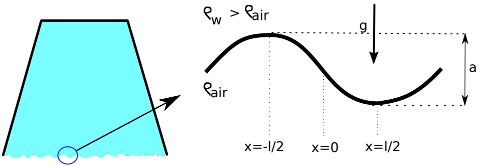


- What is the purpose of gauze ?
  - Not a mechanical barrier : no strength, porous
  - It helps surface tension “smooth” the interface.

  Gauze suppresses small ripples at the interface

Rippled interface



- Imagine that:
  - The interface is still
  - $P_{air}(x=0) = P_W(x=0) = P_0$

- Then, hydrostatic balance implies that:
  - $P_{air}(-\ell/2) = P_0 - \rho_{air}\, g\, a/2 \quad \& \quad P_W(-\ell/2) = P_0 - \rho_W\, g\, a/2$

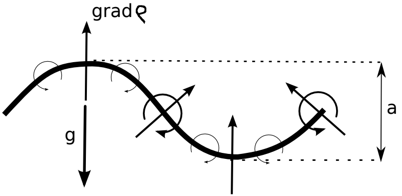
$$[P_{air} - P_W](-\ell/2) = (\rho_W - \rho_{air})\, g\, a/2 > 0$$

  - Opposite at $x = +\ell/2$:

$$[P_{air} - P_W](+\ell/2) = -(\rho_W - \rho_{air})\, g\, a/2 < 0$$

- This simple reasoning:
  - Shows that a rippled interface cannot be still
  - Suggests that air pushes water around and goes up at $x = -\ell/2$ and that water pushes air around and goes down at $x = +\ell/2$.

Rayleigh-Taylor instability (RTI)



- Velocity u_x, u_z → vorticity $\omega = \partial_x u_y - \partial_y u_x$

Euler eq. : $$\partial_t \frac{\omega}{\rho} = -\frac{\nabla\rho \wedge \nabla P}{\rho^3}$$

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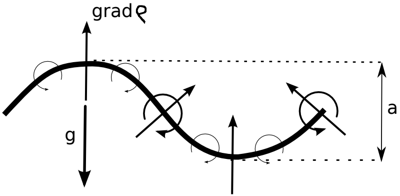

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- Normal mode analysis:

$$a(t) = a_0 e^{\sqrt{A_t g \kappa} t}$$

$A_t = \frac{\rho_W - \rho_{air}}{\rho_W + \rho_{air}}$, κ = wave number of the perturbation

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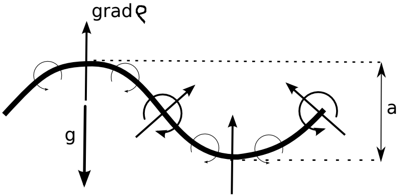

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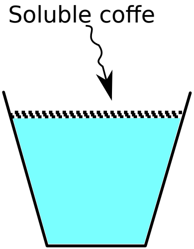
- RTI is the reason why water falls from the glass.

A simple Rayleigh-Taylor experiment



- Surface tension holds the coffe grains
- The grains mix with water
- Mixed water is denser than fresh water → RTI

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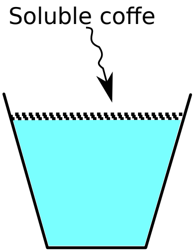

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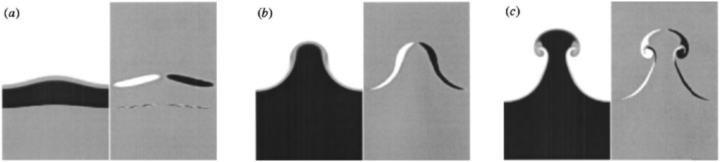
→

- Mushroom shaped structures appear
- Eventually, some chaotic, random mixing → turbulence

Non-linear stage of RTI

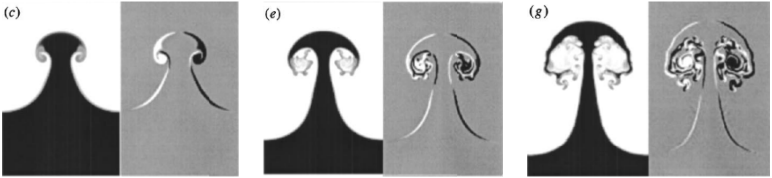


From Peng et al., Phys. Fluids, Vol. 15, No. 12, 2013

- Shear instability (Kelvin-Helmotz) at the tip of the bubble
- Creates two contra-rotative vortices

⇒ mushroom shape

Transition to turbulence



From Peng et al., Phys. Fluids, Vol. 15, No. 12, 2013

- Shear instability and RTI keep on producing smaller vortices
- Richardson's cascade:

Big whirls have little whirls
that feed on their velocity,
and little whirls have lesser whirls,
and so on to viscosity
– in the molecular sense.

- Eventually, vortices with a continuous spectrum of scales are created.
  - From $\ell \sim$ size of the largest mushroom
  - To $\eta \sim$ molecular dissipation scale
  - ℓ/η can reach values up to 10^6 and more

About RTI small scales (1/2)

- Kolmogorov-Obukhov (KO,1941) gave a more precise description of the Richardson's cascade in Homogeneous Isotropic Turbulence (HIT).

- Velocity increment between two points

$$\delta \boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x} + \boldsymbol{r}) - \boldsymbol{u}(\boldsymbol{x}) \sim \text{velocity of vortex of size } r$$

- In HIT, energy decays: $\partial_t \left\langle \frac{1}{2}|u|^2 \right\rangle = -\langle \varepsilon \rangle = -\nu \left\langle |\nabla u|^2 \right\rangle$
  - $\langle \varepsilon \rangle$ is the mean kinetic energy dissipation
  - $\langle \varepsilon \rangle$ remains finite when $\nu \to 0$

- Kolmogorov-Obukhov (but also Heisenberg, Onsager, von Weizsäcker) conjectured that, for small scales $\ell \gg r \gg \eta$:

$$\delta u \propto (\langle \epsilon \rangle r)^{1/3}$$

  - In particular: $\left\langle \delta u^2 \right\rangle = C_r \langle \epsilon \rangle^{2/3} r^{2/3}$

    or in spectral space $E_\kappa = C_0 \langle \epsilon \rangle^{2/3} \kappa^{-5/3}$

About RTI small scales (2/2)

- Kolmogorov (1941) gave one of the few (if not the sole) exact laws of turbulence:

$$\left\langle \delta u_{\parallel}^{3} \right\rangle = -\tfrac{4}{5} \langle \epsilon \rangle \, r$$

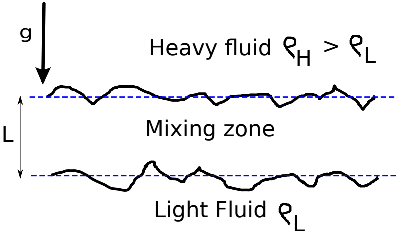
- Interpretation:
  - Energy flux Π_R flowing from scales larger than R to scales smaller than R

$$\Pi_R = -\frac{1}{4V_R} \oint_{Sphere(R)} \overline{\delta \boldsymbol{u} |\delta u|^2} \cdot \frac{\boldsymbol{r}}{|r|} dS$$

  - $4/5^{th}$ law $\Pi_R = \langle \varepsilon \rangle$
  - Energy flows from large to small scales at a constant rate $\langle \varepsilon \rangle$ $\approx$ Richardson's cascade

- In RTI, this phenomenology is almost unchanged:
  - buoyancy only creates a small inverse cascade and adds anisotropy

About RTI large scales (1/2)



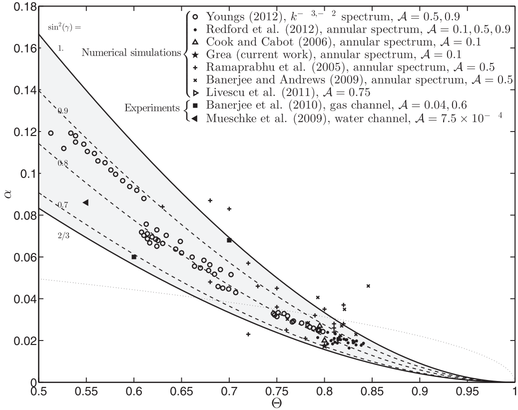
- Large scales reach a self-similar state
- Dimensional analysis: (NB: $A_t = (\rho_H - \rho_L)/(\rho_H + \rho_L)$)

$$L = 2\alpha_{(A_t)} g t^2$$

- α is the mixing width constant
  - Most theoretical/numerical/experimental works about RTI in the turbulent stage are devoted to finding the value of α.
  - Most engineering models are calibrated to reproduce a "correct" value of α.

About RTI large scales (2/2)

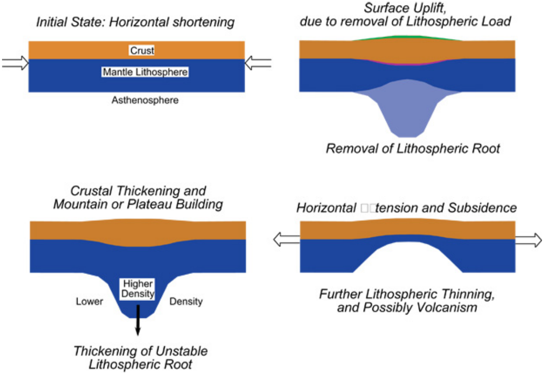
- The mixing constant α is not universal.
- α depends on the initial perturbation at very large scales, i.e. at scales larger than L, the mixing zone width.
- Very large scales have a slow evolution that can affect the flow at large times.



From Grea B.-J., Phys. Fluids, 2013

Some examples of RTI

- Geology:
  - Significant deformation can occur in plate interiors
  - Interaction between the lithosphere and underlying mantle
  - Rayleigh-Taylor is suspected to be one of these interactions

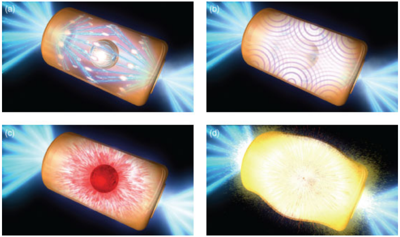


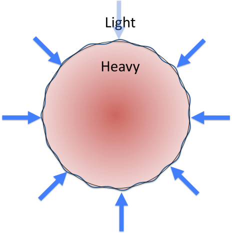
From P. Molnar, univ. colorado

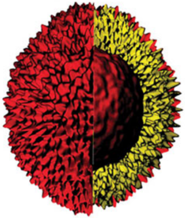
  - Density contrast due to the contraction of lithosphere, or compositional density variations.
  - Timescale: **1-10 millions of years**, Lengthscale: **100 km**

Some examples of RTI

- Inertial Confinement Fusion (ICF): (Images from LLNL, LANL)



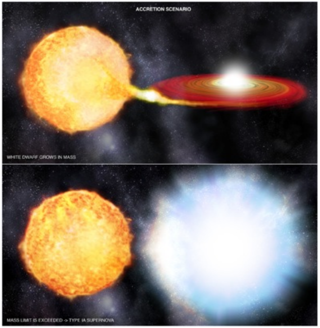


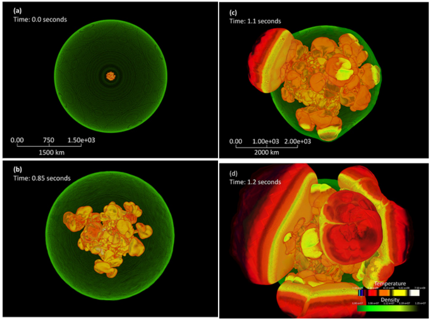


- Timescale: $\mathbf{10^{-12}}$ $\boldsymbol{s}$, Lengthscale: $< \mathbf{10^{-6}}$ $\boldsymbol{m}$

Some examples of RTI

- Type Ia supernovae: (Images from LLNL, LANL)
  - RTI is thought to be the main mechanism destabilizing the nuclear flame
  - Nuclear combustion regime: from thin to thick flames
  - Transition from deflagration to detonation ?
    - Abundancy of some heavy elements
    - Light curve: estimating distances





- Timescale: **1 s**, Lengthscale: $\mathbf{10^6}$ $\boldsymbol{m}$

Interlude: impulsive acceleration

- In RTI, acceleration is continuous in time and space
- What happens when g is impulsive ?

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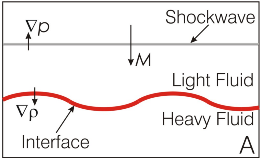
- Richtmyer-Meshkov instability



  - Linear stage: $a(t) = a_0 A_t \Delta U \kappa t$
  - Turbulent stage: $a(t) \propto t^{\theta}$

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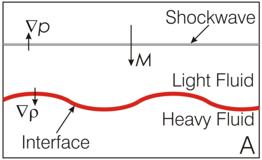

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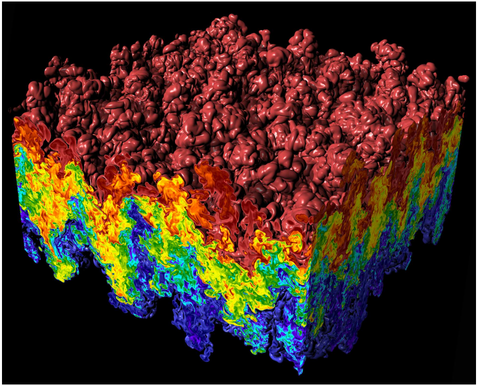
- "Balloon" instability (Dalziel & Lund, 2011)

How can we predict RTI turbulence ?



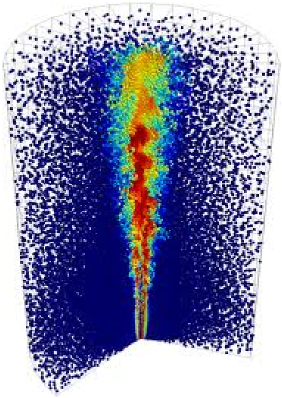
- Direct numerical simulations (DNS) of the Navier-Stokes equations
- Largest DNS of RTI by Cook & Cabot (2006):
  - $3072^3 = 29 \cdot 10^9$ numerical cells, $\approx$ 12 days on 131000 CPUs (IBM Blue Gene).
  - ℓ/η on the order of $50 - 100$ → still a small separation of scale

PDF approach

- DNS too costly for engineering applications
  $\Rightarrow$ turbulent models

- Huge variety of turbulent models
  - will only discuss so called "PDF models" (PDF is for probability density function)

- Principle:
  - The flow is decomposed into "tiny" cells of fluid
  - Model predicts the trajectory and interactions between these fluid particles



Simulation of a turbulent flame with a PDF

- Typical modelled PDF equation ≈ Fokker-Planck equation

For instance, for a one componential velocity field:

$$\partial_t f + u\partial_x f = -\partial_u \left(\partial_x \left\langle u^2 \right\rangle f - \frac{C_1}{2} \omega u f \right) + \frac{C_0 \omega \left\langle u^2 \right\rangle}{2} \partial^2_{u^2} f$$

- Objective of project TURBULENT :
  - Solve a PDF model like the one above in a simplified RT configuration
  - Work done by Nadezda Petrova, Viviana Letizia, Casimir Emako, Remi Sainct, Vincent Perrier