Optimal Control of a Microgrid with Combined Heat and Power Generator

How to manage optimally a battery and a hot water tank?

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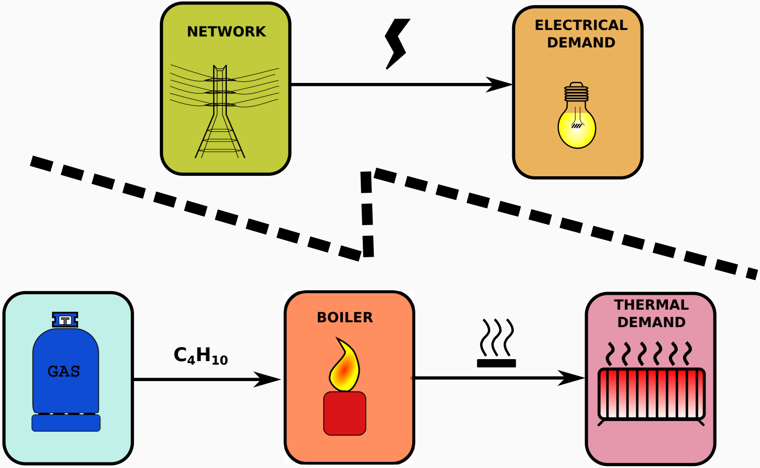
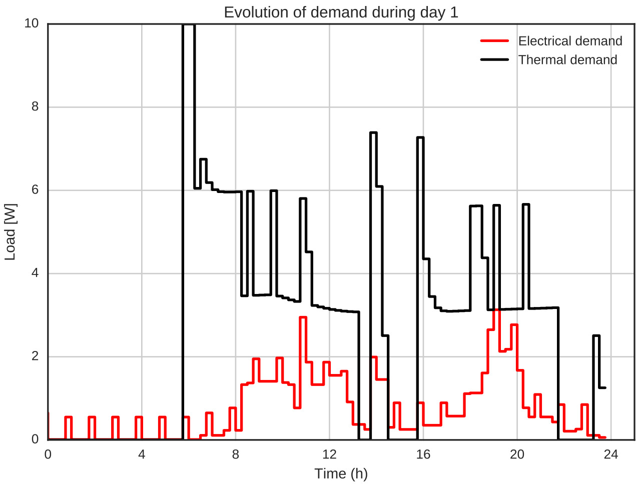
A partnership between mathematicians and thermicians

- Efficacity is a research institute for energy transition — an original mix of companies and academic researchers
- This presentation sums up a common work between Cermics and Efficacity
- This cooperation aims to apply optimization algorithms to real world problems

In a “classical” energy system, thermal and electrical energy management are usually treated apart



As electrical and thermal consumptions are correlated, we envisage a coupled management



Outline

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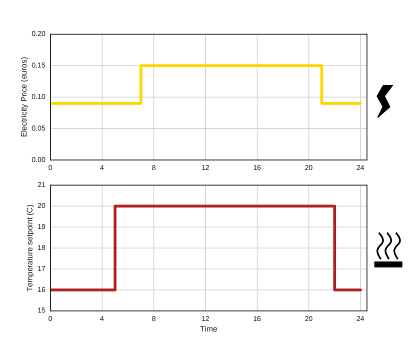
Is it worth to equip the system with
a combined heat and power generator (CHP)
together **with** or **without** a battery?

Challenge: CHP is either ON or OFF, and always produces
the same amount of electricity and heat

We turn to mathematical optimization to answer the question

Here are settings for the problem

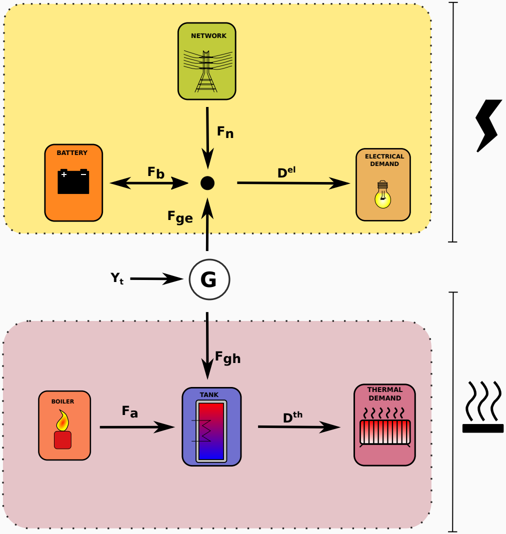
We optimize during $T_f = 24\text{h}$, with decisions taken every $\Delta T = 15\text{mn}$



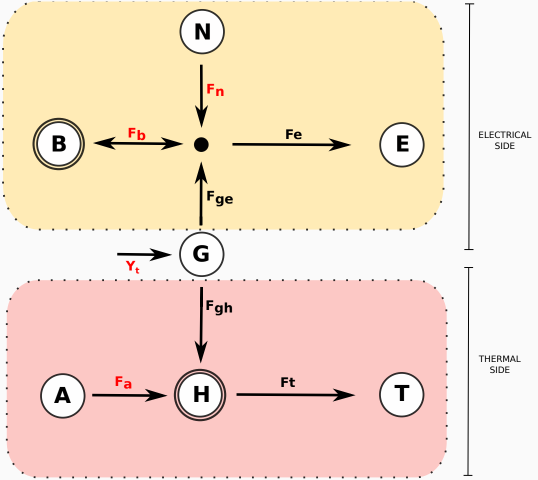
Electricity peak and off-peak hours

- $\pi_{elec} = 0.09$ or 0.15 euros/kWh
- $\pi_{gas} = 0.06$ euros/kWh

Schematic representation of a house equipped with a CHP



We sketch a graph



- B: Battery
- H: Tank
- A: Auxiliary Heater
- N: Electrical Network
- G: μ-CHP
- E: Electrical Demand
- T: Thermal Demand

We lay out an optimization model

- Optimization model captures a simplified dynamic of the physical system

- Battery and tank dynamics are modelled with simplified dynamic

$$\textbf{next state} = \textbf{previous state} + \textbf{balance flow}$$

We introduce state, control and slack variables

- Time:
  discrete periods $t \in \{0, 1, 2, ..., T_f\}$, corresponding to timestep Δt
- Stock variables:
  - B_t for the battery level
  - H_t for the hot water storage
- Control variables:
  - the boolean ON/OFF CHP generator control variable $Y_t \in \{0, 1\}$, yielding the electrical and thermal flows
    $F_{GH,t} = Y_t \times power^T$, $F_{GE,t} = Y_t \times power^E$
  - all the energy flows $F_t = (F_{B,t}, F_{A,t})$, positive or negative
  - all flows are constrained: $-F_B^\sharp \leq F_B \leq F_B^\sharp$, $0 \leq F_A \leq F_A^\sharp$
- Slack variables:
  we add two slack variables to ensure that the equality
  Production=Demand
  holds true at all times

$$\widetilde{F}_{t+1} = (\widetilde{F}_{NE,t+1}, \widetilde{F}_{H,t+1})$$

Electrical equations

- Dynamic constraints

$$B_{t+1} = \alpha_B B_t - \beta_B F_{B,t}$$

- Capacity constraints

$$B^\flat \leq B_t \leq B^\sharp$$

- Max charge/discharge constraint

$$\begin{aligned} & \Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\ \iff & \frac{\Delta B^\flat + (1-\alpha_B)B_t}{\beta_B} \leq F_{B,t} \leq \frac{\Delta B^\sharp + (1-\alpha_B)B_t}{\beta_B} \end{aligned}$$

- Offer=Demand

$$\underbrace{F_{GE,t}}_{CHP} + \underbrace{F_{B,t}}_{Battery} + \underbrace{\widetilde{F}_{NE,t+1}}_{Network} = \underbrace{D^E_{t+1}}_{Demand}$$

Thermal and CHP generator equations

- Dynamic constraints

$$H_{t+1} = \alpha_H H_t - \beta_H F_{H,t}$$

- Capacity constraints

$$H^\flat \leq H_t \leq H^\sharp$$

- Flows balance in thermal tank:

$$\underbrace{F_{H,t}}_{Tank} = \underbrace{F_{A,t}}_{Burner} + \underbrace{F_{GH,t}}_{CHP} - \underbrace{D^T_{t+1}}_{Demand} + \underbrace{\widetilde{F}_{H,t+1}}_{Comfort}$$

where $\widetilde{F}_{H,t+1}$ is a recourse variable added to ensure that the energy stored in the tank remains bounded whatever the demand.

Optimization criterion

- Two instantaneous linear costs:
  - Using gas for auxiliary burner: $\theta \pi_{gas} F_{A,t}$
  - Using the CHP generator: $\pi_{gas} P_G Y_t$
- Two instantaneous convex costs:
  - selling/buying electricity from/to the network:

$$-\underbrace{b^E \max\{0, -\widetilde{F}_{NE,t+1}\}}_{\text{selling}} + \underbrace{h^E \max\{0, \widetilde{F}_{NE,t+1}\}}_{\text{buying}}$$

    convex because we assume that $b^E < h^E = \pi_{elec}$
  - Missing heat demand:

$$-\underbrace{b^T \max\{0, -\widetilde{F}_{H,t+1}\}}_{\text{uncomfort}} + \underbrace{h^T \max\{0, \widetilde{F}_{H,t+1}\}}_{\text{uncomfort}}$$

    convex because we assume that $b^T < h^T$

Instantaneous and final costs

- The instantaneous convex costs are

$$\begin{aligned} \mathcal{C}(Y_t, F_t, \widetilde{F}_{t+1}) = & \underbrace{\pi_{chp} Y_t}_{CHP} + \underbrace{\theta \pi_{gas} F_{A,t}}_{Burner} \\ & - b_E \max\{0, -\widetilde{F}_{NE,t+1}\} + h_E \max\{0, \widetilde{F}_{NE,t+1}\} \\ & - b_T \max\{0, -\widetilde{F}_{H,t+1}\} + h_T \max\{0, \widetilde{F}_{H,t+1}\} \end{aligned}$$

- We add a final linear cost

$$-\pi_H H_{T_f} - \pi_B B_{T_f}$$

to avoid empty stocks at the final horizon T_f
(in the sequel, we do not penalize empty stocks at midnight by taking $\pi_H = 0$ and $\pi_B = 0$)

We are now able to formulate an optimization problem

$$
\min_{Y_\cdot \in \{0,1\}, F_\cdot \geq 0} \sum_{t=0}^{T_f - 1} \underbrace{\mathcal{C}(Y_t, F_t, \widetilde{F}_{t+1})}_{instantaneous \;\; cost} \underbrace{- \pi_H H_{T_f} - \pi_B B_{T_f}}_{final \;\; cost}
$$

$$
\begin{aligned}
s.t. \quad Y_\cdot = \left(Y_0, \dots, Y_{T_f - 1}\right), \quad F_\cdot = \left(F_0, \dots, F_{T_f - 1}\right) \\
B^\flat \leq B_t \leq B^\sharp \\
H^\flat \leq H_t \leq H^\sharp \\
B_{t+1} = \alpha_B B_t - \beta_B F_{B,t} \\
\widetilde{F}_{NE,t+1} = D^E_{t+1} - F_{GE,t} - F_{B,t} \\
\Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\
H_{t+1} = \alpha_H H_t - \beta_H F_{H,t} \\
\widetilde{F}_{H,t+1} = D^T_{t+1} - F_{A,t} - F_{H,t} \\
-F^\sharp_B \leq F_{B,t} \leq F^\sharp_B \;, \;\; 0 \leq F_{A,t} \leq F^\sharp_A \\
F_{GH,t} = Y_t \times power^T \;, \;\; F_{GE,t} = Y_t \times power^E
\end{aligned}
$$

This problem is a MILP (Mixed Integer Linear Programming)

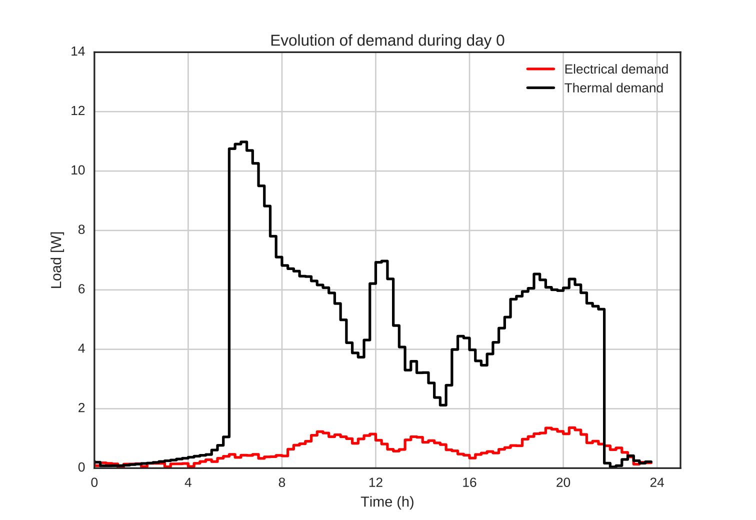
Where are we now? And where are we heading to?

- We have formulated a deterministic optimization problem, because the demands scenario $D^E(\cdot) = (D_1^E, \ldots, D_\tau^E)$,
  $D^T(\cdot) = (D_1^T, \ldots, D_\tau^T)$
  is part of the data of the above optimization problem,
  hence is supposed to be known in advance when the problem is set
- We are now going to provide numerical results
  within such deterministic setting

However, this is just a step towards
a stochastic formulation, to come later

Outline

We suppose given an average scenario for the demands



$$D^E(\cdot) = (D_1^E, \dots, D_\tau^E),\ D^T(\cdot) = (D_1^T, \dots, D_\tau^T)$$

We are going to compare different strategies (that may use the average scenario as input)

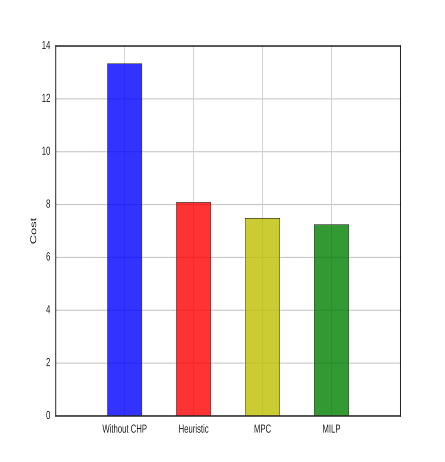
Without CHP: Import all electricity from network
and supply heat with auxiliary burner

Heuristic (CHP): Echelon based strategy:
when $H \approx H^\flat$, activate CHP till $H \approx H^\sharp$,
and, if this is not sufficient, activate auxiliary burner;
the battery is not used

MPC (CHP + Battery): Model Predictive Control with battery

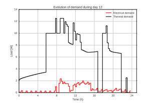
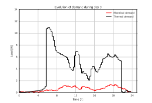
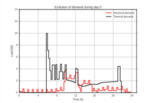
Optimal (CHP + Battery): Optimal control with battery
(MILP solved with CPLEX)

The optimal solution with battery beats the others



	euros/day
Without CHP	13.33
Heuristic	8.08
MPC	7.48
Optimal	7.24

Additional results with other scenarios of demand

Scenario	High demand	Average (previous)	Low demand
			
No CHP	19.98	13.33	7.36
Heuristics	12.30	8.08	5.63
MPC	11.34	7.48	5.02
Optimal	11.02	7.24	4.70

Algorithm running times

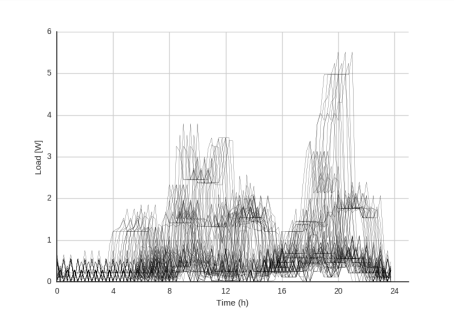
- The MILP is solved in around 10 minutes with CPLEX
- The boolean constraint over Y_t is relaxed in MPC, as it takes too long to solve a MILP at each timestep (sometimes more than 10mn, with a timestep of 15mn)
- That is why we got different results between the optimal solution and the MPC
- Without relaxation, the runtime of MPC at each iteration is too long to be used in production

Where are we now? And where are we heading to?

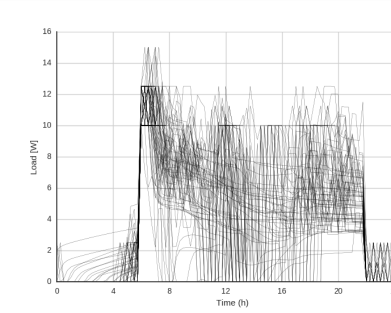
- It pays to control the system with CHP
- Compared with heuristic, the optimal strategy achieves between 5% and 10% cost reduction
- Beware: all these results were obtained supposing that the demand was known in advance...

Outline

The electrical and thermal demands are highly variable



Electrical demand



Thermal demand

We model demands as random variables

- We introduce a probabilistic setting,
  with $(\Omega, \mathcal{A}, \mathbb{P})$ a probability space
  - Ω is the sample space, or the scenario space
  - $\mathbb{P}$ is a probability
  - $\mathbb{E}$ is the mathematical expectation attached to the probability $\mathbb{P}$
- Then, we model demands as random variables

$$D_t^E : \Omega \to \mathbb{R}$$
$$D_t^T : \Omega \to \mathbb{R}$$

  so that $(D_1^E, D_1^T, \dots, D_{T_f}^E, D_{T_f}^T)$ forms a stochastic process
- Here, D_{t+1}^E and D_{t+1}^T stand for the demands
  during the time interval $[t, t+1[$

And we need to add the nonanticipativity constraints

- Filtration

$$\mathcal{A}_t = \sigma(D_1^E, D_1^T, \ldots, D_t^E, D_t^T)$$

- Control variables measurability

$$(Y_t, F_t) = (Y_t, F_{B,t}, F_{A,t})$$

is $\mathcal{A}_t$–measurable

- Recourse variables measurability

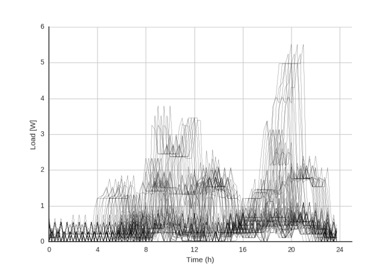
$$\widetilde{F}_{t+1} = (\widetilde{F}_{NE,t+1}, \widetilde{F}_{H,t+1}) \text{ is } \mathcal{A}_{t+1}\text{–measurable}$$

Stochasticity is contaminating and all variables turn into random variables!

$$
\begin{aligned}
\min_{Y_\cdot \in\{0,1\}, F_\cdot \geq 0} \; & \mathbb{E}\left[\sum_{t=0}^{T_f-1} \mathcal{C}(Y_t, F_t, \widetilde{F}_{t+1}) - \pi_H H_{T_f} - \pi_B B_{T_f}\right] \\
s.t. \quad & B^\flat \leq B_t \leq B^\sharp \\
& H^\flat \leq H_t \leq H^\sharp \\
& B_{t+1} = \alpha_B B_t - \beta_B F_{B,t} \\
& \widetilde{F}_{NE,t+1} = D^E_{t+1} - F_{GE,t} - F_{B,t} \\
& \Delta B^\flat \leq B_{t+1} - B_t \leq \Delta B^\sharp \\
& H_{t+1} = \alpha_H H_t - \beta_H F_{H,t} \\
& \widetilde{F}_{H,t+1} = D^T_{t+1} - F_{A,t} - F_{H,t} \\
& F_{GH,t} = Y_t \times power^T \\
& F_{GE,t} = Y_t \times power^E \\
& 0 \leq F_{A,t} \leq F^\sharp_A \;, \quad -F^\sharp_B \leq F_{B,t} \leq F^\sharp_B \\
& F_{A,t} \preceq \mathcal{A}_t \;, \quad F_{B,t} \preceq \mathcal{A}_t \\
& Y_t \preceq \mathcal{A}_t \\
& \widetilde{F}_{t+1} \preceq \mathcal{A}_{t+1}
\end{aligned}
$$

Outline

How to assess management strategies?



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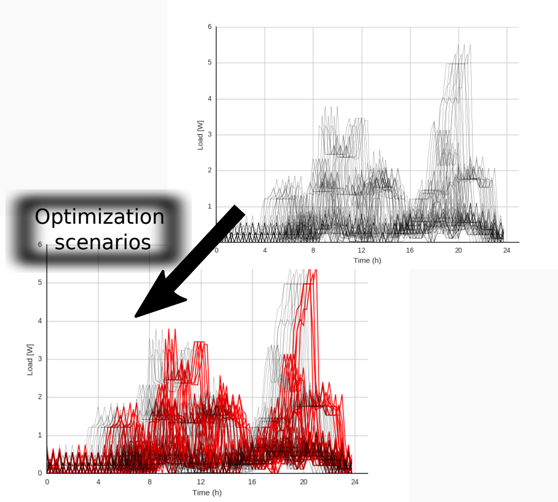

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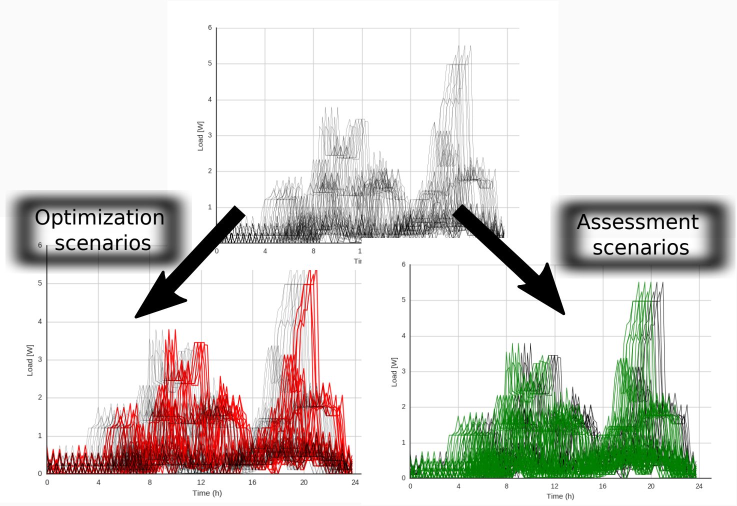


How to assess management strategies?

Online simulation

Use *assessment* scenarios

- to integrate the dynamics

$$B_{t+1} = f_b\big(B_t, Y_t^{opt}, F_{B,t}^{opt}\big)$$
$$H_{t+1} = f_h\big(H_t, Y_t^{opt}, F_{A,t}^{opt}, D_{t+1}^{th}\big)$$

  with the online controls Y_t^{opt}, F_t^{opt} provided by any strategy

- to evaluate the random costs
  and compute the mean costs attached to any strategy

We are going to compare methods to obtain strategies (online controls) (that use the the optimization scenarios as input)

Heuristic: Heuristic with CHP

MPC: Model Predictive Control (with battery)

DP: Stochastic dynamic programming (with battery)

SDDP: Stochastic dual dynamic programming (with battery)

Model Predictive Control

At the beginning of time period $[\tau, \tau + \Delta t]$, do

- Derive a deterministic scenario $(\overline{D}^{E}_{\tau}, \overline{D}^{T}_{\tau}, \ldots, \overline{D}^{E}_{T_f}, \overline{D}^{T}_{T_f})$ of demands (forecast) from the *optimization* scenarios
- Solve the deterministic optimization problem

$$
\begin{aligned}
\min_{Y. \in \{0,1\}, F. \geq 0} \quad & \sum_{t=\boldsymbol{\tau}}^{T_f - 1} \underbrace{\mathcal{C}(Y_t, F_t, \widetilde{F}_{t+1})}_{instantaneous\ cost} \underbrace{- \pi_H H_{T_f} - \pi_B B_{T_f}}_{final\ cost} \\
s.t. \quad & Y. = (Y_0, \ldots, Y_{T_f - 1}), \quad F. = (F_0, \ldots, F_{T_f - 1}) \\
& B^{\flat} \leq B_t \leq B^{\sharp} \\
& H^{\flat} \leq H_t \leq H^{\sharp} \\
& B_{t+1} = \alpha_B B_t - \beta_B F_{B,t} \\
& \widetilde{F}_{NE,t+1} = \overline{\mathbf{D}}^{\mathbf{E}}_{\mathbf{t+1}} - F_{GE,t} - F_{B,t} \\
& \Delta B^{\flat} \leq B_{t+1} - B_t \leq \Delta B^{\sharp} \\
& H_{t+1} = \alpha_H H_t - \beta_H F_{H,t} \\
& \widetilde{F}_{H,t+1} = \overline{\mathbf{D}}^{\mathbf{H}}_{\mathbf{t+1}} - F_{A,t} - F_{H,t} \\
& F_{i,t} \leq F_i^{\sharp}, \quad \forall i \in \{B, A\} \\
& F_{GH,t} = Y_t \times power^{T} \\
& F_{GE,t} = Y_t \times power^{E}
\end{aligned}
$$

- Apply only the first control F_{τ} at time τ, and iterate at time $\tau + \Delta t$

Stochastic dynamic programming: Backward offline computation of value functions

Offline computation

- Use *optimization* scenarios to derive marginal distributions, with expectation $\widehat{\mathbb{E}}$
- Use these marginal distributions, and $\widehat{\mathbb{E}}$, to compute so-called value functions, given by

$$V_t(B, H) = \min_{Y_t \in \{0,1\}, F_t \in F_t^{ad}} \widehat{\mathbb{E}}\Big[C_t(Y_t, F_t, \widetilde{F}_{t+1}) + V_{t+1}\big(B_{t+1}, H_{t+1}\big)\Big]$$

where $\big(B_{t+1}, H_{t+1}\big)$ is a shorthand for

$$B_{t+1} = f_b\big(B, Y_t, F_{B,t}\big)$$
$$H_{t+1} = f_h\big(H, Y_t, F_{A,t}, D_{t+1}^{th}\big)$$

Stochastic dynamic programming: Forward online computation of controls

Online computation of controls

Online controls Y_t, F_t are computed thanks to the value functions

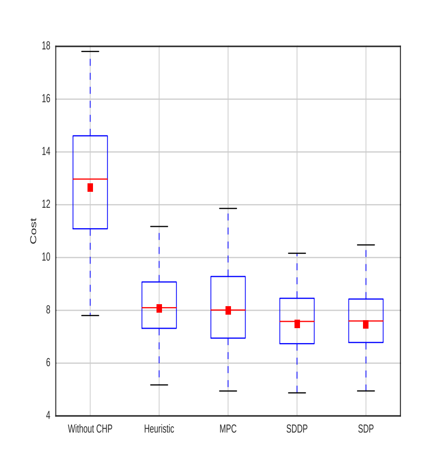
$$\big(Y_t^{opt}, F_t^{opt}\big) \in \mathop{\arg\min}_{Y_t \in \{0,1\}, F_t \in F_t^{ad}} \widehat{\widehat{\mathbb{E}}}\big[C_t(Y_t, F_t, \widetilde{F}_{t+1}) + V_{t+1}\big(B_{t+1}, H_{t+1}\big)\big]$$

where $\big(B_{t+1}, H_{t+1}\big)$ is a shorthand for

$$\begin{aligned} B_{t+1} &= f_b\big(B, Y_t, F_{B,t}\big) \\ H_{t+1} &= f_h\big(H, Y_t, F_{A,t}, D_{t+1}^{th}\big) \end{aligned}$$

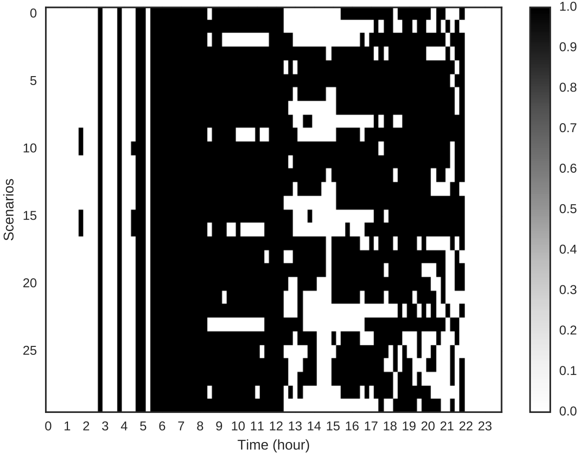
and where $\widehat{\widehat{\mathbb{E}}}$ is the expectation with respect to a possibly different marginal distribution (provided online to capture available new information)

Now, we assess the different strategies by their mean costs over assessment scenarios

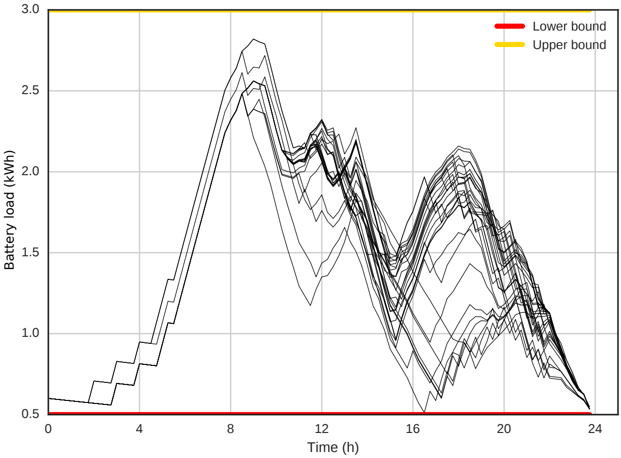


	euros/day
Without CHP	12.644
Heuristic with CHP	8.073
MPC with battery	7.993
SDDP with battery	7.488
DP with battery	7.475

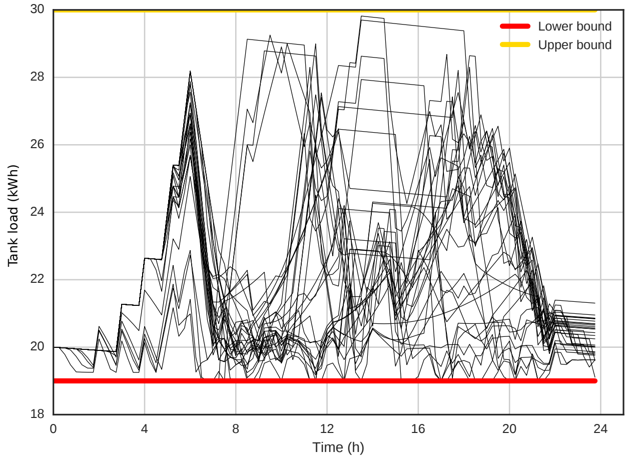
Behavior of the CHP over assessment scenarios



Behavior of the battery over assessment scenarios



Behavior of the thermal tank over assessment scenarios



Performance of the different algorithms

Which algorithm is the more efficient numerically?

MPC: Solved with CPLEX

DP: The time to interpolate future cost depends upon states and controls discretization
(Here discretization is small enough to ensure a good precision)

	Offline	Online (CPU Time)
MPC	0	2.4ms /timestep
DP	7mn30s	22.8ms /timestep

Hardware: Core i7-5500U @2.4GHz

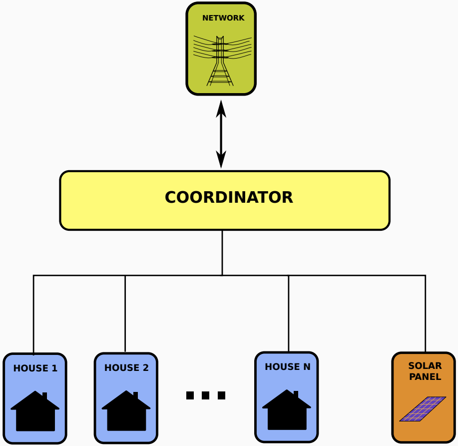
Outline

Conclusion

- Using marginal distributions for the uncertainties makes it possible to achieve at least 5% cost savings, with respect to deterministic based algorithms (MPC)
- The variability in demands can be handled, both in assessment and in optimization
- Implementation costs are low

Perspectives

Use decomposition/coordination algorithms to control an urban neighbourhood



Algorithms: SDDP, DADP