Optimal Control of a Domestic Microgrid

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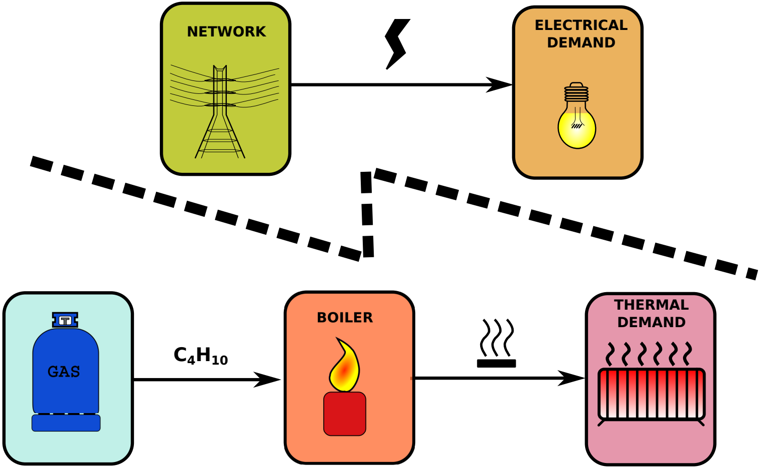
efficacity

A partnership between mathematicians and thermicians

- Efficacity is a research institute for energy transition — an original mix of companies and academic researchers
- This presentation sums up a common work between Cermics and Efficacity
- This cooperation develops optimization algorithms for real world problems

In a “classical” energy system, thermal and electrical energy management are usually treated apart



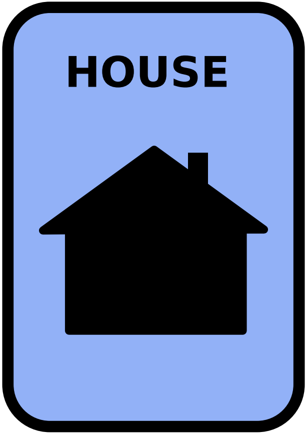
Is it worth to equip the system with
a combined heat and power generator (CHP)
together with a battery?

Challenges:

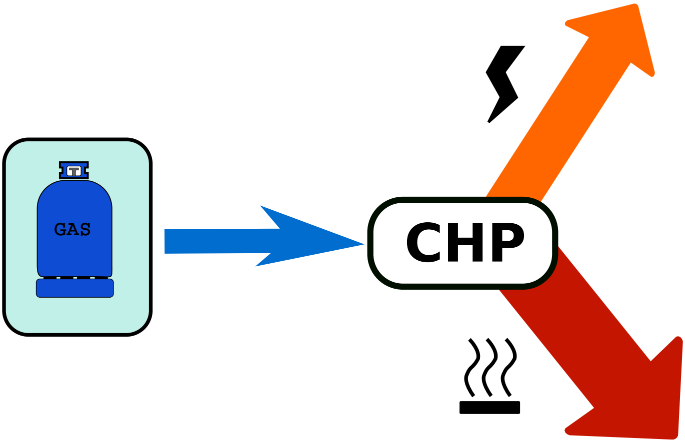
- CHP is either ON or OFF, and always produces the same amount of electricity and heat
- Thermal and electrical system are coupled with the CHP
- Two storage devices (battery and hot water tank) with a dynamic
- Prices and setpoints vary along time

We turn to mathematical optimization to answer the question

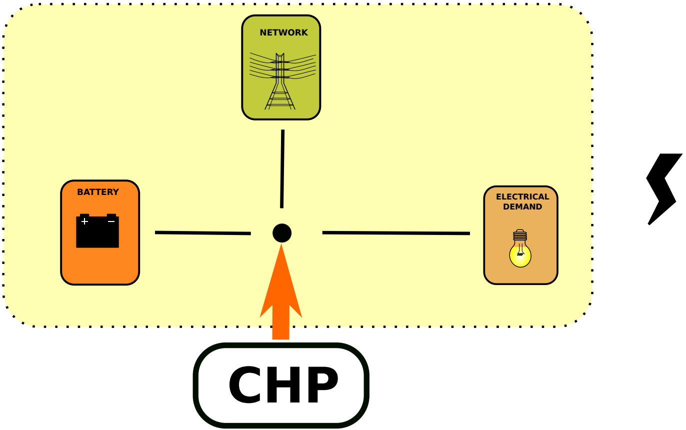
Our system



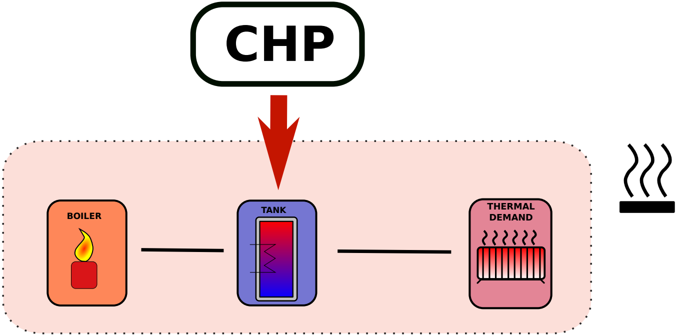
What is a Combined Heat and Power Generator?



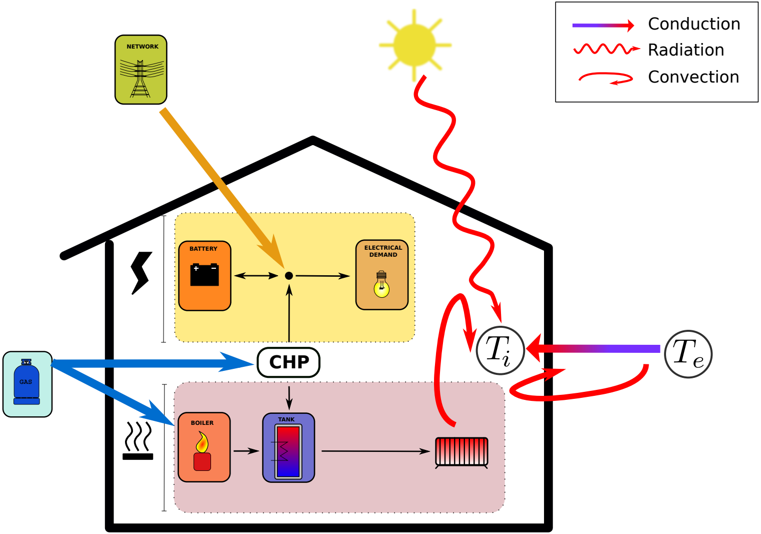
What electrical system are we considering?



What thermal system are we considering?



Problem's description



What do we aim to do?

We want to:

- Minimize costs (electricity + gas)
- Maintain a comfortable temperature inside the house

To achieve these goals, we can:

- Switch on/off the CHP
- Store electricity in battery and heat in hot water tank
- Control auxiliary boiler and heaters' inflow

We consider **15 minutes** timesteps

We formulate a multistage stochastic programming problem

Outline

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We introduce states, controls and noises

- **Stock variables** $X_t = \big(B_t, H_t, \theta^i_t, \theta^w_t\big)$
  - B_t, battery level (kWh)
  - H_t, hot water storage (kWh)
  - θ^i_t, inner temperature ($^\circ C$)
  - θ^w_t, wall's temperature ($^\circ C$)
- **Control variables** $U_t = \big(Y_t, F_{B,t}, F_{A,t}, F_{H,t}\big)$
  - $Y_t \in \{0, 1\}$ boolean ON/OFF CHP generator control variable
  - $F_{B,t}$, energy stored in the battery
  - $F_{A,t}$, energy produced by the auxiliary boiler
  - $F_{H,t}$, thermal heating
- **Perturbations** $W_t = \big(D^E_t, N_t, P^{ext}_t, \theta^e_t\big)$
  - D^E_t, electrical demand (kW)
  - N_t, occupancy (integer)
  - P^{ext}_t, external radiations (kW)
  - θ^e_t, external temperature ($^\circ C$)

Discrete time state equations

So we have the four state equations (all linear):

$$B_{t+1} = \alpha_B B_t - \beta_B F_{B,t}$$

$$H_{t+1} = \alpha_H H_t + \beta_H \big[F_{A,t} + F_{GH,t} - F_{H,t}\big]$$

$$\theta^w_{t+1} = \theta^w_t + \frac{\Delta T}{c_m}\left[\frac{\theta^i_t - \theta^w_t}{R_i + R_s} + \frac{\theta^e_t - \theta^w_t}{R_m + R_e} + \gamma F_{H,t} + \frac{R_i}{R_i + R_s} P^{int}_t + \frac{R_e}{R_e + R_m} P^{ext}_t\right]$$

$$\theta^i_{t+1} = \theta^i_t + \frac{\Delta T}{c_i}\left[\frac{\theta^w_t - \theta^i_t}{R_i + R_s} + \frac{\theta^e_t - \theta^i_t}{R_v} + \frac{\theta^e_t - \theta^i_t}{R_f} + (1-\gamma) F_{H,t} + \frac{R_s}{R_i + R_s} P^{int}_t + P_{occ}\right]$$

which will be denoted:

$$\boxed{X_{t+1} = f_t(X_t, U_t, W_{t+1})}$$

Optimization criterion

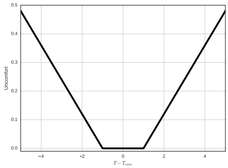
- Cost to import electricity from the network

$$-\underbrace{b_E \max\{0, -F_{NE,t+1}\}}_{\text{selling}} + \underbrace{h_E \max\{0, F_{NE,t+1}\}}_{\text{buying}}$$

where we define the recourse variable (electricity balance):

$$\underbrace{F_{NE,t+1}}_{Network} = \underbrace{D^E_{t+1}}_{Demand} - \underbrace{F_{B,t}}_{Battery} - \underbrace{P^E_{chp} \times Y_t}_{CHP}$$

- Cost to use the CHP: $\pi_{chp} \times Y_t$
- Cost to use auxiliary burner: $\pi_{gas} \times F_{A,t}$
- Virtual Cost of thermal discomfort: $\kappa_{th}(\underbrace{\theta^i_t - \bar{\theta}^i_t}_{\text{deviation from setpoint}})$



Instantaneous and final costs

- The instantaneous convex costs are

$$\mathcal{C}_t(X_t, U_t, W_{t+1}) = \underbrace{\pi_{chp} Y_t}_{CHP} + \underbrace{\pi_{gas} F_{A,t}}_{Aux.\ Burner} \underbrace{-b_E \max\{0, -F_{NE,t+1}\}}_{buying} + \underbrace{h_E \max\{0, F_{NE,t+1}\}}_{selling} + \underbrace{\kappa_{th}(\theta^i_t - \bar{\theta}^i_t)}_{discomfort}$$

- We add a final linear cost

$$-\pi_H H_{T_f} - \pi_B B_{T_f}$$

to avoid empty stocks at the final horizon T_f

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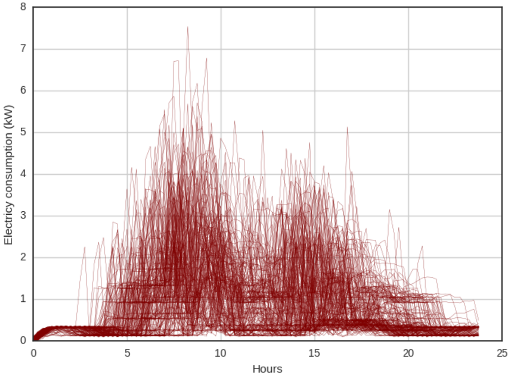
Perturbations display high variability

Different scenarios for electrical demand:



We model perturbations as random variables

- We recall that $W_t = \left(D_t^E, N_t, P_t^{ext}, \theta_t^e\right)$ with:
  - D_t^E, electrical demand (kW)
  - N_t, occupancy (integer)
  - P_t^{ext}, external radiations (kW)
  - θ_t^e, external temperature ($^\circ C$)
- We model W_t as random variables upon $\left(\Omega, \mathcal{A}, \mathbb{P}\right)$

$$W_t : \Omega \to \mathbb{R}^4$$

  so that $(W_1, \dots, , W_{T_f})$ forms a stochastic process
- We recall that W_{t+1} stand for the exogeneous perturbations during the time interval $[t, t+1[$

We need to add the nonanticipativity constraints

- σ-algebra

$$\mathcal{A}_t = \sigma(W_1, \dots, W_t)$$

- Non-anticipativity constraint

$$U_t = \big(Y_t, F_{B,t}, F_{A,t}, F_{H,t}\big)$$

is $\mathcal{A}_t$–measurable

Outline

That gives the following stochastic optimization problem

$$
\begin{aligned}
\min_{X,U} \quad & \mathbb{E}\left[\sum_{t=0}^{T_f-1} \underbrace{\mathcal{C}(X_t, U_t, W_{t+1})}_{instantaneous\ \ cost} \underbrace{-\pi_H H_{T_f} - \pi_B B_{T_f}}_{final\ \ cost}\right] \\
s.t \quad & X_{t+1} = f(X_t, U_t, W_{t+1}) \quad \text{Dynamic} \\
& B^\flat \leq B_t \leq B^\sharp \\
& H^\flat \leq H_t \leq H^\sharp \\
& \Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\
& F_i^\flat \leq F_{i,t} \leq F_i^\sharp \ , \ \ \forall i \in \{B, A, H\} \\
& U_t \preceq \mathcal{A}_t \quad \text{Non-anticipativity}
\end{aligned}
$$

Where are we now? And where are we heading to?

- The problem is formulated
- Now, we are going to present two methods to tackle this problem
- And we are going to answer whether it pays to equip the system with a CHP

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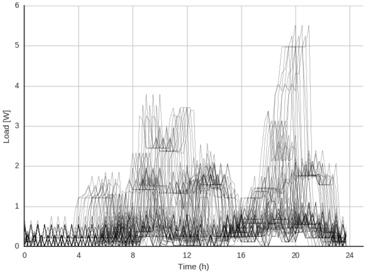
We are going to compare two methods

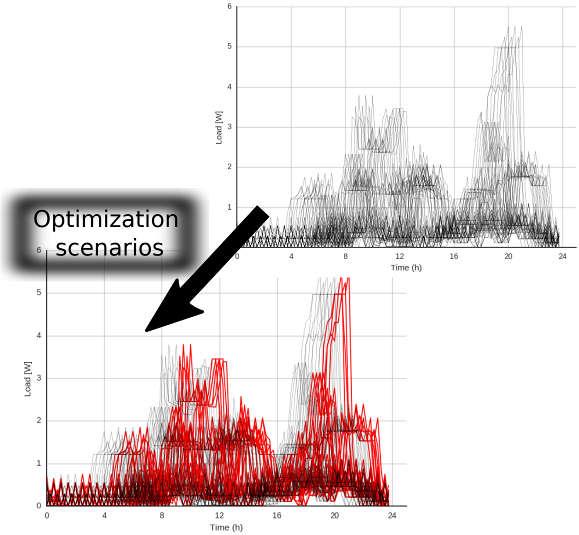
MPC
Model Predictive Control

SDDP
Stochastic Dual Dynamic Programming

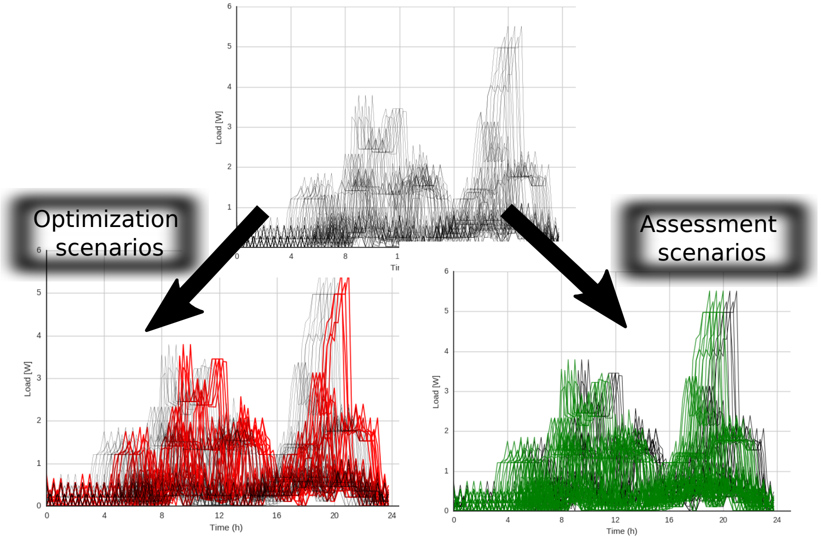
How to assess management strategies?



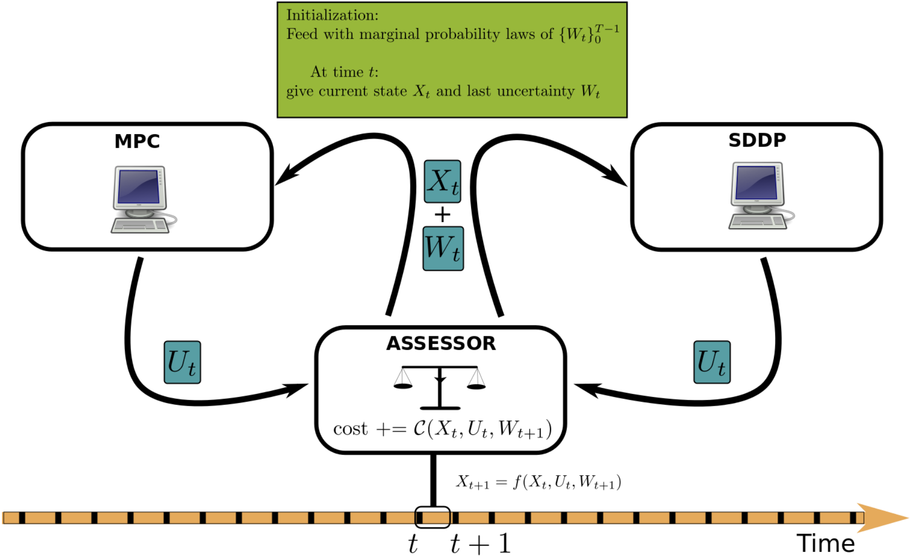
How to assess management strategies?



How to assess management strategies?



How are we going to evaluate these two methods?



Model Predictive Control

At the beginning of time period $[\tau, \tau+1]$, do

- Consider a **rolling horizon** $[\tau, \tau+H[$
- Consider a **deterministic scenario** of demands (forecast) $\left(\overline{W}_{\tau+1}, \dots, \overline{W}_{\tau+H}\right)$
- Solve the **deterministic optimization** problem

$$\min_{X,U} \left[\sum_{t=\tau}^{\tau+H} \mathcal{C}(X_t, U_t, \overline{W}_{t+1}) - \pi_H H_{T_f} - \pi_B B_{T_f} \right]$$

$$\begin{aligned} s.t. \quad X_\cdot = (X_\tau, \dots, X_{\tau+H}), \quad U_\cdot &= (U_\tau, \dots, U_{\tau+H-1}) \\ X_{t+1} &= f(X_t, U_t, \overline{W}_{t+1}) \\ B^\flat &\le B_t \le B^\sharp \\ H^\flat &\le H_t \le H^\sharp \\ \Delta B^\flat &\le B_{t+1} - B_t \le \Delta B^\sharp \\ F_{i,t} &\le F_i^\sharp, \quad \forall i \in \{B, A, H\} \end{aligned}$$

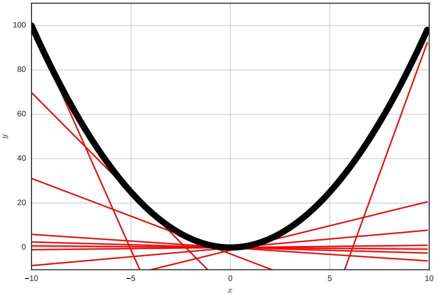
- Get optimal solution $(U_\tau, \dots, U_{\tau+H})$ over horizon $H = 24h$
- Use only control U_τ, and iterate at time $\tau + 1$

Stochastic Dual Dynamic Programming: Offline

Dynamic Programming

Use marginal laws μ_{t+1} of W_{t+1} to estimate expectation and compute **offline** value functions with the backward equation:

$$V_t(x_t) = \min_{U_t} \mathbb{E}_{\mu_t}\Big[\underbrace{\mathcal{C}_t(x_t, U_t, W_{t+1})}_{\text{current cost}} + \underbrace{V_{t+1}\Big(f(x_t, U_t, W_{t+1})\Big)}_{\text{future costs}}\Big]$$



SDDP

Convex value functions V are approximated as a supremum of a finite set of affine functions

Stochastic Dual Dynamic Programming: Online

Online computation

- We compute offline the approximated value functions $\left(\widetilde{V}_t\right)_0^{T_f}$

$$\widetilde{V}_t \leq V_t$$

- At time τ, we solve

$$U_\tau^{\#} = \arg\min_{u_\tau} \left[\mathcal{C}_\tau(X_\tau, u_\tau, w_\tau) + \widetilde{V}_{\tau+1}\Big(f_\tau(X_\tau, u_\tau, w_\tau)\Big)\right]$$

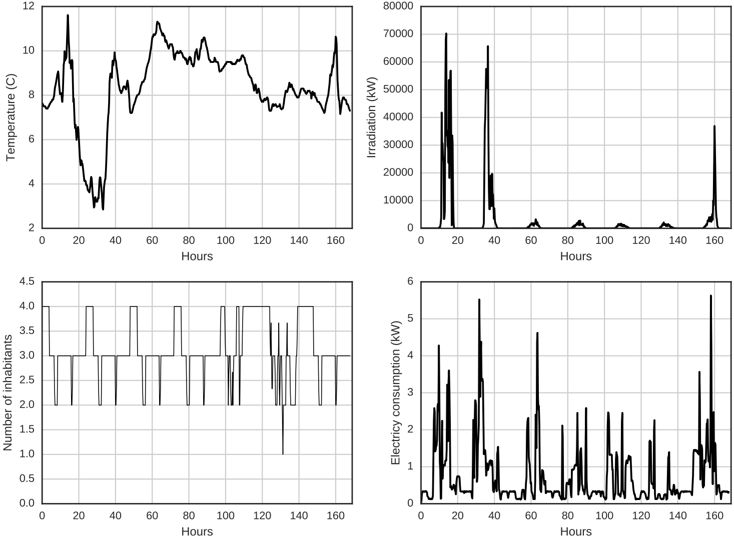
  where w_τ is the previous realization of random variable W_τ between $[\tau - 1, \tau[$ so that we respect the non anticipativity constraint $U_\tau \preceq \mathcal{A}_\tau$

- Send $U_\tau^{\#}$ to assessor

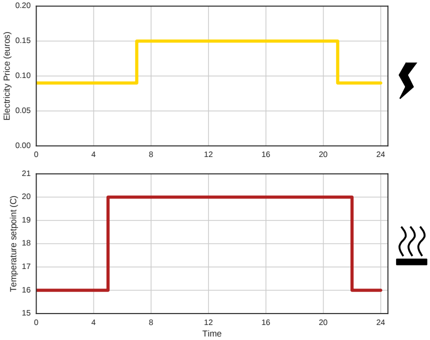
Outline

How to assess management methods?

We consider one week in winter and 200 assessment scenarios



We define settings for our problem

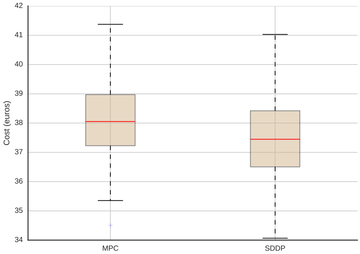


- $T_f = 24\text{h}$, $\Delta T = 15\text{mn}$
- Electricity peak and off-peak hours
  - $\pi_{elec} = 0.09$ or 0.15 euros/kWh
  - $\pi_{gas} = 0.06$ euros/kWh
- Temperature set-point
  $16^\circ C$ or $20^\circ C$
- Empty stocks at midnight

$$\pi_H = 0\,, \quad \pi_B = 0$$

Outline

Comparing MPC vs SDDP



	euros/week	%
no CHP, no battery	46.84	ref
MPC	38.08	- 18.7%
SDDP	37.46	- 20.1 %

- SDDP performs 1.4 % better than MPC
- Although small, the difference can be significative over several years and houses

Outline

Conclusion

- With SDDP, offline computations take some time (15 minutes) but online computations are straightforward
- We could use more online information (updated forecast)
- Such results have been used to perform an economic evaluation of investing in CHP and battery

Perspectives

Use decomposition/coordination algorithms to control an urban neighbourhood