METAL PAD ROLL INSTABILITIES

Jean-Frédéric GERBEAU
INRIA, Rocquencourt, B.P. 105, 78153 Le Chesnay Cedex –France

Tony LELIEVRE, Claude LE BRIS
CERMICS, Ecole Nationale des Ponts et Chaussées,
6 & 8, Avenue Pascal, 77455 Champs-sur-Marne, France

Nicolas LIGONESCHE
ALUMINIUM PECHINEY, LRF, B.P. 114
73303 – Saint-Jean-de-Maurienne Cédex – France

Abstract

We present some numerical results obtained with a code developed in order to better take into account the coupling between the magnetic field and the velocity in magnetohydrodynamic phenomena. We compare our simulations with some results obtained on linearized equations. We show that the nonlinear approach may usefully complement the linear one and even correct its conclusions in some situations.

1 - Introduction

The magnetohydrodynamic (MHD) phenomena which dominate the motions of fluids (cryolite and aluminium) in Hall-Héroult cells are extremely complicated. Indeed, the hydrodynamic and magnetic equations are deeply coupled, through the Lorentz force

$$F = J \times B \qquad (1)$$

and induced currents

$$J = \sigma\left(E + u \times B\right) \qquad (2)$$

This coupling is responsible for instabilities in the cells, which result from interaction between the velocity profile, the magnetic field and the shape of the metal/bath interface.

To avoid these instabilities is one of the most important aims of some current research. Indeed, important displacements of the interface reduce the efficiency of the cell.

One of the phenomena which is observed in industrial cells and which has been investigated a lot over the past few years is the metal pad rolling. It is an oscillation of the bath / metal interface with period ranging from five seconds up to more than one minute. The aim of most of the theoretical and applied studies of MHD cells has been to understand, forecast and avoid this phenomenon. See [1] and the references therein for a survey on the main approaches until l992.

This phenomenon has been explored through analytical studies on simplified systems or through numerical experiments on linearized systems. The originality of our approach is to solve the original physical equations in a real 3d geometry, without any simplifying assumptions. In particular, we take into account the background motion, the viscosity, the deformation of the interface, the induced currents, the induced magnetic fields and the surface tension. Using the whole system, we have obtained a result on the instability of circular cells, that we can compare to some previous analytical studies [2].

2 - The physical phenomenon

One of the explanation of the metal pad rolling is the presence of a vertical field. The famous Sele's criterion is part of this theory [3]. T. Sele has been the first one to provide a physical reason of the rotation by the interaction of the vertical magnetic field with horizontal perturbed currents. More recently, Davidson and Lindsay [2] have derived a more general linearized system. Their analysis leads to quantitative results for the instability of standing and travelling waves in rectangular and circular cells. They also suggest a mechanical analogue which provides a good physical insight into the phenomenon.

The physical phenomenon is explained on Figure 1. An initial tilting (or a long-wavelength disturbance) creates a perturbed current flow

$$j = J - J_0 \qquad (3)$$

(*Jo* is the unperturbed -or background- current and *J* the total current in the cell) which is largely vertical in the cryolite and horizontal in the aluminium. The interaction of this current with the vertical magnetic field results in a horizontal Lorentz force

$$F = j \times B_z \qquad (4)$$

in the direction perpendicular to j. It finally induces a rotating motion of the interface.

In some cases, this phenomenon can lead to an instability : when the vertical field is too large, the amplitude of the oscillation may grow with time. It has been reported on that some metal may even escape from the cell in some cases !

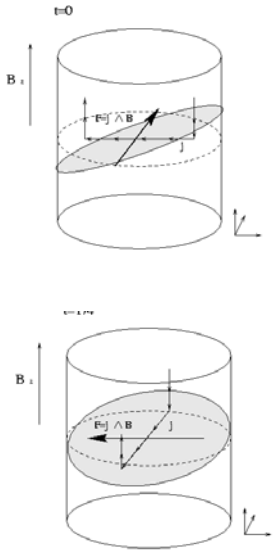


Figure 1 : Rolling phenomenon

3 - Numerical choices of our code

In this section, we want to briefly present the principal features of the code we have developed to simulate aluminium electrolysis cells, and the metal pad rolling in particular. We refer to [4-5] for more details.

3.1 - The equations of MHD

The equations we solve are deduced from three physical equations: the Navier-Stokes equations, the Maxwell equations and the Ohm's law.

$$\begin{cases} \zeta \dfrac{\partial u}{\partial t} + \zeta (u.\nabla)u + \nabla p - div\left(\dfrac{\zeta}{Re} \nabla u \right) = \dfrac{\zeta}{Fr} g + S\, curl\, B \times B \\ \dfrac{\partial \mathrm{B}}{\partial \mathrm{t}} + curl\left(\dfrac{1}{Rm} curl\,(B) \right) = curl\,(u \times B) \end{cases} \qquad (5)$$

$$div(u) = 0 \qquad (6)$$

and

$$div(B) = 0 \qquad (7)$$

The unknowns are *(u, B,p)* (5 scalar unknowns in 2d, 7 scalar unknowns in 3d). Our code also gives the possibility to add a surface tension term. For a stationary computation, we just set $\partial t = 0$ and $g = 0$ in the above equations. In practice, these equations are separately written in the two liquids : cryolite and aluminium. The adimensional numbers are :

the Reynolds number : $Re = \dfrac{\rho U L}{\eta}$ *(8)*

the magnetic Reynolds number : $R\mathrm{m} = \mu\sigma$ LU *(9)*

the coupling parameter : $S = \dfrac{B^2}{\mu U^2}$ *(10)*

the Froude number : $Fr = \dfrac{U^2}{gL}$ *(11)*

The numbers *U, L* and *B* are respectively the characteristic velocity, the length and the magnetic field. The density of the fluid is denoted by ρ, the dynamic viscosity is denoted by η, and the conductivity is denoted by σ. The parameter ζ is an adimensional density ($\zeta = 1$ in the aluminium and $\zeta = \dfrac{\rho_{cryo}}{\rho_{alu}}$ in the cryolite). The values of the parameter ζ, of the Reynolds number *Re* and of the magnetic Reynolds number *Rm* depend on the fluid.

The boundary conditions on the velocity *u* can be :

- either tangential velocity : $u.n = 0$
- or zero velocity (no-slip boundary condition) : $u = 0$

The boundary conditions on the magnetic flux *B* can be :

- either tangential[1] :

$$\begin{cases} \left(\frac{1}{Rm} curl\,(\mathrm{B}) - u \times B\right) \times n = \left(\frac{1}{Rm} curl\,(\mathrm{B}_0) - u \times B_0\right) \times n \\ B.n = B_0.n \end{cases} \qquad (12)$$

- or normal[2] :

$$\begin{cases} B \times n = B_0 \times n \\ div(B) = 0 \end{cases} \qquad (13)$$

The field *Bo* is a given magnetic field which can come from some measures or from a previous magnetostatic computatation. In practice, we have observed that tangential boundary conditions give better results.

These equations are then discretized using quadrilateral finite elements. The three unknowns are Ql fields, and we use some stabilization methods (streamline upwinding) to bypass the inf-sup condition problem and solve advection problems [6].

The position of the interface is updated at each time step, using an Arbitrary Lagrangian Eulerian (ALE) technique. The mesh at the interface is moved along the velocity at the interface. The rest of the mesh is then moved arbitrarily. We have chosen this method since the interface does not encounter big motions nor topological changes.

3.2 - Numerical assets and drawbacks of our scheme

In this section, we present the main advantages of our choices of discretization. These choices have also a few drawbacks. All the assertions of this section are detailed in [5].

3.2.1 - Coupling of magnetic fields and velocity

Our discretization couples the degrees of freedom (*u, B, p)* : the linear system we solve directly give us the three unknowns. When the problem becomes stiff, this is more robust than the segregated schemes which decouple the unknowns.

The main drawback of this choice is that it leads to large systems to be solved. However, these systems are sparse and well conditioned (thanks to the stabilization technique we use). We solve them using iterative methods (GMRES) and the incomplete factorisation as preconditioner (ILU). In the time-dependent problem, the initial guess is the result of the last time step and we obtain the desired precision in about ten iterations. The principal limitation is finally the construction of the matrix at each time step, and the upper bound on this time step.

3.2.2 - Motion of the interface

The ALE scheme we have chosen has a lot of advantages. Apart from good energetic properties (see below), it satisfies the geometric conservation law (GCL). The GCL is a property which ensures the conservation of the mass of each liquid from one time-step to the next.

3.2.3 - Time discretization and stability

We use an Euler time discretization. The main advantage of this scheme is that it ensures energetic conservation: the only additional energetic term which stems from time discretization (and which cannot be avoided) is a negative term of order δt. This means that, at worst, we lose an energy of size δt in the system. In particular, the energetic compensation between the Laplace force in the momentum equation and the term of induced currents in the equation on *B* is preserved in the discretized formulation.

This property of energetic conservation is definitely important in computations dealing with stability results : the instability can therefore not stem from a numerical *artefact.*

4 - Numerical simulations

We present in this section a result of computation on an unstable system : a circular cell submitted to a uniform vertical field.

4.1 - Choice of the test case

In [2], it is shown that it is the interaction between the different gravitational modes which causes unstable rolling waves. In particular , the authors assert that a circular cell becomes unstable whenever a vertical field *Bz* is applied.

We have chosen to reproduce this simple experiment of a circular cell. The following assumptions are made in [2] in order to derive this result (actually, these assumptions are made in order to obtain an analytically solvable linear system) :

. No background motion, which implies a uniform vertical magnetic field.

. No induced currents.

. No surface tension.

. The undisturbed surface is taken to be flat.

. The fluid is inviscid.

. Shallow water approximations (which leads to vertical j in the cryolite for instance).

---

[1] Notice that $\left(\frac{1}{Rm} curl\,(\mathrm{B}) - u \times B\right)$ is the electric field. In practice, we use this type of boundary condition where a no-slip boundary condition holds on u. The first condition is therefore reduced to : $curl(B) \times n = curl\,(\mathrm{B}_0) \times n$.

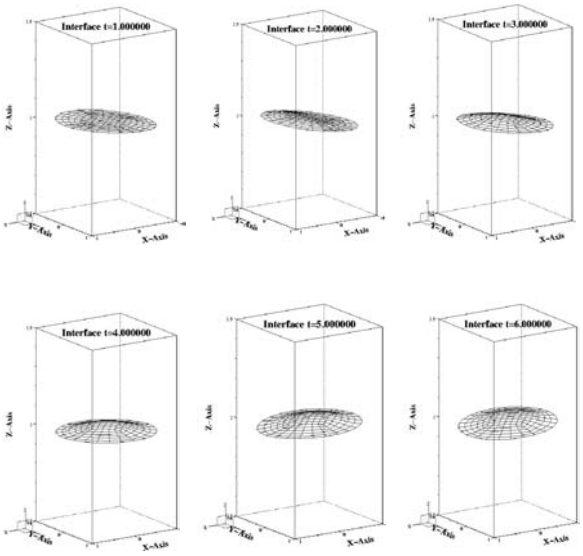
[2] The condition $div(B) = 0$ derives in fact from the variational formulation.

The advantage of a circular cell is also that the magnetic field at the boundary is easily computed (by Biot and Savart law). The idea is to numerically test the influence of some parameters which are neglected in this linear approach. We also want to check wether induced currents have a stabilizing effect, as it is often reported on in the literature. The linearized approach may well reproduce qualitatively the metal pad rolling, but some of the assumptions may be questionable as far as quantitative results are concerned.

The test case cell is a circular cell of radius equal to 1 and height equal to 2. The interface aluminium / cryolite is situated at the mean height. In order to create an initial disturbance, we have chosen to slightly tilt the cell, and then put it straight again, next apply the current *J* and the vertical field *Bz*.

4.2 - Some results

We have indeed observed the metal pad rolling on this test case. We show an example in Figure 2. The physical explanation of the phenomenon can be checked by computing the disturbed currents in the cryolite. In Figure 3, we show the disturbance of the currents created by the tilting of the interface. In order to compute this disturbance, we have subtracted from *curl(B)* the initial current *curl(Bo),* with *Bo* calculated with a flat horizontal interface. In our simulation, we have also observed that small disturbances of the initial state do not lead to instability (for example in the case of Figure 2). In the same way, a small positive *Bz* does not induce instability. This is in apparent contradiction with the results of the linear approach which claim the instability of the cell. At least our results show that if instability occurs, it will occur only across a large time frame, and therefore may not be relevant from a practical viewpoint. The result of our computation indeed shows that the amplitude of the rolling does not grow during 5 or 6 periods. Of course, definite conclusions about this comparison between the two approaches are yet to be determined.



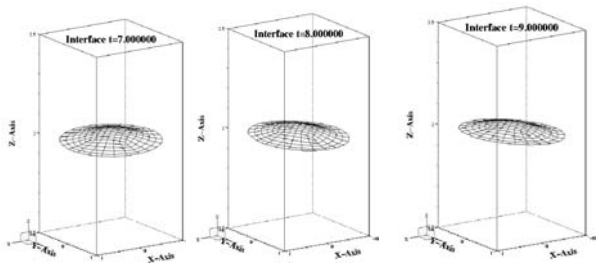


Figure 2 : The phenomenon of metal pad rolling in a circular cell. Visualization of the interface. This is a case with Bz > 0 and no instabilities

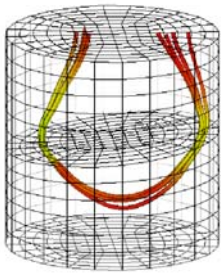


Figure 3 : Loops of currents in cryolite while metal pad rolling. We show here some streamlines of the field *j* (the perturbed current). The perturbed current goes from the right-hand side (where the elevation of the interface is the highest) to the left-hand side.

5 - Conclusion

This test case demonstrates the ability of our code to simulate complex MHD phenomena, which are usually numerically reproduced after many simplifications of the original equations. The impact of these simplifications on the result is certainly not negligible. Taking all the real physical parameters and equations into account, we hope to predict more *quantitatively* the instability of industrial cells.

This case shows that it is possible to deal with the entire original physical system and that this approach efficiently complements the linear one and helps to understand qualitatively and quantitatively the behavior of electrolysis cells.

References

[1] A.F. LACAMERA, D.P. ZIEGLER, R.L. KOZAREK, *Magnetohydrodynamics in the Hall-Heroult process, an overview,* Light Metals, 1179-1186, (1992).

[2] P.A DAVIDSON, R.I. LINDSAY, *Stability of interfacial waves in aluminium reduction cells,* J. Fluid Mech., 362:273-295, (1998).

[3] T. SELE, *Instabilities of the metal surface in electrolytic cells,* Light Metals, 7-24, (1977)

[4] J.F GERBEAU , *Problèmes mathématiques* et *numériques posés par la modélisation de l'électrolyse de l'aluminium,* Thèse de l'Ecole Nationale des Ponts et Chaussées, (1998).

[5] J.F GERBEAU, C. LEBRIS, T. LELIÈVRE, Simulations of MHD flows with moving interfaces, work in preparation.

[6] J.F GERBEAU, *A stabilized finite element method for incompressible magnetohydrodynamic equations,* Numerische Mathematik, 87:83-111, (2000).