Optimization of an urban district microgrid

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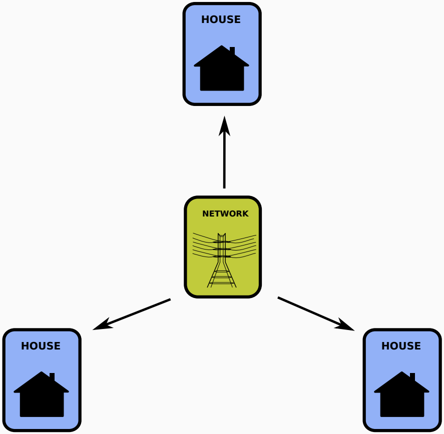
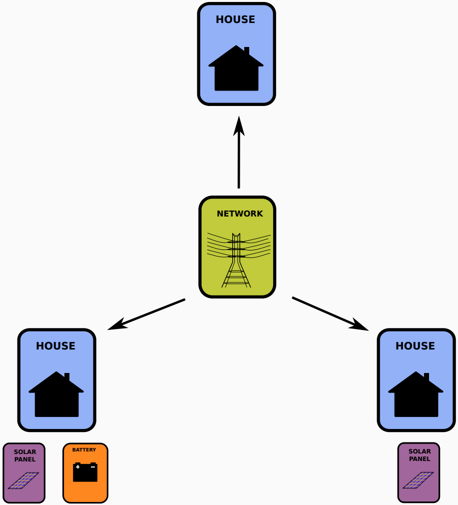
A partnership between mathematicians and thermicians

- Efficacity is a research institute for energy transition — an original mix of companies and academic researchers
- This presentation sums up a common work between CERMICS and Efficacity
- This cooperation develops optimization algorithms for real problems concerning the energy transition

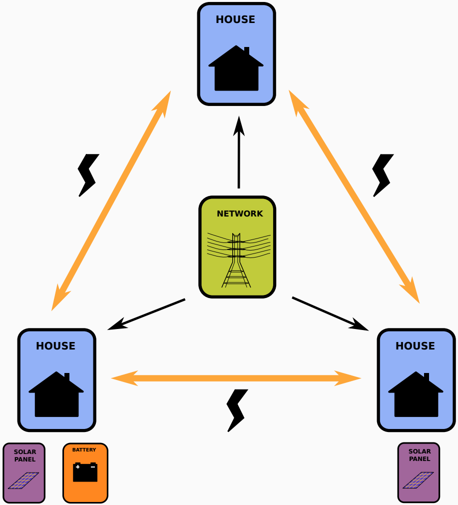
Usually houses import electricity from the grid



But more and more houses are equipped with solar panel



Is it worth to add a local grid to exchange electricity?



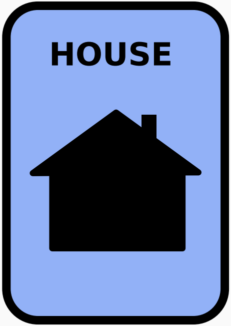
Is it worth to connect different houses together inside a district?

Challenges:

- Handle electrical exchanges between houses

We turn to mathematical optimization to answer the question

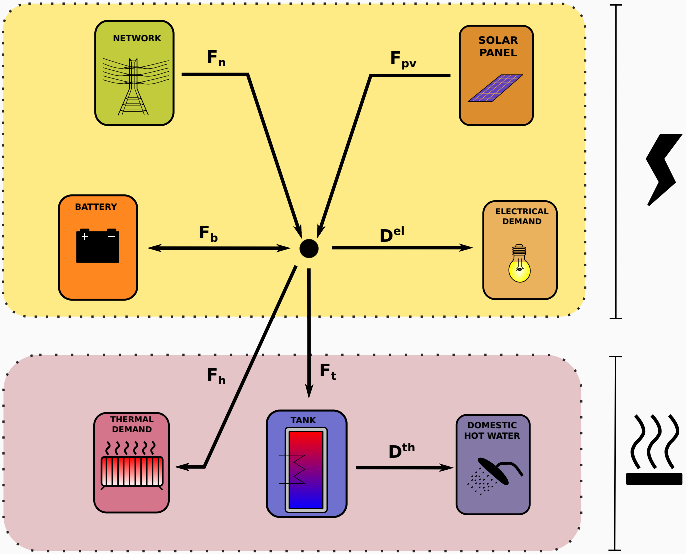
Two commandments to rule them all



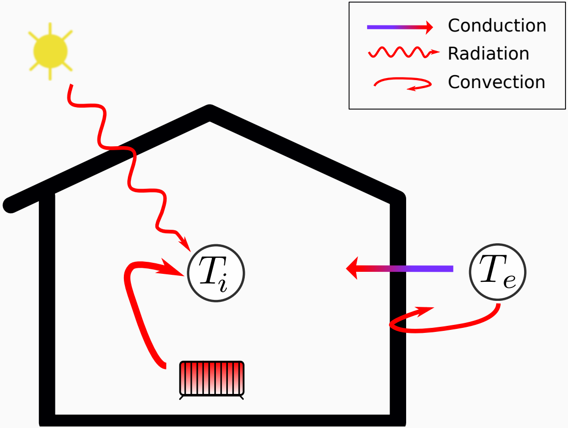
Thou shall:

- Satisfy thermal comfort
- Improve energy efficiency

For each house, we consider the electrical system...



… and the thermal enveloppe



Where do we come from?

We already solve the house's problem with 4 state variables to:

- Minimize electrical consumption
- Maintain a comfortable temperature inside the house

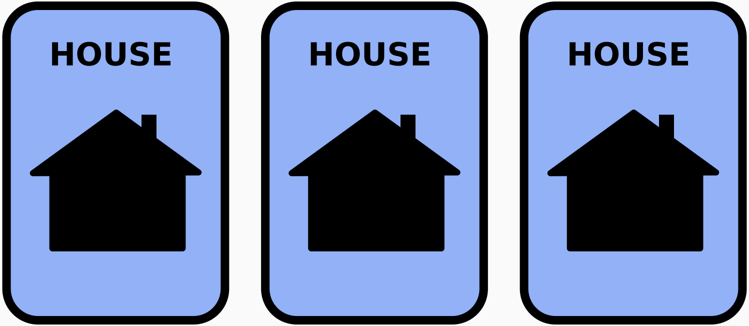
To achieve these goals, we:

- Stored electricity in battery
- Stored heat in hot water tank

We controlled the stocks every 15mn over one day.

We formulated a multistage stochastic programming problem

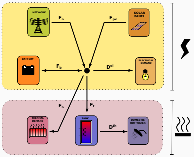
And now: we add two other houses

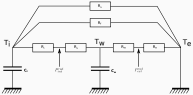


Outline

Outline

We introduce states, controls and noises

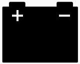




- **Stock variables** $X_t = \left(B_t, H_t, \theta^i_t, \theta^w_t\right)$
  - B_t, battery level (kWh)
  - H_t, hot water storage (kWh)
  - θ^i_t, inner temperature ($^\circ C$)
  - θ^w_t, wall's temperature ($^\circ C$)
- **Control variables** $U_t = \left(F^+_{B,t}, F^-_{B,t}, F_{T,t}, F_{H,t}\right)$
  - $F^+_{B,t}$, energy stored in the battery
  - $F^-_{B,t}$, energy taken from the battery
  - $F_{T,t}$, energy used to heat the hot water tank
  - $F_{H,t}$, thermal heating
- **Perturbations** $W_t = \left(D^E_t, D^{DHW}_t, P^{ext}_t, \theta^e_t\right)$
  - D^E_t, electrical demand (kW)
  - D^{DHW}_t, domestic hot water demand (kW)
  - P^{ext}_t, external radiations (kW)
  - θ^e_t, external temperature ($^\circ C$)

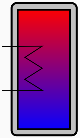
Discrete time state equations

So we have the four state equations (all linear):



$$B_{t+1} = \alpha_B B_t + \Delta T \Big(\rho_c F^{+}_{B,t} - \frac{1}{\rho_d} F^{-}_{B,t} \Big)$$

$$H_{t+1} = \alpha_H H_t + \Delta T \big[F_{T,t} - D^{DHW}_t \big]$$



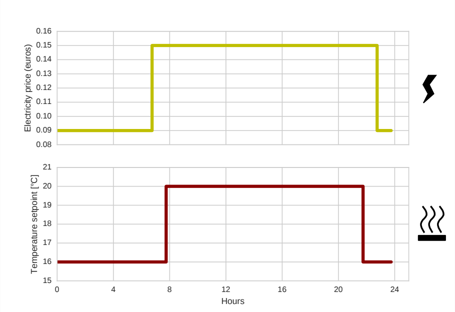
$$\theta^w_{t+1} = \theta^w_t + \frac{\Delta T}{c_m} \left[\frac{\theta^i_t - \theta^w_t}{R_i + R_s} + \frac{\theta^e_t - \theta^w_t}{R_m + R_e} + \gamma F_{H,t} + \frac{R_i}{R_i + R_s} P^{int}_t + \frac{R_e}{R_e + R_m} P^{ext}_t \right]$$

$$\theta^i_{t+1} = \theta^i_t + \frac{\Delta T}{c_i} \left[\frac{\theta^w_t - \theta^i_t}{R_i + R_s} + \frac{\theta^e_t - \theta^i_t}{R_v} + \frac{\theta^e_t - \theta^i_t}{R_f} + (1 - \gamma) F_{H,t} + \frac{R_s}{R_i + R_s} P^{int}_t \right]$$

which will be denoted:

$$\boxed{X_{t+1} = f_t(X_t, U_t, W_{t+1})}$$

Prices and temperature setpoints vary along time



- $T_f = 24\text{h}$, $\Delta T = 15\text{mn}$
- Electricity peak and off-peak hours
- $\pi_{elec} = 0.09$ or 0.15 euros/kWh
- Temperature set-point $16^\circ C$ or $20^\circ C$

The costs we have to pay

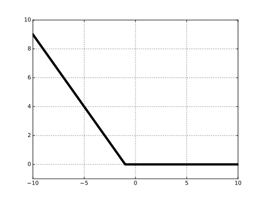
- Cost to import electricity from the network

$$-\underbrace{b_E \max\{0, -F_{NE,t+1}\}}_{\text{selling}} + \underbrace{h_E \max\{0, F_{NE,t+1}\}}_{\text{buying}}$$

where we define the recourse variable (electricity balance):

$$\underbrace{F_{NE,t+1}}_{Network} = \underbrace{D^E_{t+1}}_{Demand} - \underbrace{F^+_{B,t} + F^-_{B,t}}_{Battery} + \underbrace{F_{H,t}}_{Heating} + \underbrace{F_{T,t}}_{Tank} + \underbrace{F_{pv,t}}_{Solar\ \ panel}$$

- Virtual Cost of thermal discomfort: $\kappa_{th}(\underbrace{\theta^i_t - \bar{\theta}^i_t}_{\text{deviation from setpoint}})$



κ_{th}

Piecewise linear cost

Penalize temperature if below given setpoint

Instantaneous and final costs for a single house

- The instantaneous convex costs are

$$\mathcal{C}_t(X_t, U_t, W_{t+1}) = \underbrace{-b_E \max\{0, -F_{NE,t+1}\}}_{\text{buying}} + \underbrace{h_E \max\{0, F_{NE,t+1}\}}_{\text{selling}} + \underbrace{\kappa_{th}(\theta^i_t - \bar{\theta}^i_t)}_{\text{discomfort}}$$

- We add a final linear cost

$$-\pi_H H_{T_f} - \pi_B B_{T_f}$$

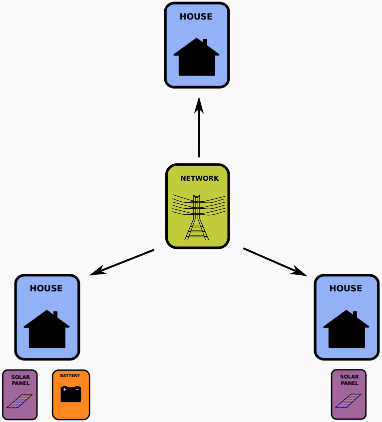
to avoid empty stocks at the final horizon T_f

That gives the following stochastic optimization problem

$$
\begin{aligned}
\min_{X,U} \quad & J(X,U) = \mathbb{E}\left[\sum_{t=0}^{T_f-1} \underbrace{\mathcal{C}(X_t, U_t, W_{t+1})}_{instantaneous \;\; cost} + \underbrace{K(X_{T_f})}_{final \;\; cost}\right] \\
s.t \quad & X_{t+1} = f_t(X_t, U_t, W_{t+1}) \quad \text{Dynamic} \\
& X^\flat \leq X_t \leq X^\sharp \\
& U^\flat \leq U_t \leq U^\sharp \\
& X_0 = X_{ini} \\
& U_t \preceq \sigma(W_1, \dots, W_t) \quad \text{Non-anticipativity}
\end{aligned}
$$

Outline

We have three different houses



Our (small) district:

- House 1: solar panel + battery
- House 2: solar panel
- House 3: nothing

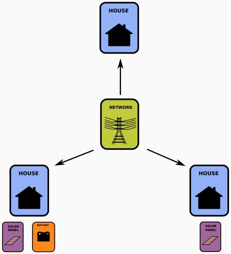
For the three houses:

- 10 stocks (= 4 + 3 + 3)
- 8 controls (= 4 + 2 + 2)
- 8 perturbations
  (2 perturbations in common)

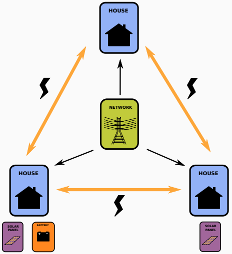
The total demand to the network is bounded:

$$\sum_{k=1}^{3} F_{NE,t+1}^{k} \leq F_{NE}^{\sharp}$$

We want to compare two configurations



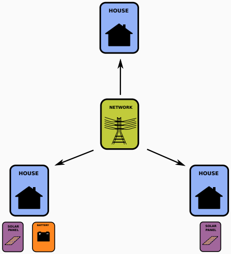
No exchange between houses



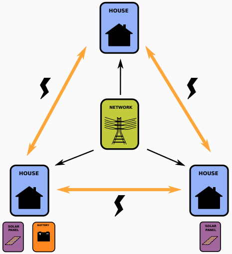
Exchange in a local grid

How much energy can we save while allowing houses to exchange energy through a local grid?

We want to compare two configurations



No exchange between houses

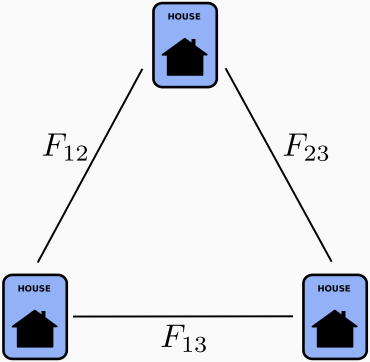


Exchange in a local grid

How much energy can we save
while allowing houses to exchange energy
through a local grid?

We show that local grid + optimization decreases costs
by 23 % during summer!

The grid adds three controls to the problem



How to solve this stochastic optimal control problem?

We recall the different parameters of our multistage stochastic problem:

- 96 timesteps
- 10 stocks
- 8 controls
- 8 perturbations (+3 flows between houses)

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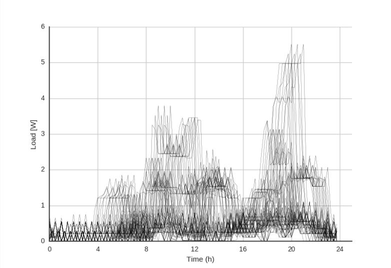
We will compare two methods that overcome this curse:

1. **Model Predictive Control** (MPC)
2. **Stochastic Dual Dynamic Programming** (SDDP)

How to assess management strategies?



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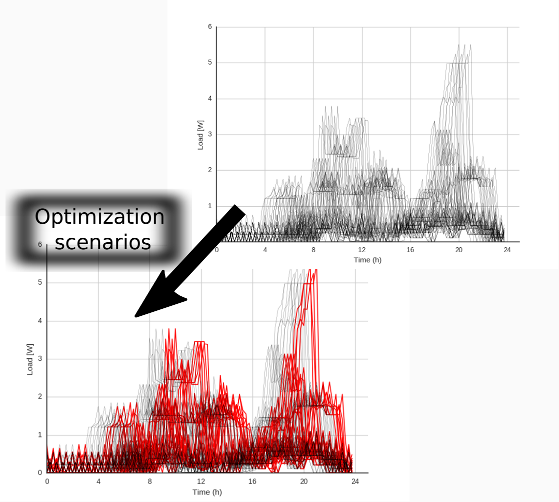

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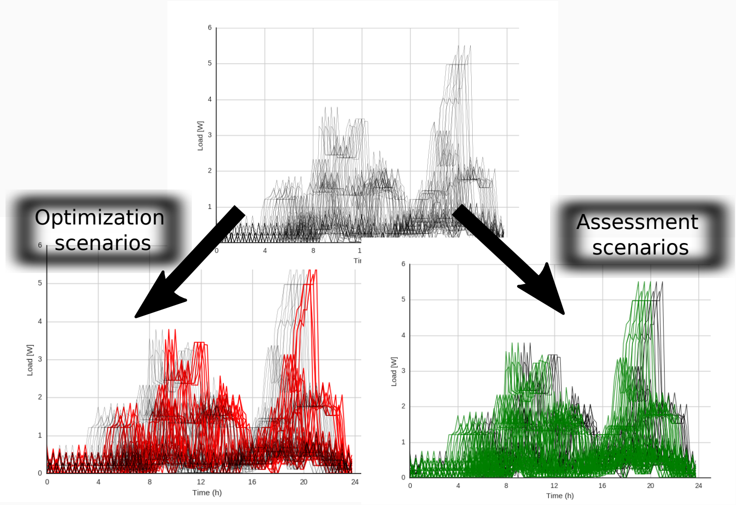
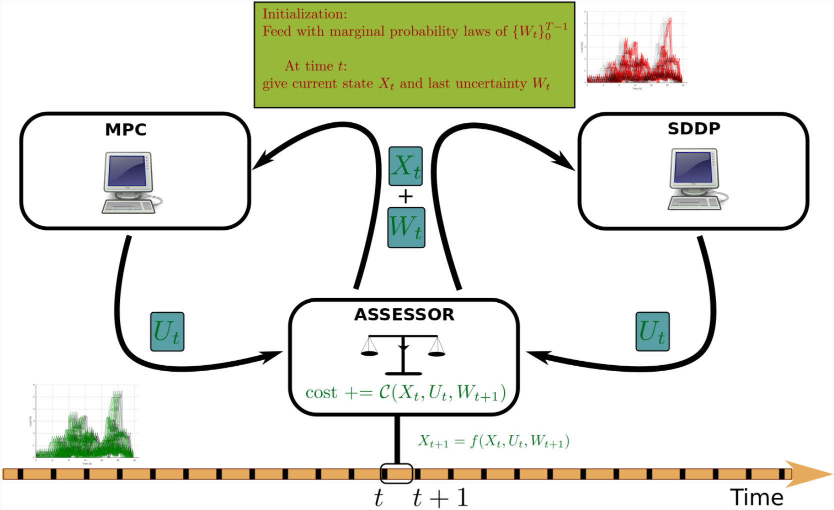


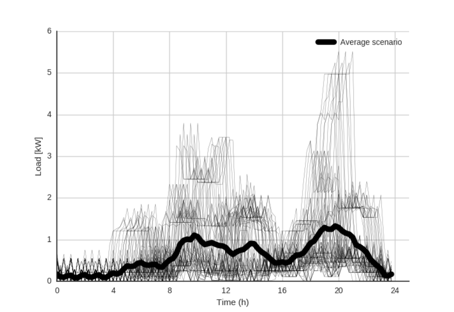
We compare SDDP and MPC with assessment scenarios



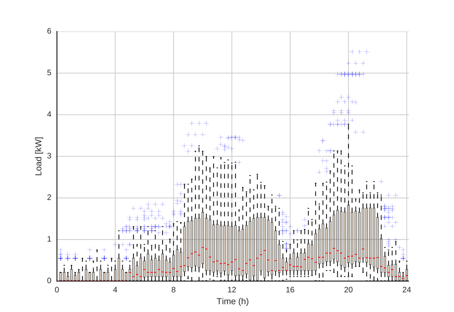
MPC vs SDDP: information structure

The two algorithms use optimization scenarios to model the perturbations:

MPC



SDDP



MPC vs SDDP: online resolution

At the beginning of time period $[\tau, \tau+1]$, do

MPC

- Consider a **rolling horizon** $[\tau, \tau + H[$
- Consider a **deterministic scenario** of demands (forecast) $(\overline{W}_{\tau+1}, \ldots, \overline{W}_{\tau+H})$
- Solve the **deterministic optimization** problem

$$\min_{X,U} \left[\sum_{t=\tau}^{\tau+H} C(X_t, U_t, \overline{W}_{t+1}) + K(X_{\tau+H}) \right]$$

$$\begin{aligned} s.t. \quad X_\cdot &= (X_\tau, \ldots, X_{\tau+H}) \\ U_\cdot &= (U_\tau, \ldots, U_{\tau+H-1}) \\ X_{t+1} &= f(X_t, U_t, \overline{W}_{t+1}) \\ x^\flat &\leq x_t^k \leq x^\sharp \\ u^\flat &\leq u_t \leq u^\sharp \end{aligned}$$

- Get optimal solution $(U_\tau, \ldots, U_{\tau+H})$ over horizon $H = 24h$
- Use only control U_τ, and iterate at time $\tau + 1$

SDDP

- We consider the approximated value functions $\left(\widetilde{V}_t\right)_0^{T_f}$

$$\underbrace{\widetilde{V}_t}_{\text{Piecewise affine functions}} \leq V_t$$

- At time τ, we solve

$$U_\tau^\# = \arg\min_{u_\tau} \mathbb{E}_{W_\tau} \Big[\mathcal{C}_\tau(X_\tau, u_\tau, w_\tau) + \widetilde{V}_{\tau+1}\Big(f_\tau(X_\tau, u_\tau, w_\tau)\Big) \Big]$$

$\Rightarrow$ this problem resumes to solve a LP at each timestep

- Send $U_\tau^\#$ to assessor

A brief recall on Dynamic Programming

Dynamic Programming

μ_t is the probability law of W_t and is being used to estimate expectation and compute **offline** value functions with the backward equation:

$$V_t(x_t) = \min_{U_t} \mathbb{E}_{\mu_t}\Big[\underbrace{\mathcal{C}_t(x_t, U_t, W_{t+1})}_{\text{current cost}} + \underbrace{V_{t+1}\Big(f(x_t, U_t, W_{t+1})\Big)}_{\text{future costs}}\Big]$$

$$V_T(x) = K(x)$$

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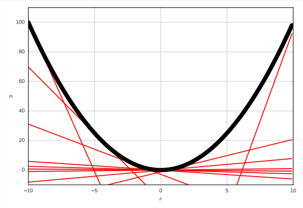
Stochastic Dual Dynamic Programming



- Convex value functions V_t are approximated as a supremum of a finite set of affine functions
- Affine functions (=cuts) are computed during forward/backward passes, till convergence
- SDDP makes an extensive use of LP solver

$$\widetilde{V}_t(x) = \max_{1 \leq k \leq K} \{\lambda_t^k x + \beta_t^k\} \leq V_t(x)$$

Outline

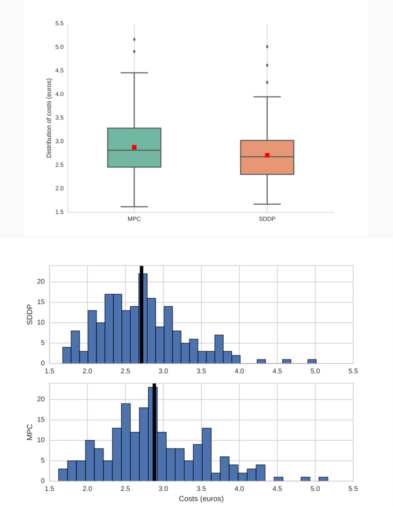
Our stack is deeply rooted in Julia language

- Modeling Language: `JuMP`
- Open-source SDDP Solver: `StochDynamicProgramming.jl`
- LP Solver: `CPLEX 12.5`

https://github.com/JuliaOpt/StochDynamicProgramming.jl

Comparison of MPC and SDDP

We compare MPC and SDDP during one day in summer over 200 assessment scenarios:



	euros/day
MPC	2.882
SDDP	2.713

SDDP is in average *6.9 %* better than MPC!

Operational costs obtained in simulation

We compare different configurations, during summer and winter:

Summer

Local Grid	Elec. bill euros/day	Self cons. %
No	3.53	48.1 %
Yes	2.71	55.2 %

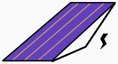
Winter

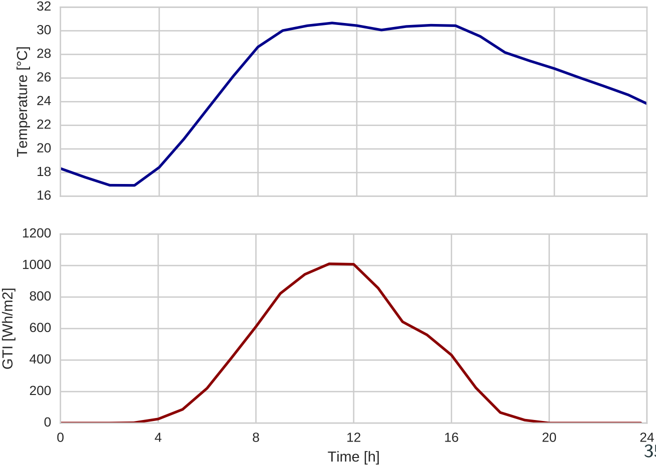
Local Grid	Elec. bill euros/day	Self cons. %
No	54.2	1.7 %
Yes	id.	id.

INPUT

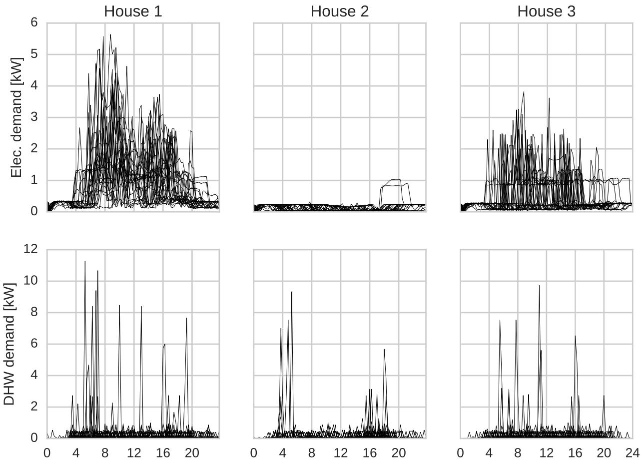
We work with real data

We consider one day during summer 2015 (data from Meteo France):





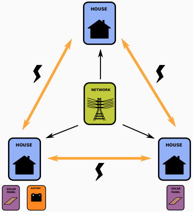
We have 200 scenarios of demands during this day

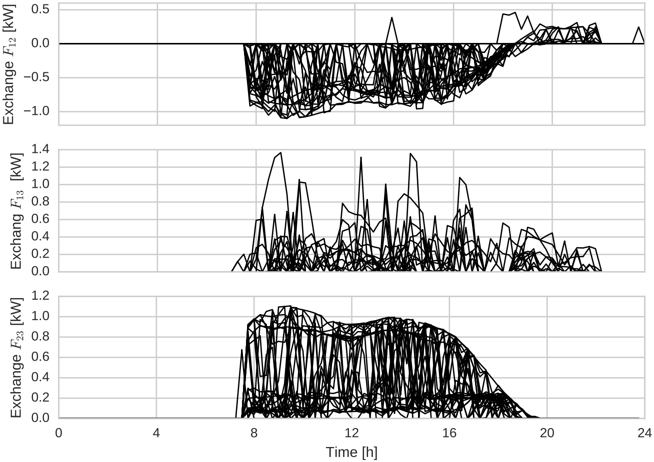


These scenarios are generated with `StRoBE`, a generator open-sourced by KU-Leuven

OUTPUT

As we gain solar energy, surplus is traded in local grid



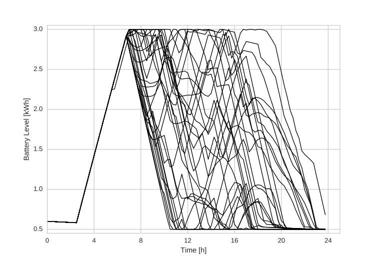


The battery is used as a global storage inside the local grid...

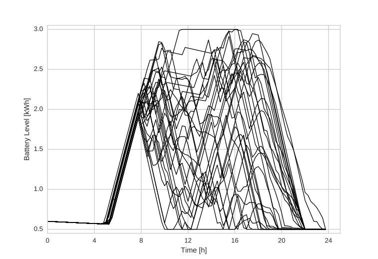
We observe that:

- the battery is more widely used
- the saturation level is reached more often (it could pay to have a bigger battery)

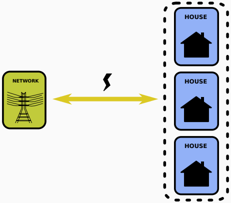
No local grid



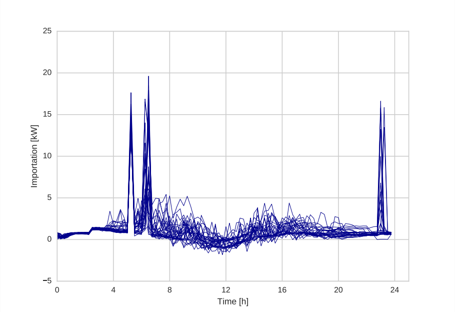
Local grid

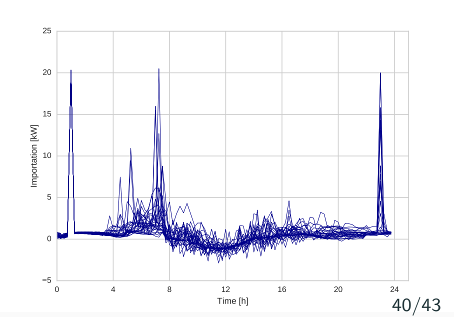


… and we minimize our average importation from the network



No local grid = 25.8 kWh	Local grid = 19.4 kWh





Outline

Conclusion

- We extend the results obtained with a single house to a small district
- This study can help to perform an economic analysis
- It pays to use stochastic optimization: SDDP is better than MPC
- We obtain promising results with SDDP, now we want to scale!

Perspectives

Mix SDDP with spatial decomposition like *Dual Approximate Dynamic Programming* (DADP) to control bigger urban neighbourhood