

constant volatility  $\sigma$ ): find  $p : \mathcal{D} \rightarrow \mathbb{R}$  such that, for all  $q : \mathcal{D} \rightarrow \mathbb{R}$ ,

$$\begin{aligned} \frac{d}{dt} \int_{\mathcal{D}} pq - \int_{\mathcal{D}} \frac{\sigma^2 S^2}{2} \frac{\partial p}{\partial S} \frac{\partial q}{\partial S} - \int_{\mathcal{D}} \frac{\sigma^2 S^2}{2} \frac{\partial p}{\partial S} \frac{\partial q}{\partial M} + \int_{\mathcal{D}} \frac{\sigma^2 S^2}{2} \frac{\partial p}{\partial M} \frac{\partial q}{\partial S} \\ + \int_{\mathcal{D}} \sigma^2 S \frac{\partial p}{\partial M} q + \int_{\mathcal{D}} (r - \sigma^2) S \frac{\partial p}{\partial S} q - r \int_{\mathcal{D}} pq = 0, \end{aligned} \quad (43)$$

$$p(T, S, M) = \phi(S, M),$$

where  $\mathcal{D} = \{(S, M) \in \mathbb{R}^2, 0 \leq S \leq M\}$ . The boundary condition  $\partial p / \partial M(t, S, S) = 0$  is naturally contained in this variational formulation since, by integration by parts over  $\mathcal{D}$ :

$$\begin{aligned} - \int_{\mathcal{D}} \frac{\sigma^2 S^2}{2} \frac{\partial p}{\partial S} \frac{\partial q}{\partial S} - \int_{\mathcal{D}} \frac{\sigma^2 S^2}{2} \frac{\partial p}{\partial S} \frac{\partial q}{\partial M} + \int_{\mathcal{D}} \frac{\sigma^2 S^2}{2} \frac{\partial p}{\partial M} \frac{\partial q}{\partial S} + \int_{\mathcal{D}} \sigma^2 S \frac{\partial p}{\partial M} q - \int_{\mathcal{D}} \sigma^2 S \frac{\partial p}{\partial S} q \\ = \int_{\mathcal{D}} \frac{\sigma^2 S^2}{2} \frac{\partial^2 p}{\partial S^2} q + \frac{1}{\sqrt{2}} \int_{\{S=M\}} \frac{\sigma^2 S^2}{2} \frac{\partial p}{\partial M} q. \end{aligned}$$

The first term corresponds to the diffusion term in (11). The second term is an integral over the boundary  $\{S = M\}$  of  $\mathcal{D}$  and naturally enforces the boundary condition  $\partial p / \partial M(t, S, S) = 0$ . On Figure 2, we represent the price of a fixed strike call, obtained using the formulation (11), an implicit Euler scheme and  $P1$  finite elements.

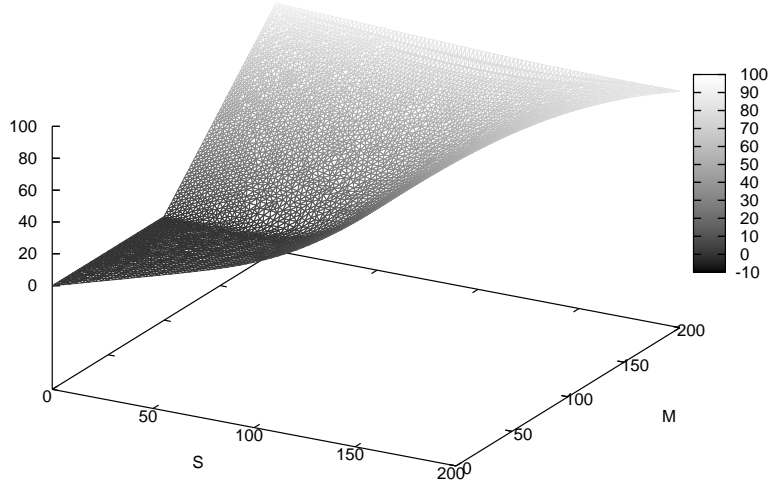


Figure 2: Price of a lookback option for a fixed strike call:  $\phi(S, M) = (M - K)_+$ . The parameters are:  $\sigma = 0.3$ ,  $r = 0.1$ ,  $K = 100$ ,  $T = 1$ . Computations made with freeFEM++ [1].

**Remark 8 (A posteriori error estimates)** *A frequently mentioned advantage of the Monte Carlo methods is that they naturally provide a posteriori error bounds through an interval of confidence, typically built upon the central limit theorem. It is also possible to obtain such a posteriori error estimates in the framework of the finite*