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Comparison of MPC and SDDP to manage a domestic microgrid.

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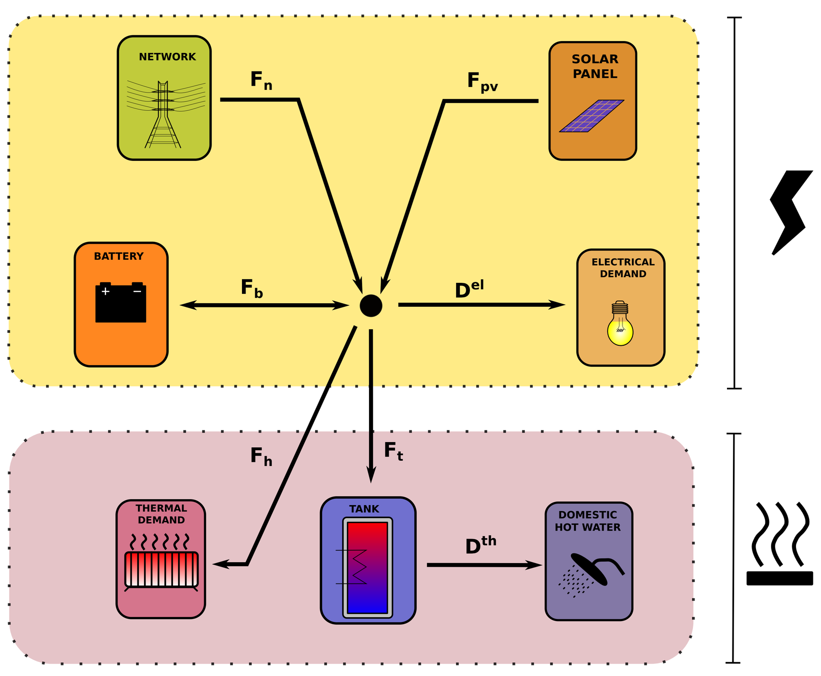
Abstract

Uncertainty are a common challenge in optimization: the fact that we cannot predict effectively the future limits the performance of optimization algorithms. In this presentation, we show a comparison of two algorithms to manage a small urban district while considering uncertainties. The first, MPC, tackles uncertainties with deterministic forecast and solve convex optimization problems to compute decisions for all time t. The contender, Stochastic Dual Dynamic Programming (SDDP), computes a lower-approximation of Bellman's value functions and solves at each time t an optimization problem that considers effectively the uncertainty. We present a numerical comparison of these two algorithms, and show that stochastic optimization algorithms may be effective to manage systems confronted to uncertainty.

Introduction

We consider the following domestic microgrid.



We formulate a discrete time optimal control problem, with a horizon $T = 24h$. We take decisions every $\Delta T = 15mn$, thus giving 96 time-steps. We consider an optimal control formulation with:

- **Stock variables** $\boldsymbol{X}_t = (\mathbf{B}_t, \mathbf{H}_t, \boldsymbol{\theta}^i_t, \boldsymbol{\theta}^w_t)$
  - $\mathbf{B}_t$, battery level (kWh)
  - $\mathbf{H}_t$, hot water storage (kWh)
  - $\boldsymbol{\theta}^i_t$, inner temperature ($°C$)
  - $\boldsymbol{\theta}^w_t$, wall's temperature ($°C$)
- **Control variables** $\boldsymbol{U}_t = (\mathbf{F}^+_{B,t}, \mathbf{F}^-_{B,t}, \mathbf{F}_{H,t}, \mathbf{F}_{T,t})$
  - $\mathbf{F}^+_{B,t}$, energy stored in the battery
  - $\mathbf{F}^-_{B,t}$, energy taken from the battery
  - $\mathbf{F}_{H,t}$, energy used to heat the hot water tank
  - $\mathbf{F}_{T,t}$, thermal heating
- **Uncertainties** $\boldsymbol{W}_t = (\boldsymbol{\Phi}^s_t)$
  - $\boldsymbol{\Phi}_t$, energy produced by the solar panel (kW)

We suppose that the electrical demand $\boldsymbol{D}^E_t$ and domestic hot water demand $\boldsymbol{D}^{DHW}_t$ are *deterministic*. However, we cannot predict in advance the random variable $\boldsymbol{\Phi}^s_t$!

We have two objectives:

1. **Minimize electricity bill**
2. **Maintain a comfortable temperature inside the house**

As we obtain a problem with random variables, we formulate a multistage stochastic programming problem

1 Physical modeling

We model the dynamics of battery $\mathbf{B}$ and hot water tank $\mathbf{H}$ with a linear model, and the dynamic of the thermal envelope with an electrical analogy (R6C2 model). That gives:

$$
\begin{aligned}
\mathbf{B}_{t+1} &= \alpha_B \mathbf{B}_t + \Delta T\Big(\rho_c \mathbf{F}^+_{B,t} - \frac{1}{\rho_d}\mathbf{F}^-_{B,t}\Big) \\
\mathbf{H}_{t+1} &= \alpha_T \mathbf{H}_t + \Delta T\big[\mathbf{F}_{H,t} - \boldsymbol{D}^{DHW}_t\big] \\
\boldsymbol{\theta}^w_{t+1} &= \boldsymbol{\theta}^w_t + \frac{\Delta T}{c_m}\left[\frac{\boldsymbol{\theta}^i_t - \boldsymbol{\theta}^w_t}{R_i + R_s} + \frac{\boldsymbol{\theta}^e_t - \boldsymbol{\theta}^w_t}{R_m + R_e} + \gamma \mathbf{F}_{T,t} + \frac{R_i}{R_i + R_s} P^{int}_t + \frac{R_e}{R_e + R_m}\boldsymbol{\Phi}^s_t\right] \\
\boldsymbol{\theta}^i_{t+1} &= \boldsymbol{\theta}^i_t + \frac{\Delta T}{c_i}\left[\frac{\boldsymbol{\theta}^w_t - \boldsymbol{\theta}^i_t}{R_i + R_s} + \frac{\boldsymbol{\theta}^e_t - \boldsymbol{\theta}^i_t}{R_v} + \frac{\boldsymbol{\theta}^e_t - \boldsymbol{\theta}^i_t}{R_f} + (1-\gamma)\mathbf{F}_{T,t} + \frac{R_s}{R_i + R_s} P^{int}_t\right]
\end{aligned}
$$

which write as a discrete state equation:

$$\boxed{\boldsymbol{X}_{t+1} = f_t(\boldsymbol{X}_t, \boldsymbol{U}_t, \boldsymbol{W}_{t+1})}$$

2 Statistical modeling

We now describe the statistical modeling of the random uncertainty $\boldsymbol{\Phi}_t$. We have available at midnight a forecast $\hat{\Phi}$, with error bounds $(\boldsymbol{\varepsilon}_0, \dots, \boldsymbol{\varepsilon}_T)$. The realization of $\boldsymbol{\Phi}_t$ is equal to

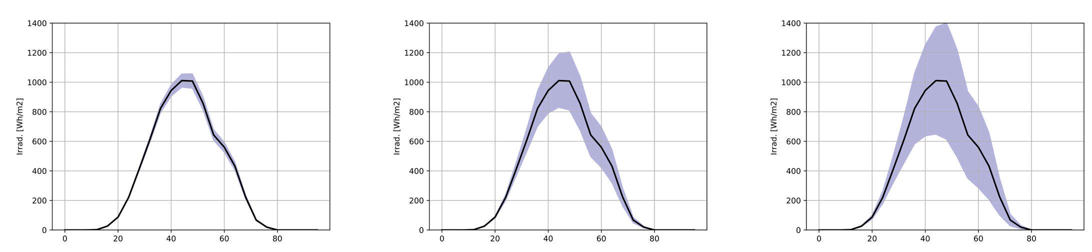
$$\boldsymbol{\Phi}_t = \hat{\Phi}_t \times (1 + \boldsymbol{\varepsilon}_t) \ ,$$

where for all t, ε_t is Gaussian:

$$\boldsymbol{\varepsilon}_t \sim \mathcal{N}(0, \sigma_t) \ .$$

We suppose that the standard-deviation increases linearly over time

$$\sigma_t = \sigma_0 + (\sigma_T - \sigma_0)\frac{t}{T}$$



$\sigma_T = 5$ % $\qquad$ $\sigma_T = 20$ % $\qquad$ $\sigma_T = 40$%

3 Optimization problem

We penalize

- the cost to import the power $\mathbf{F}_{NE,t+1}$ from the network

$$\underbrace{\mathbf{F}_{NE,t+1}}_{Network} = \underbrace{\boldsymbol{D}^E_{t+1}}_{Demand} + \underbrace{\mathbf{F}^+_{B,t} - \mathbf{F}^-_{B,t}}_{Battery} + \underbrace{\mathbf{F}_{T,t}}_{Heating} + \underbrace{\mathbf{F}_{A,t}}_{Tank} - \underbrace{\boldsymbol{\Phi}_t}_{Solar\ panel}$$

- the virtual cost of thermal discomfort: $\kappa_{th}(\underbrace{\boldsymbol{\theta}^i_t - \bar{\boldsymbol{\theta}}^i_t}_{\text{deviation from setpoint}})$

The instantaneous convex costs write

$$L_t(\boldsymbol{X}_t, \boldsymbol{U}_t, \boldsymbol{W}_{t+1}) = \underbrace{-b^E_t \max\{0, -\mathbf{F}_{NE,t+1}\}}_{buying} + \underbrace{\pi^E_t \max\{0, \mathbf{F}_{NE,t+1}\}}_{selling} + \underbrace{\kappa_{th}(\boldsymbol{\theta}^i_t - \bar{\boldsymbol{\theta}}^i_t)}_{discomfort}$$

which gives the following stochastic optimization problem

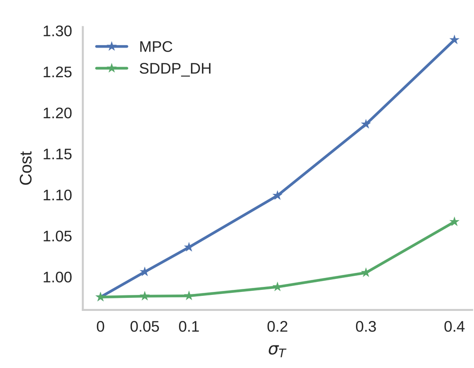
$$
\begin{aligned}
\min_{\boldsymbol{X},\boldsymbol{U}} \ & \mathbb{E}\left[\sum_{t=0}^{T-1} \underbrace{L_t(\boldsymbol{X}_t, \boldsymbol{U}_t, \boldsymbol{W}_{t+1})}_{instantaneous\ cost} + \underbrace{K(X_T)}_{final\ cost}\right] \\
s.t \ \ & \boldsymbol{X}_{t+1} = f_t(\boldsymbol{X}_t, \boldsymbol{U}_t, \boldsymbol{W}_{t+1}) \ \text{Dynamic} \\
& X^\flat \leq \boldsymbol{X}_t \leq X^\sharp \\
& U^\flat \leq \boldsymbol{U}_t \leq U^\sharp \\
& \boldsymbol{X}_0 = x_{ini} \\
& \sigma(\boldsymbol{U}_t) \subset \sigma(\boldsymbol{W}_1, \dots, \boldsymbol{W}_t) \ \text{Non-anticipativity}
\end{aligned}
$$

4 Numerical results

We compare two algorithms, namely

- Model Predictive Control (MPC)
- Stochastic Dual Dynamic Programming (SDDP)

We manage the microgrid during a particular day in summer, and we compare the costs obtained by MPC and SDDP in average. We compare different level of uncertainties, corresponding to different final standard-deviation σ_T. For each σ_T, we draw S scenarios $w^s = (\phi^s_0, \cdots, \phi^s_T)$ and assess the two algorithms upon the same set of scenarios.



σ_T	SDDP	MPC
5 %	0.984	1.006
10 %	0.984	1.038
20 %	1.034	1.104
30 %	1.077	1.187
40 %	1.202	1.296

Conclusion

- It pays to use stochastic optimization!
- How to extend these results to bigger microgrids (curse of dimensionality)?

References

[1] Bertsekas, D. P., Dynamic Programming and Optimal Control, Athena Scientific, v.3, 2000.

[2] Carpentier, P. and Cohen, G. and Chancelier, J. and De Lara, M., Stochastic Multi-Stage Optimization, 2015.