Scheduling for moving vehicles with guarantee
Proportional Fairness between users

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Short Bio

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Scheduling for moving vehicles with guarantee Proportional Fairness
1 Introduction

2 The Mathematical Model

3 Objective: Guarantee Proportional Fair and High Throughput

4 Some Existing Algorithms

5 Simulation
Current scheduling algorithms use current/past information for decision.

Connected vehicles technology will give access to future information (path prediction + Signal-to-noise ratio (SNR) maps → predict future rate)

Can data on future improve efficiency of algorithms?
1 Introduction

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5 Simulation
For each time the base station (BS) allows at most one vehicle to allocate data.
Objective: Guarantee Proportional Fair and High Throughput

\[
\text{maximize } C = \sum_{i=1}^{K} \log \left( \sum_{j=1}^{T} \alpha_{ij} r_{ij} \right)
\]

subject to \( \sum_{i=1}^{K} \alpha_{ij} = 1, \alpha_{ij} \in \{0, 1\} \)

- \( K \) is number of cars, \( T \) is number of time slots, \( \alpha_{i,j} \) is the allocation, \( r_{i,j} \) is the rate.
- \( \log \) objective function stands for Proportional fairness between users.

\( r_{i,j} \) is \textbf{given}, \( \alpha_{i,j} \) is \textbf{variable}. 
1. Introduction

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5. Simulation
Some Existing Algorithms

- **Greedy.** Current info only: choose the vehicle with best current rate.

- **PF-EXP**[3]. Current + past info: choose user with best
  \[
  \frac{\text{current rate}}{\text{total rate in the past}}
  \]

  Used in 3G network; optimal in some cases (not necessarily true for road traffic)

- **Discrete Gradient**[2]. Use current + past + future info: choose user with the best
  \[
  \frac{\text{current rate}}{\text{total rate in the past + current + estimated future rate}}
  \]
Relax the integer constraints.

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subject to \( \sum_{i=1}^{K} \alpha_{ij} = 1, \alpha_{ij} \in [0, 1] \)

Benchmark algorithm for comparison
Update rule

- Start with $\alpha^{(0)} \in D$ where $D$ is the feasible set.
- $\alpha^{(n+1)} = \Pi_D(\alpha^{(n)} + \epsilon_n \nabla C(\alpha^{(n)}))$, with $\epsilon_n \in (0, 1)$ is learning rate at step $n$.

→ This update rule can approach the global optimal.
In this below we compute the projection $\Pi_D(\alpha^{(n)} + \epsilon_n \nabla C(\alpha^{(n)}))$. 

Formula for projected gradient

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Lemma

Given $A, B$ two finite sets, $A$ is nonempty set. Then there exists a decomposition $B = B^1 \sqcup B^2$ such that $\text{mean}(A \cup B^1) \leq i$ for every $i \in B^1$ and $\text{mean}(A \cup B^1) > i$ for every $i \in B^2$. With convention that boolean on empty set is always true.

Ex: $A = \{5\}, B = \{1, 4, 6\}$ then $B^1 = \{6\}$ and $B^2 = \{1, 4\}$. 
Fix $\alpha^{(n)}$, denote $\nabla_{i,j} C = \partial C / \partial \alpha_{i,j}(\alpha^{(n)})$.

**Proposition**

$\alpha^{(n+1)}$ is given in the following formula: For each $j$, define

$A(j) = \{ \nabla_{i,j} C | \text{for } i \text{ s.t } \alpha^{(n)}_{i,j} > 0 \}$ and

$B(j) = \{ \nabla_{i,j} C | \text{for } i \text{ s.t } \alpha^{(n)}_{i,j} = 0 \}$.

$B^1(j), B^2(j)$ are defined as lemma 4.1 for two set $A(j), B(j)$ and $\bar{m} := \text{mean}(A(j) \cap B^1(j))$.

Then

$$
\alpha^{(n+1)}_{i,j} = \begin{cases} 
\alpha^{(n)}_{i,j} & \text{if } \alpha^{(n)}_{i,j} = 0 \text{ and } \nabla_{i,j} C \in B^2(j) \\
\alpha^{(n)}_{i,j} + \epsilon_n(\nabla_{i,j} C - \bar{m}) & \text{otherwise}.
\end{cases}
$$
Proposition

If $\alpha^* \in D$ and $\tilde{\nabla} C(\alpha^*) = 0$ then $\alpha^*$ is the optimal value of the relax problem, where

$$\tilde{\nabla} C(\alpha^*) = \begin{cases} 0 & \text{if } \alpha^*_{i,j} = 0 \text{ and } \nabla_{i,j} C(\alpha^*) \in B^2(j) \\ \nabla_{i,j} C(\alpha^*) - \bar{m} & \text{otherwise} \end{cases}$$

With this notation, $\alpha^{(n+1)} = \alpha^{(n)} + \epsilon_n \tilde{\nabla} C(\alpha^{(n)})$. So if we do algorithm until $\tilde{\nabla} C(\alpha^{(n)})$ converges to 0, it implies $\alpha^{(n)}$ converges to $\alpha^*$ at which $\tilde{\nabla} C(\alpha^*) = 0$, i.e, $\alpha^*$ is the optimal.

$\Rightarrow$ We do recursion until $\tilde{\nabla} C(\alpha^{(n)})$ converges to 0.
Simulations
Figure: Rate in coverage range of one Base Station
Figure: Rate in coverage range of one Base Station
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Thank you for your attention!
Any question?