Pricing d'options asiatiques americaines par une méthode binomiale

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We consider a market model where the evolution of a risky asset is governed by the Black-Scholes stochastic differential equation

$$dS_t = rS_t dt + \sigma S_t dB_t, \quad S_0 = s_0,$$

where $(B_t)_{0 \leq t \leq T}$ is a standard Brownian motion, under the risk neutral measure Q. We will consider the problem of pricing an American Asian option. The price of an American Asian option of initial time 0 and maturity T is:

$$P(0, S_0, A_0) = \sup_{\tau \in \mathcal{T}_{0,T}} E\left[e^{-r\tau}\psi(S_\tau, A_\tau) | S_0 = s_0, A_0 = s_0\right],$$

where: $\mathcal{T}_{0,T}$ is the set of all stopping times with values in $[0, T]$, ψ denotes the payoff function and A_τ is the arithmetic average of the price of the underlying asset over the period $[0, \tau]$, i.e. $A_\tau = \frac{1}{\tau}\int_0^\tau S_t dt$.

The project focus on the implementation of a binomial numerical approach, called "Singular Points Method"[1]. This method, based on a continuous representation of the price at every node of the binomial tree, allows to obtain very precise upper and lower bounds of the pure discrete binomial price reducing drastically the time of computation. The method allows also to provide a-priori estimates of the difference between an upper and a lower bound.

References

[1] Gaudenzi M., Lepellere M.A., Zanette A.: The singular point method for pricing path-dependent options. Working paper DFIMF No. 1 (2007).