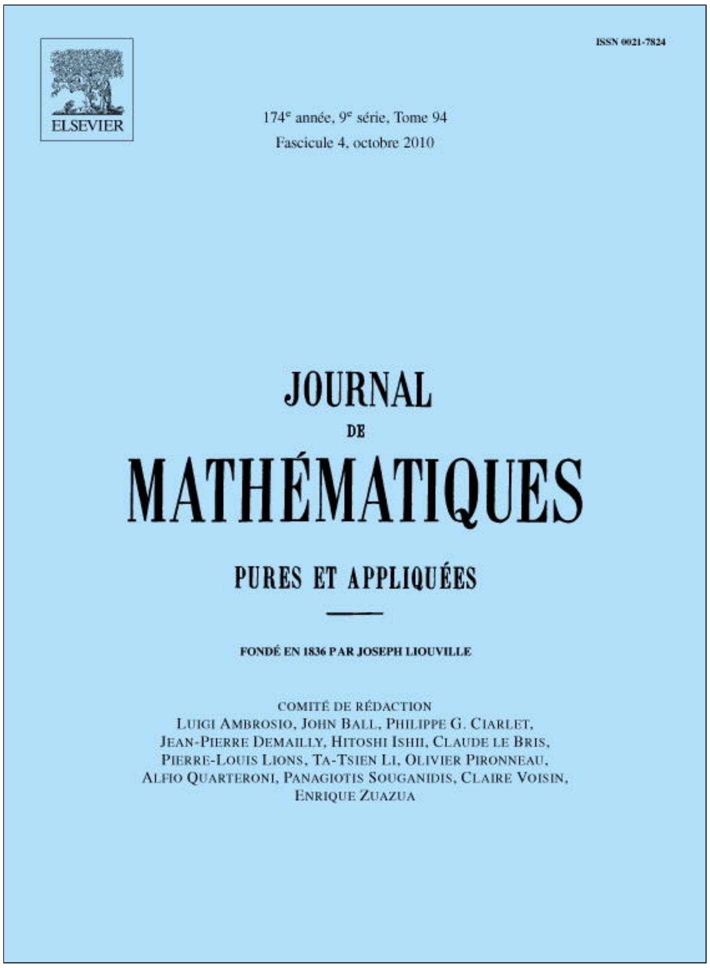

ISSN 0021-7824

174e année, 9e série, Tome 94
Fascicule 4, octobre 2010

JOURNAL DE MATHÉMATIQUES PURES ET APPLIQUÉES

FONDÉ EN 1836 PAR JOSEPH LIOUVILLE

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J. Math. Pures Appl. 94 (2010) 447–449
JOURNAL DE MATHÉMATIQUES PURES ET APPLIQUÉES
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Erratum

Erratum to "On the continuity of the time derivative of the solution to the parabolic obstacle problem with variable coefficients" [J. Math. Pures Appl. 85 (3) (2006) 371–414]

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Received 29 June 2010

Abstract

In "On the continuity of the time derivative of the solution to the parabolic obstacle problem with variable coefficients" our statement on Harnack's inequality is incorrect. This statement was used to establish *a priori* estimates. In this erratum we give a direct proof of these *a priori* estimates.

Résumé

Dans l'article « On the continuity of the time derivative of the solution to the parabolic obstacle problem with variable coefficients » l'énoncé sur l'inégalité de Harnack est incorrect. L'énoncé a été utilisée pour établir des estimations « a priori ». Dans cet erratum on donne une démonstration directe des estimations « a priori ».

MSC: 35R35

Keywords: Parabolic obstacle problem; Free boundary; *A priori* estimates

Consider a domain D of $\mathbb{R}^2$ and denote by $\mathcal{H}^\alpha(D)$ the set of functions $f \in \mathcal{C}^0 \cap L^\infty(D)$ such that

$$\sup_{\substack{(x,t),(y,s)\in D\\(x,t)\neq(y,s)}} \frac{|f(x,t)-f(y,s)|}{(|x-y|^2+|t-s|)^{\alpha/2}} < \infty,$$

DOI of original article: 10.1016/j.matpur.2005.08.007.
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0021-7824/$ – see front matter
doi:10.1016/j.matpur.2010.07.002

with $\alpha \in (0,1)$, and by $W^{2,1;q}_{x,t}(D)$ the set of functions $u \in L^q(D)$ such that $\frac{\partial u}{\partial x}$, $\frac{\partial^2 u}{\partial x^2}$, and $\frac{\partial u}{\partial t}$ are in $L^q(D)$, with $q \in [1,\infty]$. To any point $P_0 = (x_0, t_0) \in \mathbb{R}^2$ and $R > 0$, we associate the parabolic cylinder $Q_R(P_0) := \{(x,t) \in \mathbb{R}^2\colon |x - x_0| < R \text{ and } |t - t_0| < R^2\}$.

Let a, b and c and f be given functions. We consider the non-negative solutions in $W^{2,1;1}_{x,t}(Q_R(P_0))$ to

$$Lu(x,t) = f(x,t)\mathbb{1}_{\{u>0\}}(x,t) \quad (x,t) \in Q_R(P_0) \text{ a.e.} \tag{1}$$

with $Lu(x,t) := a(x,t)\frac{\partial^2 u}{\partial x^2} + b(x,t)\frac{\partial u}{\partial x} + c(x,t)u - \frac{\partial u}{\partial t}$.

Theorem 1. *Assume that a, b, c and f belong to $\mathcal{H}^\alpha(Q_R(P_0))$ for some $\alpha \in (0,1)$, and that there exists a constant $\delta_0 > 0$ such that for any $(x,t) \in Q_R(P_0)$, $a(x,t) \geqslant \delta_0$ and $f(x,t) \geqslant \delta_0$. Consider a nonnegative solution u of* (1). *For all $R' < R$, u is bounded in $W^{2,1;\infty}_{x,t}(Q_{R'}(P_0))$.*

The proof of these *a priori* estimates in [1, Theorem 2.1] uses Harnack's inequality and the Schauder interior estimates. However the statement of Harnack's inequality [1, Lemma 2.2] is not correct. In the constant coefficients case, the proof of Theorem 1 can be found in [3], as a consequence of [3, Theorem 4.1]. This method also applies to our case once the following result has been established.

Lemma 2. *Under the assumptions of Theorem* 1, *if $0 \in Q_R(P_0) \cap \partial\{u = 0\}$, then there exists a positive constant C such that*

$$\sup_{Q_r(0)} u \leqslant Cr^2,$$

for any $r > 0$ such that $Q_r(0) \subset Q_R(P_0)$.

Proof. The first part of the proof goes as for [3, Lemma 4.2], which was itself adapted from [2].

Up to a scaling, we can assume that $Q_r(0) \subset Q_R(P_0)$ if and only if $r \leqslant 1$. We introduce $S_k(u) := \sup_{Q_{2^{-k}}(0)} u$ and $N(u) := \{k \in \mathbb{N}\colon 2^2 S_{k+1}(u) \geqslant S_k(u)\}$. Let $M := \sup_{Q_R(P_0)} u$. If there exists $C_0 > 0$ such that $S_{k+1}(u) \leqslant C_0 M 2^{-2k}$ for any $k \in N(u)$, then we also have $S_{k+1}(u) \leqslant C_0 M 2^{-2k}$ for any $k \in \mathbb{N}$. The result then holds with $C := 16 M C_0$. Assume therefore that there is no such C_0: for any $j \in \mathbb{N}$, there exists $k_j \in N(u)$ such that

$$S_{k_j+1}(u) \geqslant j 2^{-2k_j}. \tag{2}$$

We define $L^j v(x,t) := a(2^{-k_j}x, 2^{-2k_j}t)\frac{\partial^2 v}{\partial x^2} + 2^{-k_j} b(2^{-k_j}x, 2^{-2k_j}t)\frac{\partial v}{\partial x} + 2^{-2k_j} c(2^{-k_j}x, 2^{-2k_j}t)v - \frac{\partial v}{\partial t}$ and

$$u_j(x,t) := \frac{1}{S_{k_j+1}(u)} u\big(2^{-k_j}x, 2^{-2k_j}t\big) \quad \text{for all } (x,t) \in Q_1(0).$$

By regularity of a, b, c and f and by (2), the functions u_j satisfy $\lim_{j\to\infty} \sup_{Q_1(0)} |L^j u_j| = 0$, $\sup_{Q_1(0)} u \leqslant 4$ and $\sup_{Q_{1/2}(0)} u_j = 1$. Moreover u_j is non-negative and $u_j(0,0) = 0$. By L^p parabolic estimates, up to the extraction of a sub-sequence, $(u_j)_{j\in\mathbb{N}}$ converges to a function u_∞ locally uniformly on compact sets.

In [3] the authors use Caffarelli's monotonicity formula to obtain a contradiction. This monotonicity formula is not valid for the variable coefficients case. Here the sign condition on the solution allows to conclude directly by the maximum principle in the following way: the function u_∞ is a bounded non-negative function such that $a(0,0)\frac{\partial^2 u_\infty}{\partial x^2} - \frac{\partial u_\infty}{\partial t} = 0$ in $Q_1(0)$ and achieves its minimum in 0. By the strong maximum principle u_∞ is constant and equal to zero which contradicts $\sup_{Q_{1/2}(0)} u_\infty = 1$. $\square$

Finally, let us mention that in [1, p. 375], just before Theorem 1.4, the backward heat kernel has to be defined as $G(x,t) = (4\pi(-t))^{-1/2}\exp(-x^2/(-4t))$.

Acknowledgements

We thank H. Shahgholian for suggesting us the above method.

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